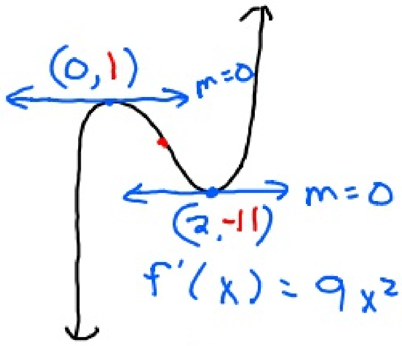
1-29-20 1st Trig

① Find the critical points on $f(x) = 3x^3 - 9x^2 + 1$



$f'(x) = 9x^2 - 18x$

$9x^2 - 18x = 0$

$9x(x-2) = 0$

$9x = 0$ $\qquad$ $x - 2 = 0$

$x = 0$ $\qquad$ $\underline{\;+2\;\;+2\;}$

$\qquad\qquad$ $x = 2$

Plug these x values into the Original equation $f(x) = 3x^3 - 9x^2 + 1$

$f(0) = 3 \cdot 0^3 - 9 \cdot 0^2 + 1 = 1 \quad (0,1)$ rel. max

$f(2) = 3 \cdot 2^3 - 9 \cdot 2^2 + 1 = -11 \quad (2,-11)$ rel. min.

Point of Inflection

Take the 2nd derivative

$f(x) = 3x^3 - 9x^2 + 1$

$f'(x) = 9x^2 - 18x$

$f''(x) = 18x - 18$

$18x - 18 = 0$

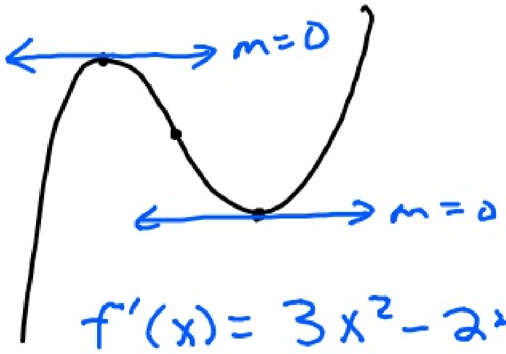
$x = 1$

Plug $x=1$ into original eq.

