

UNIT 1

Foundations for Functions

Focus

Use symbols to express relationships and solve real-world problems.

CHAPTER 1

The Language and Tools of Algebra

BIG Idea Identify and use the arithmetic properties of subsets of integers and rational, irrational, and real numbers.

BIG Idea Use properties of numbers to demonstrate whether assertions are true or false and to construct simple, valid arguments (direct or indirect) for, or formulate counterexamples to claimed assertions.

CHAPTER 2

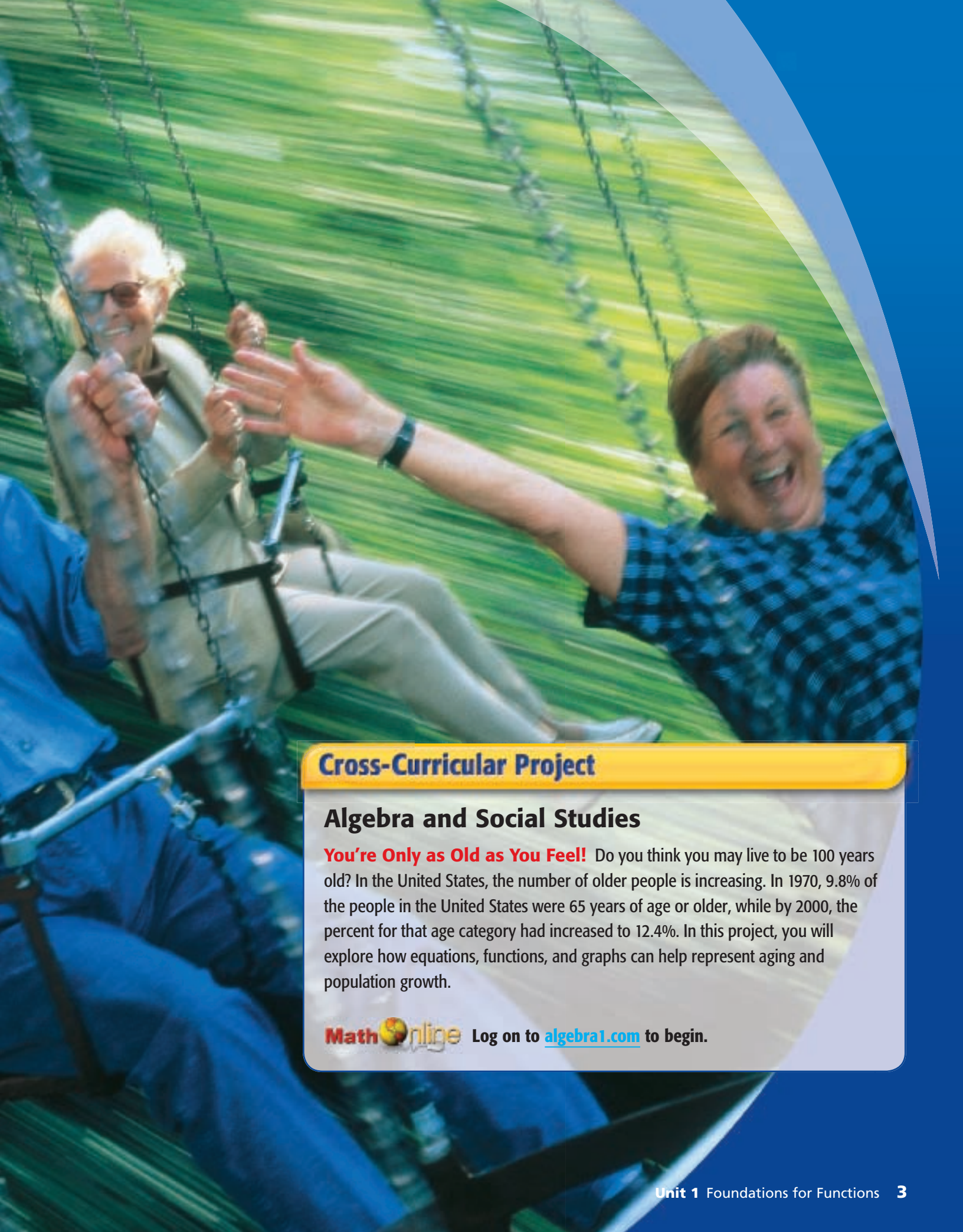
Solving Linear Equations

BIG Idea Simplify expressions before solving linear equations and inequalities in one variable.

BIG Idea Solve multistep problems, including word problems, involving linear equations and linear inequalities in one variable.

BIG Idea Apply algebraic techniques to solve rate problems, work problems, and percent mixture problems.





Cross-Curricular Project

Algebra and Social Studies

You're Only as Old as You Feel! Do you think you may live to be 100 years old? In the United States, the number of older people is increasing. In 1970, 9.8% of the people in the United States were 65 years of age or older, while by 2000, the percent for that age category had increased to 12.4%. In this project, you will explore how equations, functions, and graphs can help represent aging and population growth.

Math **Online** Log on to algebra1.com to begin.

The Language and Tools of Algebra

BIG Ideas

- Write algebraic expressions.
- Evaluate expressions and solve open sentences.
- Use algebraic properties of identity and equality.
- Use conditional statements and counterexamples.

Key Vocabulary

algebraic expression (p. 6)

coefficient (p. 29)

equation (p. 15)

function (p. 53)

Real-World Link

Architecture Architects can use algebraic expressions to describe the shapes of the structures they design. A few of the shapes these buildings can resemble are a rectangle, a triangle, or even a pyramid.

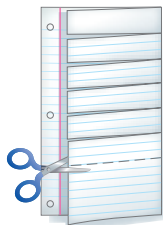
FOLDABLES™ Study Organizer

The Language and Tools of Algebra Make this Foldable to help you organize information about algebraic properties. Begin with a sheet of notebook paper.

- 1** Fold lengthwise to the holes.



- 2** Cut along the top line and then cut 10 tabs.



- 3** Label the tabs using the lesson numbers and concepts.

1-1	Variables and Equations
1-2	Order of Operations
1-3	Open Sentences
1-4	Identify and Equality Properties
1-5	Distributive Property
1-6	Commutative & Associative Property
1-7	Logic
1-8	Number Systems
1-9	Functions & Graphs

GET READY for Chapter 1

Diagnose Readiness You have two options for checking Prerequisite Skills.

Option 2



Take the Online Readiness Quiz at algebra1.com.

Option 1

Take the Quick Check below. Refer to the Quick Review for help.

QUICK Check

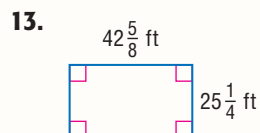
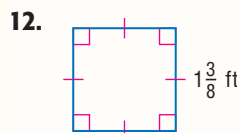
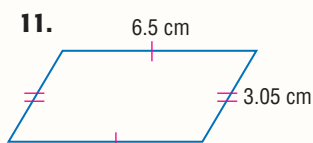
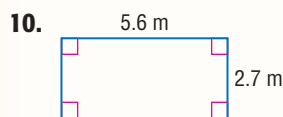
Write each fraction in simplest form. If the fraction is already in simplest form, write *simplest form*. (Prerequisite Skill)

1. $\frac{52}{13}$ 2. $\frac{6}{18}$ 3. $\frac{9}{15}$ 4. $\frac{13}{25}$
5. $\frac{26}{100}$ 6. $\frac{3}{81}$ 7. $\frac{17}{1}$ 8. $\frac{15}{75}$

9. **SURVEY** Thirty-three out of 198 students surveyed said that they preferred hockey to all other sports. What fraction of students surveyed is this?

Find the perimeter of each figure.

(Prerequisite Skill)



14. **HOMES** The dimensions of a rectangular backyard are 45 feet by 84 feet. What is its perimeter?

Find each product or quotient.

(Prerequisite Skill)

15. $6 \cdot 1.12$ 16. $0.5 \cdot 3.9$
17. $3.24 \div 1.8$ 18. $10.64 \div 1.4$
19. $\frac{3}{4} \cdot 12$ 20. $1\frac{2}{3} \cdot \frac{3}{4}$
21. $\frac{5}{16} \div \frac{9}{12}$ 22. $\frac{5}{6} \div \frac{2}{3}$

QUICK Review

EXAMPLE 1

Write $\frac{30}{36}$ in simplest form.

Find the greatest common factor (GCF) of 30 and 36.

factors of 30: 1, 2, 3, 5, **6**, 10, 15, 30

factors of 36: 1, 2, 3, 4, **6**, 9, 12, 18, 36

The GCF of 30 and 36 is 6.

$$\frac{30 \div 6}{36 \div 6} = \frac{5}{6} \quad \text{Divide the numerator and denominator by their GCF, 6.}$$

EXAMPLE 2

Find the perimeter of the figure.



$$P = 2\ell + 2w$$

$$P = 2(1.5) + 2(0.75) \quad \ell = 1.5 \text{ and } w = 0.75$$

$$P = 3 + 1.5 \text{ or } 4.5 \quad \text{Simplify.}$$

The perimeter is 4.5 centimeters.

