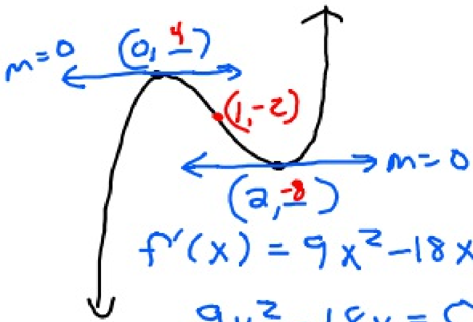
1-30-20 4th Trig

① Find the critical points on $f(x) = 3x^3 - 9x^2 + 4$



$$f'(x) = 9x^2 - 18x$$

$$9x^2 - 18x = 0$$

$$9x(x-2) = 0$$

$9x = 0$ $\qquad$ $x - 2 = 0$

$\qquad\qquad\quad$ $+2 \quad +2$

$x = 0$ $\qquad$ $x = 2$

Plug the x values back into the original equation.

$$f(x) = 3x^3 - 9x^2 + 4$$

$f(0) = 3(0)^3 - 9(0)^2 + 4 = 4$ $\quad$ rel. max (0,4)

$f(2) = 3(2)^3 - 9(2)^2 + 4 = -8$ $\quad$ rel. min. (2,-8)

Point of Inflection

$$f(x) = 3x^3 - 9x^2 + 4$$

$$f'(x) = 9x^2 - 18x$$

$$f''(x) = 18x - 18$$

$$18x - 18 = 0$$

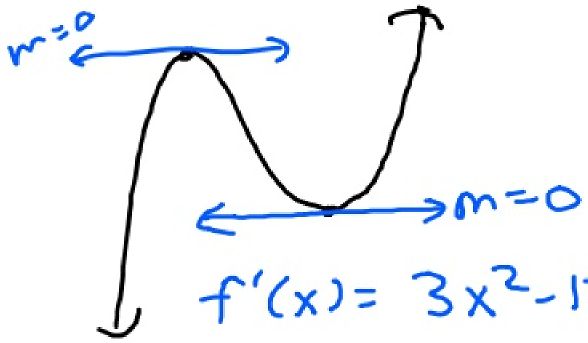
$$x = 1$$

original

$f(1) = 3(1)^3 - 9(1)^2 + 4 = -2$

Point of Inflection at (1, -2)

② $f(x) = x^3 - 6x^2 + 1$



$$f'(x) = 3x^2 - 12x$$

$$3x^2 - 12x = 0$$

$$3x(x-4) = 0$$

$$\frac{3x}{3} = \frac{0}{3} \qquad\qquad \begin{array}{r} x - 4 = 0 \\ +4 \ \ +4 \\ \hline x = 4 \end{array}$$

$$x = 0$$

$$f(x) = x^3 - 6x^2 + 1$$

$f(0) = 0^3 - 6 \cdot 0^2 + 1 = 1 \qquad (0, 1)$

$f(4) = 4^3 - 6 \cdot 4^2 + 1 = -31 \qquad (4, -31)$

$\quad 64 - 96 + 1$

Point of Inflection

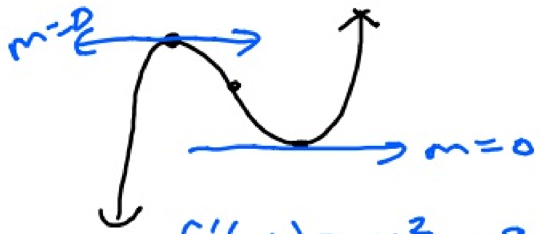
$$f'(x) = 3x^2 - 12x$$

$$f''(x) = 6x - 12$$

$$\begin{array}{r} 6x - 12 = 0 \\ +12 \ \ +12 \\ \hline 6x = 12 \\ x = 2 \end{array}$$

$f(2) = 2^3 - 6(2)^2 + 1 = -15 \qquad (2, -15)$

③ $f(x) = \frac{1}{3}x^3 + 4x^2 + 12x + 1$



$f'(x) = x^2 + 8x + 12$

$x^2 + 8x + 12 = 0$

$(x+2)(x+6) = 0$

$x + 2 = 0$ $\qquad$ $x + 6 = 0$

$x = -2$ $\qquad$ $x = -6$

$f(x) = \frac{1}{3}x^3 + 4x^2 + 12x + 1$

$f(-2) = \frac{1}{3}(-2)^3 + 4(-2)^2 + 12(-2) + 1 = -9.\overline{6}$

$-\frac{8}{3} + 16 - 24 + 1$

$(-2, -9\tfrac{2}{3})$

$f(-6) = \frac{1}{3}(-6)^3 + 4(-6)^2 + 12(-6) + 1 = 1$

$(-6, 1)$

Point of Inflection

$f(x) = \frac{1}{3}x^3 + 4x^2 + 12x + 1$

$f'(x) = x^2 + 8x + 12$

$f''(x) = 2x + 8$

$2x + 8 = 0$

$x = -4$

$f(-4) = \frac{1}{3}(-4)^3 + 4(-4)^2 + 12(-4) + 1 = -4\tfrac{1}{3}$

$(-4, -4\tfrac{1}{3})$