$f(1) = 3(1)^3 - 9 \cdot (1)^2 + 1 = -5$

$(1,-5) \rightarrow$ point of inflection

② $f(x) = x^3 - 12x^2 + 2$



$f'(x) = 3x^2 - 24x$

$3x^2 - 24x = 0$

$3x(x-8) = 0$

$3x = 0 \qquad x - 8 = 0$

$x = 0 \qquad x = 8$

$f(x) = x^3 - 12x^2 + 2$

$f(0) = 0^3 - 12 \cdot 0^2 + 2 = 2$ $(0,2)$

$f(8) = 8^3 - 12(8)^2 + 2 = -254$ $(8,-254$

Point of Inflection

$f(x) = x^3 - 12x^2 + 2$

$f'(x) = 3x^2 - 24x$

$f''(x) = 6x - 24$

$6x - 24 = 0$

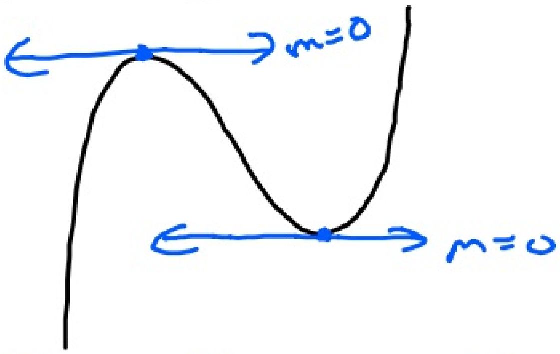
$x = 4$

$f(4) = 4^3 - 12(4)^2 + 2 = -126$

Point of Inflection is

$(4,-126)$

③ $f(x) = \frac{1}{3}x^3 + 4x^2 + 12x + 1$



$f'(x) = x^2 + 8x + 12$

$x^2 + 8x + 12 = 0$

$(x+6)(x+2) = 0$

$x+6=0 \qquad x+2=0$

$x=-6 \qquad x=-2$

$(-6, \underline{1}) \qquad (-2, \underline{-9\tfrac{2}{3}})$

$f(x) = \frac{1}{3}x^3 + 4x^2 + 12x + 1$

$f(-6) = \frac{1}{3}(-6)^3 + 4(-6)^2 + 12(-6) + 1 = 1$

$f(-2) = \frac{1}{3}(-2)^3 + 4(-2)^2 + 12(-2) + 1 = -9\tfrac{2}{3}$

Point of Inflection

$f''(x) = 2x + 8$

$2x + 8 = 0$

$x = -4$

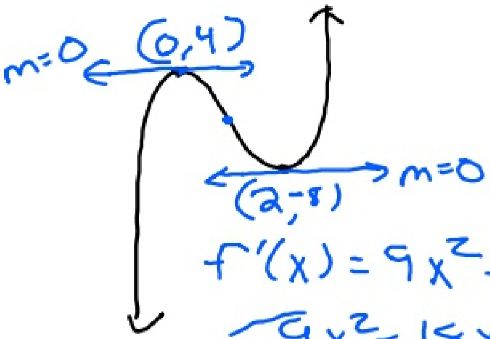
Plug $x=-4$ into original equation

$f(-4) = \frac{1}{3}(-4)^3 + 4(-4)^2 + 12(-4) + 1 = -4\tfrac{1}{3}$

$(-4, -4\tfrac{1}{3})$

1-29-20 3rd Trig

① Find the critical points on $f(x) = 3x^3 - 9x^2 + 4$



$f'(x) = 9x^2 - 18x$

$9x^2 - 18x = 0$

$9x(x-2) = 0$

$9x = 0$ $\quad$ $x - 2 = 0$

$x = 0$ $\quad$ $x = 2$

Plug $x=0$ and $x=2$ into the original equation to find the y values.

$f(x) = 3x^3 - 9x^2 + 4$

$f(0) = 3 \cdot 0^3 - 9 \cdot 0^2 + 4 = 4$ $\quad (0,4)$ rel. max

$f(2) = 3 \cdot 2^3 - 9 \cdot 2^2 + 4 = -8$ $\quad (2,-8)$ rel. min.

Point of Inflection

$f(x) = 3x^3 - 9x^2 + 4$

$f'(x) = 9x^2 - 18x$

$f''(x) = 18x - 18$

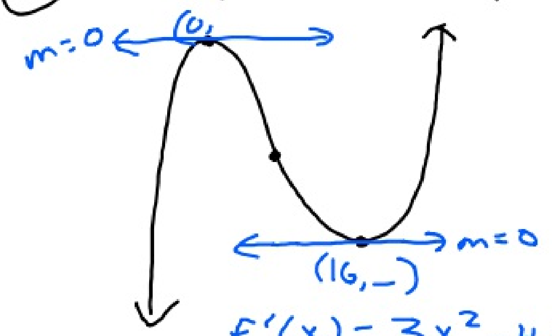
$18x - 18 = 0$

$x = 1$

$f(1) = 3 \cdot 1^3 - 9 \cdot 1^2 + 4 = -2$

Point of Inflection $\rightarrow (1,-2)$

② $f(x) = x^3 - 24x^2 + 2$



$f'(x) = 3x^2 - 48x$

$3x^2 - 48x = 0$

$3x(x-16) = 0$

$3x = 0$ $\qquad$ $x - 16 = 0$

$\qquad\qquad\qquad +16 \quad +16$

$x = 0$ $\qquad$ $x = 16$

$f(x) = x^3 - 24x^2 + 2$

$f(0) = 0^3 - 24 \cdot 0^2 + 2 = 2$ $\quad (0, 2)$

$f(16) = 16^3 - 24(16)^2 + 2 = -2046$ $\quad (16, -2046)$

$f(x) = x^3 - 24x^2 + 2$

$f'(x) = 3x^2 - 48x$

$f''(x) = 6x - 48$

$6x - 48 = 0$

$\quad +48 \; +48$

$6x = 48$

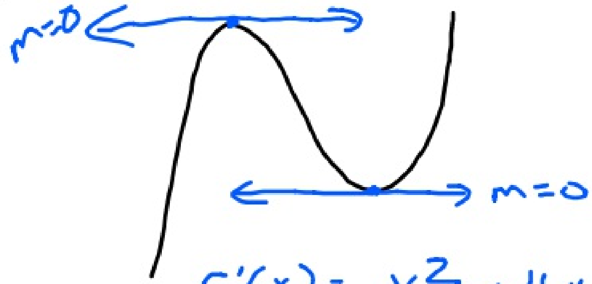
$x = 8$

Plug $x = 8$ into the original eq.

$f(8) = 8^3 - 24(8)^2 + 2 = -1022$

Point of Inflection $(8, -1022)$

③ $f(x) = \frac{1}{3}x^3 + 2x^2 + 3x + 3$



$$f'(x) = x^2 + 4x + 3$$

$$x^2 + 4x + 3 = 0$$

$$(x+1)(x+3) = 0$$

$x + 1 = 0$ $\quad\quad$ $x + 3 = 0$

$x = -1$ $\quad\quad$ $x = -3$

$$f(x) = \frac{1}{3}x^3 + 2x^2 + 3x + 3$$

$$f(-1) = \frac{1}{3}\cdot(-1)^3 + 2(-1)^2 + 3(-1) + 3 = 1\tfrac{2}{3}$$

$$f(-3) = \frac{1}{3}(-3)^3 + 2(-3)^2 + 3(-3) + 3 = 3$$

$(-1, 1\tfrac{2}{3})$ rel. min.

$(-3, 3)$ rel. max

Point of Inflection

$$f''(x) = 2x + 4$$

$$\frac{5}{3}$$

$$2x + 4 = 0$$

$$x = -2$$

$$f(-2) = \frac{1}{3}(-2)^3 + 2(-2)^2 + 3(-2) + 3 = 2\tfrac{1}{3}$$

Point of Inflection is $(-2, 2\tfrac{1}{3})$