EXAMPLE 3

Find $\frac{4}{5} \div \frac{12}{15}$.

$$\frac{4}{5} \div \frac{12}{15} = \frac{4}{5} \left(\frac{15}{12} \right) \quad \text{Multiply } \frac{4}{5} \text{ by } \frac{15}{12}, \text{ the reciprocal of } \frac{12}{15}.$$

$$= \frac{4(15)}{5(12)} \quad \text{Multiply the numerators and the denominators.}$$

$$= \frac{60}{60} \text{ or } 1 \quad \text{Simplify.}$$

Variables and Expressions

Main Ideas

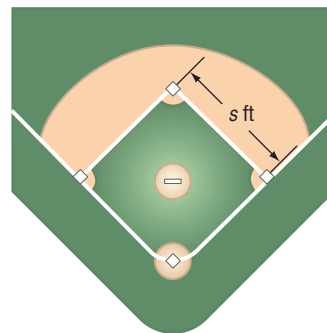
- Write mathematical expressions for verbal expressions.
- Write verbal expressions for mathematical expressions.

New Vocabulary

variables
 algebraic expression
 factors
 product
 power
 base
 exponent
 evaluate

GET READY for the Lesson

A baseball infield is a square with a base at each corner. Each base lies the same distance from the next one. Suppose s represents the length of each side. Since the infield is a square, you can use the expression 4 times s , or $4s$, to find the perimeter.



Write Mathematical Expressions In the algebraic expression $4s$, the letter s is called a variable. In algebra, **variables** are symbols used to represent unspecified numbers or values. Any letter may be used as a variable. *The letter s was used above because it is the first letter of the word side.*

An **algebraic expression** consists of one or more numbers and variables along with one or more arithmetic operations. Here are some examples of algebraic expressions.

$$5x \quad 3x - 7 \quad 4 + \frac{p}{q} \quad m \times 5n \quad 3ab \div 5cd$$

In algebraic expressions, a raised dot or parentheses are often used to indicate multiplication as the symbol \times can be easily mistaken for the letter x . Here are several ways to represent the product of x and y .

$$xy \quad x \cdot y \quad x(y) \quad (x)y \quad (x)(y)$$

In each expression, the quantities being multiplied are called **factors**, and the result is called the **product**.

An expression like x^n is raised is called a **power**. The variable x is called the **base**, and n is called the **exponent**. The word *power* can also refer to the exponent. The exponent indicates the number of times the base is used as a factor. The expression x^n is read " x to the n th power."

Reading Math

First Power

When no exponent is shown, it is understood to be 1. For example, $a = a^1$.

Symbols	Words	Meaning
3^1	3 to the first power	3
3^2	3 to the second power or 3 squared	$3 \cdot 3$
3^3	3 to the third power or 3 cubed	$3 \cdot 3 \cdot 3$
3^4	3 to the fourth power	$3 \cdot 3 \cdot 3 \cdot 3$
$2b^6$	2 times b to the sixth power	$2 \cdot b \cdot b \cdot b \cdot b \cdot b \cdot b$
x^n	x to the n th power	$\underbrace{x \cdot x \cdot x \cdot \dots \cdot x}_{n \text{ factors}}$

By definition, $x^0 = 1$ for any nonzero number x .

It is often necessary to translate verbal expressions into algebraic expressions.

EXAMPLE Write Algebraic Expressions

1 Write an algebraic expression for each verbal expression.

a. eight more than a number

The words *more than* suggest addition. Let n represent the number.

Thus, the algebraic expression is $n + 8$.

b. 7 less the product of 4 and a number x

Less implies subtract, and *product* implies multiply. So the expression can be written as $7 - 4x$.

c. one third of the original area a

The word *of* implies multiply. The expression can be written as $\frac{1}{3}a$ or $\frac{a}{3}$.

d. the product of 7 and m to the fifth power

$$7m^5$$

Reading Math

Subtraction

5 less x is $5 - x$.

5 less than x is $x - 5$.

CHECK Your Progress

1A. 13 less than a number

1C. three-fourths of the perimeter p

1B. 9 more than the quotient of b and 5

1D. n cubed divided by 2

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To **evaluate** an expression means to find its value.

EXAMPLE Evaluate Powers

2 Evaluate 2^6 .

$$\begin{aligned} 2^6 &= 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 && \text{Use 2 as a factor 6 times.} \\ &= 64 && \text{Multiply.} \end{aligned}$$

CHECK Your Progress

2. Evaluate 4^3 .

Study Tip

Multiple Translations

There may be more than one way to translate an algebraic expression into a verbal expression.

$4m^3 \rightarrow$
4 times m cubed

$c^2 + 21d \rightarrow$
 c squared plus the product of 21 and d

Write Verbal Expressions Another important skill is translating algebraic expressions into verbal expressions.

EXAMPLE Write Verbal Expressions

3 Write a verbal expression for each algebraic expression.

a. $4m^3$
4 times m to the third power

b. $c^2 + 21d$
the sum of c squared and 21 times d

CHECK Your Progress

3. Write a verbal expression for $x^4 - \frac{y}{9}$.



CHECK Your Understanding

Example 1
(p. 7)

Write an algebraic expression for each verbal expression.

1. the sum of a number and 14
2. 6 less a number t
3. 24 less than 3 times a number
4. 1 minus the quotient of r and 7
5. two-fifths of a number j squared
6. n cubed increased by 5
7. **MONEY** Lorenzo bought a bag of peanuts that cost p dollars, and he gave the cashier a \$20 bill. Write an expression for the amount of change that he will receive.

Example 2
(p. 7)

Evaluate each expression.

8. 9^2
9. 4^4

Example 3
(p. 7)

Write a verbal expression for each algebraic expression.

10. $2m$
11. $\frac{1}{2}n^3$
12. $a^2 - 18b$

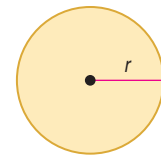
Exercises

HOMEWORK HELP	
For Exercises	See Examples
13–25	1
26–29	2
30–37	3

Write an algebraic expression for each verbal expression.

13. x more than 7
14. a number less 35
15. 5 times a number
16. one third of a number
17. f divided by 10
18. the quotient of 45 and r
19. 49 increased by twice a number
20. 18 decreased by 3 times d
21. k squared minus 11
22. 20 divided by t to the fifth power

23. **GEOMETRY** The area of a circle is the number π times the square of the radius. Write an expression that represents the area of a circle with a radius r .



RECYCLING For Exercises 24 and 25, use the following information.

Each person in the United States produces about 3.5 pounds of trash a day.

24. Write an expression to describe the pounds of trash produced per day by a family with m members.
25. Use the expression you wrote to predict the amount of trash produced by a family of four each day.

Evaluate each expression.

26. 8^2
27. 10^6
28. 3^5
29. 15^3

Write a verbal expression for each algebraic expression.

30. $7p$
31. $\frac{1}{8}y$
32. $15 + r$
33. $w - 24$
34. $3x^2$
35. $\frac{r^4}{9}$
36. $2a + 6$
37. $n^3 \cdot p^5$

Write a verbal expression for each algebraic expression.

38. $17 - 4m^5$
39. $\frac{12z^2}{5}$
40. $3x^2 - 2x$

EXTRA PRACTICE

See pages 714, 744.

Math online

Self-Check Quiz at algebra1.com

41. **SAVINGS** Kendra is saving to buy a new computer. Write an expression to represent the total amount of money she will have if she has s dollars saved and she adds d dollars per week for the next 12 weeks.

H.O.T. Problems

42. **MUSIC** Mario has 55 CDs. Write an expression to represent the total number of CDs he will have after 18 months if he buys x CDs per month.

43. **REASONING** Determine whether the product given by the expression $-3a$ is *always*, *sometimes*, or *never* a negative value. Explain.

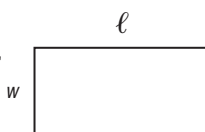
44. **CHALLENGE** In the square, x represents a positive whole number. Find the value of x such that the area and the perimeter of the square have the same value.



45. **Writing in Math** Use the data about baseball found on page 6 to explain how expressions can be used to find the perimeter of a baseball diamond. Include two different verbal expressions and an algebraic expression other than $4s$ to represent the perimeter of a square.

STANDARDIZED TEST PRACTICE

46. Which expression best represents the perimeter of the rectangle?

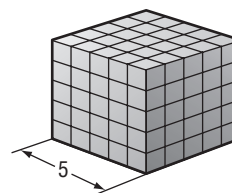


- A $2lw$ C $2l + 2w$
 B $l + w$ D $4(l + w)$

47. The yards of fabric needed to make curtains is 3 times the width of a window in inches, divided by 36. Which expression best represents the yards of fabric needed in terms of the width of the window w ?

- F $\frac{3 + w}{36}$ H $\frac{3}{36w}$
 G $\frac{3w}{36}$ J $3w(36)$

48. **REVIEW** Which expression best represents the volume of the cube?



- A the product of five and three
 B three to the fifth power
 C five squared
 D five cubed

GET READY for the Next Lesson

PREREQUISITE SKILL Evaluate each expression. (Pages 696–697)

49. $10 - 3.24$

50. 1.04×4.3

51. $15.36 \div 4.8$

52. $\frac{1}{3} + \frac{2}{5}$

53. $\frac{3}{8} \times \frac{4}{9}$

54. $\frac{7}{10} \div \frac{3}{5}$

Order of Operations

Main Ideas

- Evaluate numerical expressions by using the order of operations.
- Evaluate algebraic expressions by using the order of operations.

New Vocabulary

order of operations

GET READY for the Lesson

Nicole's Internet service provider charges \$4.95 a month, which includes 100 hours of access. If she is online more than 100 hours, she pays an additional \$0.99 per hour. Suppose Nicole is online 117 hours this month. The expression below represents what she must pay for the month.

$$4.95 + 0.99(117 - 100)$$



Evaluate Numerical Expressions Numerical expressions often contain more than one operation. A rule is needed to let you know which operation to perform first. This rule is called the **order of operations**.

KEY CONCEPT

Order of Operations

- Step 1** Evaluate the expressions inside grouping symbols.
- Step 2** Evaluate all powers.
- Step 3** Multiply and/or divide in order from left to right.
- Step 4** Add and/or subtract in order from left to right.

EXAMPLE Evaluate Expressions

1 Evaluate $15 \div 3 \cdot 6 - 4^2$.

$$\begin{aligned} 15 \div 3 \cdot 6 - 4^2 &= 15 \div 3 \cdot 6 - 16 && \text{Evaluate power.} \\ &= 5 \cdot 6 - 16 && \text{Divide 15 by 3.} \\ &= 30 - 16 && \text{Multiply 5 by 6.} \\ &= 14 && \text{Subtract 16 from 30.} \end{aligned}$$

CHECK Your Progress

Evaluate each expression.

1A. $8 - 6 \cdot 4 \div 3$

1B. $32 + 7^2 - 5 \cdot 2$

Grouping symbols such as parentheses (), brackets [], and braces { } are used to clarify or change the order of operations. They indicate that the expression within the grouping symbol is to be evaluated first. A fraction bar also acts as a grouping symbol. It indicates that the numerator and denominator should each be treated as a single value.

EXAMPLE Grouping Symbols

2 Evaluate each expression.

a. $2(5) + 3(4 + 3)$

$$\begin{aligned} 2(5) + 3(4 + 3) &= 2(5) + 3(7) && \text{Evaluate inside parentheses.} \\ &= 10 + 21 && \text{Multiply expressions left to right.} \\ &= 31 && \text{Add 10 and 21.} \end{aligned}$$

b. $2[5 + (30 \div 6)^2]$

$$\begin{aligned} 2[5 + (30 \div 6)^2] &= 2[5 + (5)^2] && \text{Evaluate innermost expression first.} \\ &= 2[5 + 25] && \text{Evaluate power.} \\ &= 2[30] && \text{Evaluate expression inside grouping symbols.} \\ &= 60 && \text{Multiply.} \end{aligned}$$

c. $\frac{6 + 4}{3^2 \cdot 4}$

$$\begin{aligned} \frac{6 + 4}{3^2 \cdot 4} &= \frac{10}{3^2 \cdot 4} && \text{Add 6 and 4 in the numerator.} \\ &= \frac{10}{9 \cdot 4} && \text{Evaluate the power in the denominator.} \\ &= \frac{10}{36} \text{ or } \frac{5}{18} && \text{Multiply 9 and 4 in the denominator. Then simplify.} \end{aligned}$$

Study Tip

Grouping Symbols

When more than one grouping symbol is used, start evaluating within the innermost grouping symbols.

CHECK Your Progress

2A. $(15 - 9) + 3 \cdot 6$

2B. $45 + [(1 + 1)^3 \div 4]$

2C. $\frac{6^2 - 8}{4(3 + 7)}$

Evaluate Algebraic Expressions To evaluate an algebraic expression, replace the variables with their values. Then, find the value of the numerical expression using the order of operations.

EXAMPLE Evaluate an Algebraic Expression

3 Evaluate $a^2 - (b^3 - 4c)$ if $a = 7$, $b = 3$, and $c = 5$.

$$\begin{aligned} a^2 - (b^3 - 4c) &= 7^2 - (3^3 - 4 \cdot 5) && \text{Replace } a \text{ with } 7, b \text{ with } 3, \text{ and } c \text{ with } 5. \\ &= 49 - (27 - 4 \cdot 5) && \text{Evaluate } 7^2 \text{ and } 3^3. \\ &= 49 - (27 - 20) && \text{Multiply } 4 \text{ and } 5. \\ &= 49 - 7 && \text{Subtract } 20 \text{ from } 27. \\ &= 42 && \text{Subtract.} \end{aligned}$$

CHECK Your Progress

3. Evaluate $x(y^3 + 8) \div 12$ if $x = 3$ and $y = 4$.



EXAMPLE

- 4 ARCHITECTURE** The Pyramid Arena in Memphis, Tennessee, is the third largest pyramid in the world. The area of its base is 360,000 square feet, and it is 321 feet high. The volume of a pyramid is one third of the product of the area of the base B and its height h .

- a. Write an expression that represents the volume of a pyramid.

Words	one third	of	the product of area of base and height
Variables	B = area of base and h = height		
Expression	$\frac{1}{3}$	\times	$(B \cdot h)$ or $\frac{1}{3}Bh$

- b. Find the volume of the Pyramid Arena.

$$\begin{aligned}
 V &= \frac{1}{3}(Bh) && \text{Volume of a pyramid} \\
 &= \frac{1}{3}(360,000 \cdot 321) && \text{Replace } B \text{ with } 360,000 \text{ and } h \text{ with } 321. \\
 &= \frac{1}{3}(115,560,000) && \text{Multiply } 360,000 \text{ by } 321. \\
 &= 38,520,000 && \text{Multiply } \frac{1}{3} \text{ by } 115,560,000.
 \end{aligned}$$

The volume of the Pyramid Arena is 38,520,000 cubic feet.

CHECK Your Progress

According to market research, the average consumer spends \$78 per trip to the mall on weekends and only \$67 per trip during the week.

- 4A.** Write an algebraic expression to represent how much the average consumer spends at the mall in x weekend trips and y weekday trips.
- 4B.** Evaluate the expression to find what the average consumer spends after going to the mall twice during the week and 5 times on the weekends.

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CHECK Your Understanding

Examples 1, 2
(pp. 10–11)

Evaluate each expression.

- | | | |
|-----------------------------------|----------------------------------|-----------------------------|
| 1. $30 - 14 \div 2$ | 2. $5 \cdot 5 - 1 \cdot 3$ | 3. $6^2 + 8 \cdot 3 + 7$ |
| 4. $(4 + 6)7$ | 5. $50 - (15 + 9)$ | 6. $[8(2) - 4^2] + 7(4)$ |
| 7. $\frac{11 - 8}{1 + 7 \cdot 2}$ | 8. $\frac{(4 \cdot 3)^2}{9 + 3}$ | 9. $\frac{3 + 2^3}{5^2(6)}$ |

Example 3
(p. 11)

Evaluate each expression if $a = 4$, $b = 6$, and $c = 8$.

- | | | |
|--------------|-------------------------|----------------------------|
| 10. $8b - a$ | 11. $2a + (b^2 \div 3)$ | 12. $\frac{b(9 - c)}{a^2}$ |
|--------------|-------------------------|----------------------------|

Example 4
(p. 12)

- 13. GEOMETRY** Write an algebraic expression to represent the area of the rectangle. Then evaluate it to find the area when $n = 4$ centimeters.



$$2n + 3$$

Exercises

HOMEWORK HELP	
For Exercises	See Examples
14–17	1
18–25	2
26–31	3
32–33	4

Evaluate each expression.

14. $22 + 3 \cdot 7$ 15. $18 \div 9 + 2 \cdot 6$ 16. $10 + 8^3 \div 16$
 17. $12 \div 3 \cdot 5 - 4^2$ 18. $(11 \cdot 7) - 9 \cdot 8$ 19. $29 - 3(9 - 4)$
 20. $(12 - 6) \cdot 5^2$ 21. $3^5 - (1 + 10^2)$ 22. $108 \div [3(9 + 3^2)]$
 23. $[(6^3 - 9) \div 23]4$ 24. $\frac{8 + 3^3}{12 - 7}$ 25. $\frac{(1 + 6)9}{5^2 - 4}$

Evaluate each expression if $r = 2$, $s = 3$, and $t = 11$.

26. $r + 6t$ 27. $7 - rs$ 28. $(2t + 3r) \div 4$
 29. $s^2 + (r^3 - 8)5$ 30. $t^2 + 8st + r^2$ 31. $3r(r + s)^2 - 1$

32. **BOOKS** At a bookstore, Luna bought one new book for \$20 and three used books for \$4.95 each. Write and evaluate an expression to find how much money the books cost, not including sales tax.

33. **ENTERTAINMENT** Derrick sold tickets for the school musical. He sold 50 tickets for floor seats and 90 tickets for balcony seats. Write and evaluate an expression to find how much money Derrick collected.

School Musical Tickets	
Type of Seat	Cost per Ticket (\$)
floor	7.50
balcony	5.00

Evaluate each expression.

34. $\frac{2 \cdot 8^2 - 2^2 \cdot 8}{2 \cdot 8}$ 35. $6 - \left[\frac{2 + 7}{3} - (2 \cdot 3 - 5) \right]$ 36. $7^3 - \frac{2}{3}(13 \cdot 6 + 9)4$

Evaluate each expression if $x = 12$, $y = 8$, and $z = 3$.

37. $\frac{2xy - z^3}{z}$ 38. $\left(\frac{x}{y}\right)^2 - \frac{z}{(x - y)^2}$ 39. $\frac{x - z^2}{xy} + \frac{2y - x}{y^2}$

40. **BIOLOGY** The cells of a certain type of bacteria double in number every 20 minutes. Suppose 100 of these cells are in one culture dish and 250 cells are in another culture dish. Write and evaluate an expression to find the total number of bacteria cells in both dishes after 20 minutes.

BUSINESS For Exercises 41 and 42, use the following information.

A sales representative receives an annual salary s , an average commission each month c , and a bonus b for each sales goal that she reaches.

41. Write an algebraic expression to represent her total earnings in one year if she receives four equal bonuses.
 42. Suppose her annual salary is \$52,000 and her average commission is \$1225 per month. If each bonus is \$1150, how much does she earn in a year?

EXTRA PRACTICE
 See pages 717, 744.
Math online
 Self-Check Quiz at algebra1.com

H.O.T. Problems

43. **FIND THE ERROR** Leonora and Chase are evaluating $3[4 + (27 \div 3)]^2$. Who is correct? Explain your reasoning.

Leonora

$$\begin{aligned} 3[4 + (27 \div 3)]^2 &= 3(4 + 9^2) \\ &= 3(4 + 81) \\ &= 3(85) \\ &= 255 \end{aligned}$$

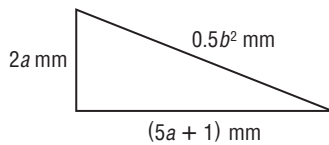
Chase

$$\begin{aligned} 3[4 + (27 \div 3)]^2 &= 3(4 + 9)^2 \\ &= 3(13)^2 \\ &= 3(169) \\ &= 507 \end{aligned}$$

44. **OPEN ENDED** Write a numerical expression involving division in which the first step in evaluating the expression is addition. Discuss why addition rather than division is the first step.
45. **CHALLENGE** Choose three numbers from 1 to 6. Using each of the numbers exactly once in each expression, write five expressions that have different results when they are evaluated. Justify your choices.
46. *Writing in Math* Use the information about the Internet on page 10 to explain how expressions can be used to determine the monthly cost of Internet service. Include an expression for the cost of service if Nicole has a coupon for \$25 off her base rate for her first six months.

STANDARDIZED TEST PRACTICE

47. **REVIEW** What is the perimeter of the triangle if $a = 9$ and $b = 10$?



- A 164 mm
 B 114 mm
 C 28 mm
 D 4 mm

48. Simplify: $[10 + 15(2^3)] \div [7(2^2) - 2]$

Step 1 $[10 + 15(8)] \div [7(4) - 2]$

Step 2 $[10 + 120] \div [28 - 2]$

Step 3 $130 \div 26$

Step 4 $\frac{1}{5}$

Which is the first *incorrect* step?

- F Step 1
 G Step 2
 H Step 3
 J Step 4

Spiral Review

Write an algebraic expression for each verbal expression. (Lesson 1-1)

49. the product of 13 and p 50. one eighth of a number b
 51. 20 increased by twice a number 52. 6 less than the square of y
53. **TRAVEL** Sari's car has 23,500 miles on the odometer. She takes a trip and drives an average of m miles each day for two weeks. Write an expression that represents the mileage on Sari's odometer after her trip. (Lesson 1-1)

Write a verbal expression for each algebraic expression. (Lesson 1-1)

54. $5 + \frac{n}{2}$ 55. $q^2 - 12$ 56. $\frac{x^3}{9}$

GET READY for the Next Lesson

PREREQUISITE SKILL Find the value of each expression. (pages 696-697, 700-701)

57. $0.5 - 0.075$ 58. $5.6 + 1.612$ 59. $2.4(6.425)$
 60. $4\frac{1}{8} - 1\frac{1}{2}$ 61. $\frac{3}{5} + 2\frac{5}{7}$ 62. $8 \div \frac{2}{9}$

Main Ideas

- Solve open sentence equations.
- Solve open sentence inequalities.

New Vocabulary

open sentence
 solving the open sentence
 solution
 equation
 replacement set
 set
 element
 solution set
 inequality

GET READY for the Lesson

The Daily News sells garage sale ads and kits. Spring Creek residents are planning a community garage sale, and their budget for advertising is \$135. The expression $15.50 + 5n$ represents the cost of an ad and n kits. The open sentence below can be used to ensure that the budget is met.

$$15.50 + 5n \leq 135$$



Solve Equations A mathematical statement with one or more variables is called an **open sentence**. An open sentence is neither true nor false until the variables have been replaced by specific values. The process of finding a value for a variable that results in a true sentence is called **solving the open sentence**. This replacement value is called a **solution**. A sentence that contains an equals sign, $=$, is called an **equation**.

A set of numbers from which replacements for a variable may be chosen is called a **replacement set**. A **set** is a collection of objects or numbers. It is often shown using braces, $\{ \}$. Each number in the set is called an **element**, or member. The **solution set** of an open sentence is the set of elements from the replacement set that make the open sentence true.

EXAMPLE Use a Replacement Set to Solve an Equation

1 Find the solution set for each equation if the replacement set is $\{3, 4, 5, 6, 7\}$.

a. $6n + 7 = 37$

Replace n in $6n + 7 = 37$ with each value in the replacement set.

n	$6n + 7 = 37$	True or False?
3	$6(3) + 7 \stackrel{?}{=} 37 \rightarrow 25 \neq 37$	false
4	$6(4) + 7 \stackrel{?}{=} 37 \rightarrow 31 \neq 37$	false
5	$6(5) + 7 \stackrel{?}{=} 37 \rightarrow 37 = 37$	true ✓
6	$6(6) + 7 \stackrel{?}{=} 37 \rightarrow 43 \neq 37$	false
7	$6(7) + 7 \stackrel{?}{=} 37 \rightarrow 49 \neq 37$	false

Since $n = 5$ makes the equation true, the solution of $6n + 7 = 37$ is 5. The solution set is $\{5\}$.

b. $5(x + 2) = 40$

Replace x in $5(x + 2) = 40$ with each value in the replacement set.

x	$5(x + 2) = 40$	True or False?
3	$5(3 + 2) \stackrel{?}{=} 40 \rightarrow 25 \neq 40$	false
4	$5(4 + 2) \stackrel{?}{=} 40 \rightarrow 30 \neq 40$	false
5	$5(5 + 2) \stackrel{?}{=} 40 \rightarrow 35 \neq 40$	false
6	$5(6 + 2) \stackrel{?}{=} 40 \rightarrow 40 = 40$	true ✓
7	$5(7 + 2) \stackrel{?}{=} 40 \rightarrow 45 \neq 40$	false

The solution of $5(x + 2) = 40$ is 6. The solution set is $\{6\}$.

CHECK Your Progress

Find the solution set for each equation if the replacement set is $\{0, 1, 2, 3\}$.

1A. $8m - 7 = 17$

1B. $28 = 4(1 + 3d)$

You can often solve an equation by applying the order of operations.

EXAMPLE Use Order of Operations to Solve an Equation

2 Solve $\frac{13 + 2(4)}{3(5 - 4)} = q$.

$$\frac{13 + 2(4)}{3(5 - 4)} = q \quad \text{Original equation}$$

$$\frac{13 + 8}{3(1)} = q \quad \begin{array}{l} \text{Multiply 2 and 4 in the numerator.} \\ \text{Subtract 4 from the 5 in the denominator.} \end{array}$$

$$\frac{21}{3} = q \quad \text{Simplify.}$$

$$7 = q \quad \text{Divide.} \quad \text{The solution is 7.}$$

CHECK Your Progress

Solve each equation.

2A. $t = 9^2 \div (2 + 1)$

2B. $x = \frac{3^2 - (7 - 5)}{3(4) + (2 + 1)}$

Reading Math

Symbols Inequality symbols are read as follows.

$<$ is less than

\leq is less than or equal to

$>$ is greater than

\geq is greater than or equal to

Solve Inequalities An open sentence that contains the symbol $<$, \leq , $>$, or \geq is called an **inequality**. Inequalities can be solved in the same way as equations.

Real-World EXAMPLE

3 **SHOPPING** Meagan had \$18. After she went to the used book store, she had less than \$10 left. Could she have spent \$8, \$9, \$10, or \$11? Find the solution set for $18 - y < 10$ if the replacement set is $\{8, 9, 10, 11\}$.

Replace y in $18 - y < 10$ with each value in the replacement set.

y	$18 - y < 10$	True or False?
8	$18 - 8 \stackrel{?}{<} 10 \rightarrow 10 \not< 10$	false
9	$18 - 9 \stackrel{?}{<} 10 \rightarrow 9 < 10$	true ✓
10	$18 - 10 \stackrel{?}{<} 10 \rightarrow 8 < 10$	true ✓
11	$18 - 11 \stackrel{?}{<} 10 \rightarrow 7 < 10$	true ✓

The solution set is $\{9, 10, 11\}$. So, Meagan could have spent \$9, \$10, or \$11.

CHECK Your Progress

Find the solution set for each inequality if the replacement set is $\{5, 6, 7, 8\}$.

3A. $30 + n \geq 37$

3B. $19 > 2y - 5$

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Real-World EXAMPLE

4 FUND-RAISING Refer to the application at the beginning of the lesson. If the residents buy an ad, what is the maximum number of garage sale kits they can buy and stay within their budget?

Explore The residents can spend no more than \$135. Let n = the number of ads. The situation can be represented by the inequality $15.50 + 5n \leq 135$.

Plan Estimate to find reasonable values for the replacement set.

Solve Start by letting $n = 10$ and then adjust values as needed.

$$15.50 + 5n \leq 135 \quad \text{Original inequality}$$

$$15.50 + 5(10) \stackrel{?}{\leq} 135 \quad \text{Replace } n \text{ with } 10.$$

$$15.50 + 50 \stackrel{?}{\leq} 135 \quad \text{Multiply } 5 \text{ and } 10.$$

$$65.50 \leq 135 \quad \text{Add } 15.50 \text{ and } 50.$$

The inequality is true, but the estimate is too low. Increase the value of n .

n	$15.50 + 5n \leq 135$	Reasonable?
20	$15.50 + 5(20) \stackrel{?}{\leq} 135 \rightarrow 115.50 \leq 135$	too low
25	$15.50 + 5(25) \stackrel{?}{\leq} 135 \rightarrow 140.50 \leq 135$	too high
23	$15.50 + 5(23) \stackrel{?}{\leq} 135 \rightarrow 130.50 \leq 135$	almost
24	$15.50 + 5(24) \stackrel{?}{\leq} 135 \rightarrow 135.50 \leq 135$	too high

Check The solution set is $\{0, 1, 2, 3, \dots, 21, 22, 23\}$. In addition to the ad, the residents can buy as many as 23 garage sale kits.


Study Tip

Ellipsis

In $\{1, 2, 3, 4, \dots\}$, the three dots are an *ellipsis*. In math, an ellipsis is used to indicate that numbers continue in the same pattern.

CHECK Your Progress

4. **ENTERTAINMENT** Trevor and his brother have a total of \$15. They plan to buy 2 movie tickets at \$6.50 each and then play video games in the arcade for \$0.50 each. Write and solve an inequality to find the greatest number of video games v that they can play.

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Example 1
(p. 15)

Find the solution of each equation if the replacement set is {11, 12, 13, 14, 15}.

1. $n + 10 = 23$
2. $7 = \frac{c}{2}$
3. $29 = 3x - 7$
4. $(k - 8)12 = 84$

Find the solution of each equation using the given replacement set.

5. $36 = 18 + a$; {14, 16, 18, 20}
6. $\frac{d+5}{11} = 2$; {4, 17, 23, 30, 45}

Example 2
(p. 16)

Solve each equation.

7. $x = 4(6) + 3$
8. $\frac{14 - 8}{2} = w$
9. $\frac{3(9) - 2}{1 + 4} = d$
10. $j = 15 \div 3 \cdot 5 - 4^2$

Example 3
(pp. 16–17)

Find the solution set of each inequality using the given replacement set.

11. $\frac{a}{5} \geq 2$; {5, 10, 15, 20, 25}
12. $24 - 2x \geq 13$; {0, 1, 2, 3, 4, 5, 6}

Example 4
(p. 17)

13. ANALYZE TABLES Suppose you have \$102.50 to buy sweaters from an online catalog. Using the information in the table, write and solve an inequality to find the maximum number of sweaters that you can purchase.

Online Catalog Prices	
Item	Cost (\$)
sweaters	39.00 each
shipping	10.95 per order

Exercises

HOMEWORK	HELP
For Exercises	See Examples
14–25	1
26–33	2
34–37	3
38–39	4

Find the solution of each equation if the replacement sets are $a = \{0, 3, 5, 8, 10\}$ and $b = \{12, 17, 18, 21, 25\}$.

14. $b - 12 = 9$
15. $22 = 34 - b$
16. $\frac{15}{a} = 3$
17. $68 = 4b$
18. $31 = 3a + 7$
19. $5(a - 1) = 10$
20. $\frac{40}{a} - 4 = 0$
21. $27 = a^2 + 2$

Find the solution of each equation using the given replacement set.

22. $t - 13 = 7$; {10, 13, 17, 20}
23. $14(x + 5) = 126$; {3, 4, 5, 6, 7}
24. $22 = \frac{n}{3}$; {62, 64, 66, 68, 70}
25. $35 = \frac{g - 8}{2}$; {78, 79, 80, 81}

Solve each equation.

26. $a = 32 - 9(2)$
27. $w = 56 \div (2^2 + 3)$
28. $\frac{27 + 5}{16} = g$
29. $\frac{12 \cdot 5}{15 - 3} = y$
30. $r = \frac{9(6)}{(8 + 1)3}$
31. $a = \frac{4(14 - 1)}{3(6) - 5} + 7$

32. FOOD During a lifetime, the average American drinks 15,579 glasses of milk, 6220 glasses of juice, and 18,995 glasses of soda. Write and solve an equation to find g , the total number of glasses of milk, juice, and soda that the average American drinks in a lifetime.

33. ENERGY A small electric generator can power 3550 watts of electricity. Write and solve an equation to find the most 75-watt light bulbs one small generator could power.

Find the solution set for each inequality using the given replacement set.

34. $s - 2 < 6$; {6, 7, 8, 9, 10, 11}

35. $5a + 7 > 22$; {3, 4, 5, 6, 7}

36. $3 \geq \frac{25}{m}$; {1, 3, 5, 7, 9, 11}

37. $\frac{2a}{4} \leq 8$; {12, 14, 16, 18, 20, 22}

ENTERTAINMENT For Exercises 38 and 39, use the table.

38. Mr. and Mrs. Conkle are taking their three children to an amusement park. Write and solve an inequality to determine whether they can all go to the park for under \$200. Describe what the variables in your inequality represent and explain your answer.

Amusement Park Admission Prices	
Person	Cost (\$)
Adult	41.99
Child	26.99

39. Write and solve an inequality to find how many children can go with three adults if the budget is \$300. Determine whether your answer is reasonable.

Find the solution of each equation or inequality using the given replacement set.

40. $x + \frac{2}{5} = 1\frac{3}{20}$; $\left\{\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, 11, 4\right\}$

41. $\frac{2}{5}(x + 1) = \frac{8}{15}$; $\left\{\frac{1}{6}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}\right\}$

42. $2.7(x + 5) = 17.28$; {1.2, 1.3, 1.4, 1.5}

43. $16(x + 2) = 70.4$; {2.2, 2.4, 2.6, 2.8}

44. $4a - 3 \geq 10.6$; {3.2, 3.4, 3.6, 3.8, 4}

45. $3(12 - x) + 2 \leq 28$ {2.5, 3, 3.5, 4}

EXTRA PRACTICE
See pages 717, 744.
Math online
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NUTRITION For Exercises 46 and 47, use the following information.

A person must burn 3500 Calories to lose one pound of weight.

46. Define a variable and write an equation for the number of Calories that a person would have to burn each day to lose four pounds in two weeks.

47. How many Calories would the person have to burn each day?

H.O.T. Problems

48. **REASONING** Describe the difference between an expression and an open sentence.

49. **OPEN ENDED** Write an inequality that has a solution set of {8, 9, 10, 11, ...}. Explain your reasoning.

50. **REASONING** Explain why an open sentence always has at least one variable.

51. **CHALLENGE** Describe the solution set for x if $3x \leq 1$.

52. **Writing in Math** Use the information about budgets on page 15 to explain how you can use open sentences when you have to stay within a budget. Also explain and give examples of real-world situations in which you would use inequalities and equations.

53. What is the solution set of the inequality $(5 + n^2) - n < 50$ if the replacement set is $\{5, 7, 9\}$?

- A $\{5\}$ C $\{7\}$
 B $\{5, 7\}$ D $\{7, 9\}$

54. $27 \div 3 + (12 - 4) =$

- F $-\frac{11}{5}$ H 17
 G $\frac{27}{11}$ J 25

55. **REVIEW** A box in the shape of a rectangular prism has a volume of 56 cubic inches. If the length of each side is multiplied by 2, what will be the approximate volume of the resulting box?

- A 112 in^3 C 336 in^3
 B 224 in^3 D 448 in^3

56. **REVIEW** Ms. Beal had 1 bran muffin, 16 ounces of orange juice, 3 ounces of sunflower seeds, 2 slices of turkey, and a half cup of spinach. According to the table, which equation best represents the total grams of protein that she consumed?

Protein Content	
Food	Protein (g)
bran muffin (1)	3
orange juice (8 oz)	2
sunflower seeds (1 oz)	6
turkey (1 slice)	12
spinach (1 c)	5

- F $P = 3 + 2 + 6 + 12 + 5$
 G $P = 3 + \frac{1}{2}(2) + \frac{1}{3}(6) + \frac{1}{2}(12) + 2(5)$
 H $P = 3 + 16(2) + 3(6) + 2(12) + 2(5)$
 J $P = 3 + 2(2) + 3(6) + 2(12) + \frac{1}{2}(5)$

Spiral Review

Evaluate each expression. (Lesson 1-2)

57. $5 + 3(4^2)$

58. $\frac{38 - 12}{2 \cdot 13}$

59. $[5(1 + 1)]^3 + 4$

60. **RING TONES** Andre downloaded three standard ringtones for \$1.99 each and two premium ringtones at \$3.49 each. Write and evaluate an expression to find how much the ringtones cost. (Lesson 1-2)

Write a verbal expression for each algebraic expression. (Lesson 1-1)

61. $n^5 - 8$

62. $r^2 + 3s$

63. $b \div 5a$

Write an algebraic expression for each verbal expression.

64. two-thirds the square of a number

65. 6 increased by one half of a number n

66. one-half the cube of x

67. one fourth of the cube of a number

GET READY for the Next Lesson

PREREQUISITE SKILL Find each product. Express answers in simplest form. (pages 700–701)

68. $\frac{1}{6} \cdot \frac{2}{5}$

69. $\frac{4}{9} \cdot \frac{3}{7}$

70. $\frac{5}{6} \cdot \frac{15}{16}$

71. $\frac{6}{14} \cdot \frac{12}{18}$

72. $\frac{2}{5} \cdot \frac{3}{4}$

73. $\frac{11}{12} \cdot \frac{4}{5}$

Identity and Equality Properties

Main Ideas

- Recognize the properties of identity and equality.
- Use the properties of identity and equality.

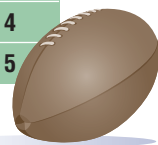
New Vocabulary

additive identity
multiplicative identity
multiplicative inverse
reciprocal

GET READY for the Lesson

During the college football season, teams are ranked weekly. The table shows the last three rankings of the top five teams for the 2005 football season. The open sentence below represents the change in rank of Texas from Week 6 to Week 7.

College Football Team	Week 6	Week 7	Final Rank
Texas	2	2	1
Southern California	1	1	2
Penn State	16	5	3
Ohio State	6	15	4
West Virginia	34	26	5



$$\begin{array}{ccccccc} \text{Rank in} & & \text{increase} & & \text{rank for} \\ \text{Week 6} & & \text{in rank} & & \text{Week 7.} \\ 2 & + & r & = & 2 \end{array}$$

The solution of this equation is 0. Texas' rank changed by 0 from Week 6 to Week 7. In other words, $2 + 0 = 2$.

Identity and Equality Properties The sum of any number and 0 is equal to the number. Thus, 0 is called the **additive identity**.

KEY CONCEPT

Additive Identity

Words For any number a , the sum of a and 0 is a .

Symbols $a + 0 = a$, $0 + a = a$

Examples $5 + 0 = 5$, $0 + 5 = 5$

There are also special properties associated with multiplication. Consider the following equations.

$7 \cdot n = 7$ The solution of the equation is 1. Since the product of any number and 1 is equal to the number, 1 is called the **multiplicative identity**.

$9 \cdot m = 0$ The solution of the equation is 0. The product of any number and 0 is equal to 0. This is called the **Multiplicative Property of Zero**.

$\frac{1}{p} \cdot p = 1$ Two numbers whose product is 1 are called **multiplicative inverses** or **reciprocals**. Zero has no reciprocal because any number times 0 is 0.

The multiplicative properties are summarized in the following table.

Study Tip

Properties

These properties are true for all *real numbers*. You will learn more about real numbers in Lesson 1-8.

KEY CONCEPT		Multiplication Properties	
Property	Words	Symbols	Examples
Multiplicative Identity	For any number a , the product of a and 1 is a .	$a \cdot 1 = a$, $1 \cdot a = a$	$12 \cdot 1 = 12$, $1 \cdot 12 = 12$
Multiplicative Property of Zero	For any number a , the product of a and 0 is 0.	$a \cdot 0 = 0$, $0 \cdot a = 0$	$8 \cdot 0 = 0$, $0 \cdot 8 = 0$
Multiplicative Inverse	For every number $\frac{a}{b}$, where $a, b \neq 0$, there is exactly one number $\frac{b}{a}$ such that the product of $\frac{a}{b}$ and $\frac{b}{a}$ is 1.	$\frac{a}{b} \cdot \frac{b}{a} = 1$ $\frac{b}{a} \cdot \frac{a}{b} = 1$	$\frac{2}{3} \cdot \frac{3}{2} = \frac{6}{6} = 1$, $\frac{3}{2} \cdot \frac{2}{3} = \frac{6}{6} = 1$

EXAMPLE Identify Properties

- Find the value of n in each equation. Then name the property that is used.
 - $42 \cdot n = 42$
 $n = 1$, since $42 \cdot 1 = 42$. This is the Multiplicative Identity Property.
 - $n \cdot 9 = 1$
 $n = \frac{1}{9}$, since $\frac{1}{9} \cdot 9 = 1$. This is the Multiplicative Inverse Property.

CHECK Your Progress

- Find the value of n in the equation $28n = 0$. Then name the property that is used.

Several properties of equality are summarized below.

KEY CONCEPT		Properties of Equality	
Property	Words	Symbols	Examples
Reflexive	Any quantity is equal to itself.	For any number a , $a = a$.	$7 = 7$, $2 + 3 = 2 + 3$
Symmetric	If one quantity equals a second quantity, then the second quantity equals the first.	For any numbers a and b , if $a = b$, then $b = a$.	If $9 = 6 + 3$, then $6 + 3 = 9$.
Transitive	If one quantity equals a second quantity and the second quantity equals a third quantity, then the first quantity equals the third quantity.	For any numbers a, b , and c , if $a = b$, and $b = c$, then $a = c$.	If $5 + 7 = 8 + 4$, and $8 + 4 = 12$, then $5 + 7 = 12$.
Substitution	A quantity may be substituted for its equal in any expression.	If $a = b$, then a may be replaced by b in any expression.	If $n = 15$, then $3n = 3(15)$.

Use Identity and Equality Properties The properties of identity and equality can be used to justify each step when evaluating an expression.

EXAMPLE Evaluate Using Properties

2 Evaluate $2(3 \cdot 2 - 5) + 3 \cdot \frac{1}{3}$. Name the property used in each step.

$$\begin{aligned} 2(3 \cdot 2 - 5) + 3 \cdot \frac{1}{3} &= 2(6 - 5) + 3 \cdot \frac{1}{3} && \text{Substitution; } 3 \cdot 2 = 6 \\ &= 2(1) + 3 \cdot \frac{1}{3} && \text{Substitution; } 6 - 5 = 1 \\ &= 2 + 3 \cdot \frac{1}{3} && \text{Multiplicative Identity; } 2 \cdot 1 = 2 \\ &= 2 + 1 && \text{Multiplicative Inverse; } 3 \cdot \frac{1}{3} = 1 \\ &= 3 && \text{Substitution; } 2 + 1 = 3 \end{aligned}$$

CHECK Your Progress

2. Evaluate $8 + (15 - 3 \cdot 5)$. Name the property used in each step.

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CHECK Your Understanding

Example 1
(p. 22)

Find the value of n in each equation. Then name the property that is used.

1. $13n = 0$

2. $17 + 0 = n$

3. $1 = \frac{1}{6}n$

Example 2
(p. 23)

Evaluate each expression. Name the property used in each step.

4. $11 + 2(8 - 7)$

5. $6(12 - 48 \div 4)$

6. $(15 \cdot \frac{1}{15} + 8 \cdot 0) \cdot 12$

7. HISTORY Abraham Lincoln's Gettysburg Address began "Four score and seven years ago...." Since a score is equal to 20, the expression $4(20) + 7$ represents this quote. Evaluate the expression to find how many years Lincoln was referring to. Name the property used in each step.

Exercises

HOMEWORK HELP	
For Exercises	See Examples
8-17	1
18-25	2

Find the value of n in each equation. Then name the property that is used in each step.

8. $12n = 12$

9. $n \cdot 1 = 5$

10. $8 \cdot n = 8 \cdot 5$

11. $7 + 1.5 = n + 1.5$

12. $6 = 6 + n$

13. $1 = 2n$

14. $n + 0 = \frac{1}{3}$

15. $4 \cdot \frac{1}{4} = n$

16. $(9 - 7)(5) = 2(n)$

17. $n + 10 = 3 + (2 + 8)$

Evaluate each expression. Name the property used in each step.

18. $(1 \div 5)5 \cdot 14$

19. $7 + (9 - 3^2)$

20. $\frac{3}{4}[4 \div (7 - 4)]$

21. $[3 \div (2 \cdot 1)] \frac{2}{3}$

22. $2(3 \cdot 2 - 5) + 3 \cdot \frac{1}{3}$

23. $6 \cdot \frac{1}{6} + 5(12 \div 4 - 3)$



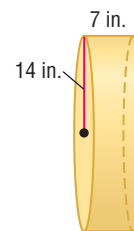
Real-World Link

Nationally organized football began in 1920 and originally included five teams. In 2005, there were 32 professional teams.

Source: www.infoplease.com

24. MILITARY PAY An enlisted member of the military at grade E-2 earns \$1427.40 per month in the first year of service. After 5 years of service, a person at grade E-2 earns \$1427.40 per month. Write and solve an equation using addition that shows the change in pay from 1 year of service to 5 years. Name the property or identity used.

25. GEOMETRY The expression $2 \cdot \frac{22}{7} \cdot 14^2 + 2 \cdot \frac{22}{7} \cdot 14 \cdot 7$ represents the surface area of the cylinder at the right. Evaluate this expression to find the surface area. Name the property used in each step.



Find the value of n in each equation. Then name the property that is illustrated.

26. $1 = \frac{(9 - 5)}{3^2} n$

27. $n(5^2 \cdot \frac{1}{25}) = 3$

28. $6(\frac{1}{2} \cdot n) = 6$

Evaluate each expression. Name the property used in each step.

29. $3 + 5(4 - 2^2) - 1$

30. $7 - 8(9 - 3^2)$

31. $[\frac{5}{8}(1 + \frac{3}{8})] \cdot 17$

32. FOOTBALL The table shows various bonus plans for the NFL in a recent year. Write an expression that could be used to determine what a team owner would pay in bonuses for the following:

NFL Bonuses	
Goal	Bonuses(\$)
Average 4.5 yards per carry	50,000
12 touchdowns	50,000
76 points scored	50,000
Keep weight below 240 pounds	100,000

Source: ESPN Sports Almanac

- eight players who keep their weight below 240 pounds and have averaged 4.5 yards per carry, and
- three players who score 12 touchdowns and score 76 points.

Name the property used in each step.

ANALYZE TABLES For Exercises 33 and 34, use the following information.

The spirit club at Marshall High School is selling school bumper stickers, buttons, and caps. The profit for each item is the difference between the selling price and the cost.

School Spirit items		
Item	Cost (\$)	Selling Price (\$)
Bumper Sticker	0.30	2.00
Button	1.00	2.50
Cap	6.00	10.00

- 33.** Write an expression that represents the profit for selling 25 bumper stickers, 80 buttons, and 40 caps.
- 34.** Evaluate the expression, indicating the property used in each step.

35. CHALLENGE The **Transitive Property of Inequality** states that if $a < b$ and $b < c$, then $a < c$. Use this property to determine whether the following statement is *sometimes*, *always*, or *never* true. Give examples to support your answer.

If $x > y$ and $z > w$, then $xz > yw$.

EXTRA PRACTICE
See pages, 718, 744.
Math online
Self-Check Quiz at algebra1.com

H.O.T. Problems

36. **REASONING** Explain whether 1 can be an additive identity. Give an example to justify your answer.
37. **OPEN ENDED** Write two equations showing the Transitive Property of Equality. Justify your reasoning.
38. **REASONING** Explain why 0 has no multiplicative inverse.
39. *Writing in Math* Use the data about football on page 21 to explain how properties can be used to compare data. Include an example of the Transitive Property using three teams' rankings as an example.



STANDARDIZED TEST PRACTICE

40. Which illustrates the Symmetric Property of Equality?
- A If $a = b$, then $b = a$.
- B If $a = b$ and $b = c$, then $a = c$.
- C If $a = b$, then $b = c$.
- D If $a = a$, then $a + 0 = a$.
41. Which property is used below?
 If $4xy^2 = 8y^2$ and $8y^2 = 72$,
 then $4xy^2 = 72$.
- F Reflexive Property
- G Substitution Property
- H Symmetric Property
- J Transitive Property

Spiral Review

Find the solution set for each inequality using the given replacement set. (Lesson 1-3)

42. $10 - x > 6$; {3, 5, 6, 8}
43. $4x + 2 < 58$; {11, 12, 13, 14, 15}

44. **EXERCISE** It takes about 2000 steps to walk one mile. Use the table to determine how many miles of walking it would take to burn all the Calories contained in a cheeseburger and two 12-ounce sodas. (Lesson 1-3)

Steps Needed to Burn Calories	
Food	Number of Steps
cheeseburger	7590
12 oz. soda	3450

45. **SHOPPING** In a recent year, the average U.S. household spent \$213 on toys and games. In San Jose, California, the average spending was \$59 less than twice this amount. Write and evaluate an expression to find the average spending on toys and games in San Jose during that year. (Lesson 1-2)
46. Write an algebraic expression for the sum of twice a number squared and 7. (Lesson 1-1)

GET READY for the Next Lesson

PREREQUISITE SKILL Evaluate each expression. (Lesson 1-2)

47. $10(6) + 10(2)$
48. $(15 - 6) \cdot 8$
49. $12(4) - 5(4)$
50. $3(4 + 2)$

The Distributive Property

Main Ideas

- Use the Distributive Property to evaluate expressions.
- Use the Distributive Property to simplify algebraic expressions.

New Vocabulary

term
like terms
equivalent expressions
simplest form
coefficient

GET READY for the Lesson

Instant Replay Video Games sells new and used games. During a sale, the first 8 customers each bought a bargain game and a new release. To calculate the total sales for these customers, you can use the Distributive Property.



Sale Prices	
Used Games	\$9.95
Bargain Games	\$14.95
Regular Games	\$24.95
New Releases	\$34.95

Evaluate Expressions There are two methods you could use to calculate the video game sales.

Method 1			Method 2		
sales of bargain games	plus	sales of new releases	number of customers	times	each customer's purchase price
$8(14.95)$	+	$8(34.95)$	8	×	$(14.95 + 34.95)$
$= 119.60$		$+ 279.60$	$= 8(49.90)$		
$= 399.20$			$= 399.20$		

Either method gives total sales of \$399.20 because the following is true.

$$8(14.95) + 8(34.95) = 8(14.95 + 34.95)$$

This is an example of the **Distributive Property**.

Vocabulary Link

Distribute

Everyday Use

to divide among several or many

Distributive

Math Use

property that allows you to multiply each number in a sum or difference by a number outside the parentheses

KEY CONCEPT

Distributive Property

Symbols For any numbers a , b , and c ,
 $a(b + c) = ab + ac$ and $(b + c)a = ba + ca$ and
 $a(b - c) = ab - ac$ and $(b - c)a = ba - ca$.

Examples $3(2 + 5) = 3 \cdot 2 + 3 \cdot 5$ $4(9 - 7) = 4 \cdot 9 - 4 \cdot 7$
 $3(7) = 6 + 15$ $4(2) = 36 - 28$
 $21 = 21$ ✓ $8 = 8$ ✓

Notice that it does not matter whether a is placed on the right or the left of the expression in the parentheses. The Symmetric Property of Equality allows the Distributive Property to be written as follows.

$$\text{If } a(b + c) = ab + ac, \text{ then } ab + ac = a(b + c).$$



Real-World Link

Ramona is America's oldest continuing outdoor drama. It has been performed since 1923 for more than 2 million people. Its location, called the *Ramona Bowl*, has been designated a California State Historical Landmark.

Source: ramonabowl.com

EXAMPLE Distribute Over Addition or Subtraction

- 1 Tickets for a play are \$8. A group of 10 adults and 4 children are planning to go. Rewrite $8(10 + 4)$ using the Distributive Property. Then evaluate to find the total cost for the group.

$$\begin{aligned} 8(10 + 4) &= 8(10) + 8(4) && \text{Distributive Property} \\ &= 80 + 32 && \text{Multiply.} \\ &= 112 && \text{Add.} \end{aligned}$$

CHECK Your Progress

Rewrite each expression using the Distributive Property. Then evaluate.

1A. $(5 + 1)9$

1B. $3(11 - 8)$

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The Distributive Property can be used to simplify mental calculations involving multiplication. To use this method, rewrite one factor as a sum or difference. Then use the Distributive Property to multiply. Finally, find the sum or difference.

EXAMPLE The Distributive Property and Mental Math

- 2 Use the Distributive Property to rewrite $15 \cdot 99$. Then evaluate.

$$\begin{aligned} 15 \cdot 99 &= 15(100 - 1) && \text{Think: } 99 = 100 - 1 \\ &= 15(100) - 15(1) && \text{Distributive Property} \\ &= 1500 - 15 && \text{Multiply} \\ &= 1485 && \text{Subtract.} \end{aligned}$$

CHECK Your Progress

Use the Distributive Property to rewrite each expression. Then evaluate.

2A. $402(12)$

2B. $60 \cdot 7\frac{2}{3}$

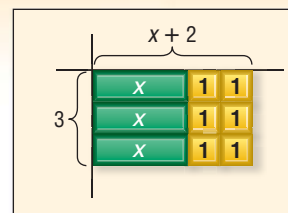
Simplify Expressions You can use algebra tiles to investigate how the Distributive Property relates to algebraic expressions.

ALGEBRA LAB

The Distributive Property

Use a product mat and algebra tiles to model $3(x + 2)$ as the area of a rectangle with dimensions of 3 and $(x + 2)$.

Make a rectangle with algebra tiles that is 3 units wide and $x + 2$ units long. The rectangle has 3 x -tiles and 6 1-tiles. The area of the rectangle is $x + 1 + 1 + x + 1 + 1 + x + 1 + 1$ or $3x + 6$. Therefore, $3(x + 2) = 3x + 6$.



(continued on the next page)

MODEL AND ANALYZE

Find each product by using algebra tiles.

1. $2(x + 1)$

2. $5(x + 2)$

3. $2(2x + 1)$

Tell whether each statement is *true* or *false*. Justify your answer with algebra tiles and a drawing.

4. $3(x + 3) = 3x + 3$

5. $x(3 + 2) = 3x + 2x$

6. $2(2x + 1) = 2x + 2$

You can apply the Distributive Property to algebraic expressions.

EXAMPLE Algebraic Expressions

3 Rewrite each product using the Distributive Property. Then simplify.

a. $5(g - 9)$

$$\begin{aligned} 5(g - 9) &= 5 \cdot g - 5 \cdot 9 && \text{Distributive Property} \\ &= 5g - 45 && \text{Multiply.} \end{aligned}$$

b. $3(2x^2 + 4x - 1)$

$$\begin{aligned} 3(2x^2 + 4x - 1) &= 3(2x^2) + 3(4x) - 3(1) && \text{Distributive Property} \\ &= 6x^2 + 12x - 3 && \text{Simplify.} \end{aligned}$$

CHECK Your Progress

3A. $(8 + n)2$

3B. $-6(r + 3s - t)$

Reading Math

Algebraic Expressions

The expression $5(g - 9)$ is read *5 times the quantity g minus 9* or *5 times the difference of g and 9*.

A **term** is a number, a variable, or a product or quotient of numbers and variables. For example, y , p^3 , $4a$, and $5g^2h$ are all terms. **Like terms** contain the same variables, with corresponding variables having the same power.

$$2x^2 + 6x + 5$$

three terms

$$3a^2 + 5a^2 + 2a$$

like terms unlike terms

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The Distributive Property and the properties of equality can be used to show that $5n + 7n = 12n$. In this expression, $5n$ and $7n$ are like terms.

$$\begin{aligned} 5n + 7n &= (5 + 7)n && \text{Distributive Property} \\ &= 12n && \text{Substitution} \end{aligned}$$

The expressions $5n + 7n$ and $12n$ are called **equivalent expressions** because they denote the same number. An expression is in **simplest form** when it is replaced by an equivalent expression having no like terms or parentheses.

EXAMPLE Combine Like Terms

4 a. Simplify $15x + 18x$.

$$\begin{aligned} 15x + 18x &= (15 + 18)x && \text{Distributive Property} \\ &= 33x && \text{Substitution} \end{aligned}$$



b. Simplify $10n + 3n^2 + 9n^2$.

$$\begin{aligned} 10n + 3n^2 + 9n^2 &= 10n + (3 + 9)n^2 && \text{Distributive Property} \\ &= 10n + 12n^2 && \text{Substitution} \end{aligned}$$

CHECK Your Progress

Simplify each expression. If not possible, write *simplified*.

4A. $6t - 4t$

4B. $b^2 + 13b + 13$

Reading Math

Like terms may be defined as terms that are the same or vary only by the coefficient.

The **coefficient** of a term is the numerical factor. For example, in $17xy$, the coefficient is 17, and in $\frac{3y^2}{4}$, the coefficient is $\frac{3}{4}$. In the term m , the coefficient is 1 since $1 \cdot m = m$ by the Multiplicative Identity Property.

CHECK Your Understanding

Example 1
(p. 27)

Rewrite each expression using the Distributive Property. Then evaluate.

1. $6(12 - 3)$

2. $8(1 + 5)$

3. $(19 + 3)10$

4. **COSMETOLOGY** A hair stylist cut 12 customers' hair. She earned \$29.95 for each haircut and received an average tip of \$4 for each. Write and evaluate an expression to determine the total amount that she earned.

Example 2
(p. 27)

Use the Distributive Property to rewrite each expression. Then find the product.

5. $16(103)$

6. $(3\frac{1}{17})(34)$

Example 3
(p. 28)

Rewrite each expression using the Distributive Property. Then simplify.

7. $2(4 + t)$

8. $(g - 9)5$

Example 4
(pp. 28–29)

Simplify each expression. If not possible, write *simplified*.

9. $13m + m$

10. $14a^2 + 13b^2 + 27$

11. $3(x + 2x)$

Exercises

HOMEWORK HELP	
For Exercises	See Examples
12–19	1
20–23	2
24–27	3
28–33	4

Rewrite each expression using the Distributive Property. Then evaluate.

12. $(5 + 7)8$

13. $7(13 + 12)$

14. $6(6 - 1)$

15. $(3 + 8)15$

16. $12(9 - 5)$

17. $(10 - 7)13$

18. **COMMUNICATION** A consultant keeps a log of all contacts she makes. In a typical week, she averages 5 hours using e-mail, 18 hours on the telephone, and 12 hours of meetings in person. Write and evaluate an expression to predict how many hours she will spend on these activities over the next 12 weeks.

19. **OLYMPICS** The table shows the average daily attendance for two venues at the 2004 Summer Olympics. Write and evaluate an expression to estimate the total number of people at these venues over a 4-day period.

Average Olympic Attendance	
Venue	Number of People
Olympic Stadium	110,000
Aquatic Center	17,500

Source: www.olympic.org

Use the Distributive Property to rewrite each expression. Then find the product.

20. $5 \cdot 97$

21. $8(990)$

22. $18 \cdot 2\frac{1}{9}$

23. $(3\frac{1}{6})48$

Rewrite each expression using the Distributive Property. Then simplify.

24. $2(x + 4)$

25. $(5 + n)3$

26. $(4 - 3m)8$

27. $-3(2x - 6)$

Simplify each expression. If not possible, write *simplified*.

28. $2x + 9x$

29. $4b - 1 + 5b$

30. $5n^2 - 7n$

31. $3a^2 + a + 14a^2$

32. $12(4 + 3c)$

33. $(3x - 5)15$

ANALYZE TABLES For Exercises 34 and 35, use the table that shows the monthly cost of a company health plan.

Available Insurance Plans - Monthly Charges			
Coverage	Medical	Dental	Vision
Employee	\$78	\$20	\$12
Family (additional charge)	\$50	\$15	\$7

34. Write and evaluate an expression to calculate the total cost of medical, dental, and vision insurance for an employee for 6 months.

35. How much would an employee expect to pay for family medical and dental coverage per year?

Rewrite each expression using the Distributive Property. Then simplify.

36. $(\frac{1}{3} - 2b)27$

37. $4(8p + 4q - 7r)$

38. $-6(2c - cd^2 + d)$

39. $(6m^3 + 4n - 3n)5$

Simplify each expression. If not possible, write *simplified*.

40. $6x^2 + 14x - 9x$

41. $4y^3 + 3y^3 + y^4$

42. $a + \frac{a}{5} + \frac{2}{5}a$

EXTRA PRACTICE
See pages 718, 744.
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Self-Check Quiz at algebra1.com

H.O.T. Problems

43. **REASONING** Explain why the Distributive Property is sometimes called the Distributive Property of Multiplication Over Addition.

44. **OPEN ENDED** Write an expression that has five terms, three of which are like terms and one that has a coefficient of 1. Describe how to simplify the expression.

45. **FIND THE ERROR** Courtney and Che are simplifying $3(x + 4)$. Who is correct? Explain your reasoning.

Courtney
 $3(x + 4) = 3(x) + 3(4)$
 $= 3x + 12$

Che
 $3(x + 4) = 3(x) + 4$
 $= 3x + 4$

46. **CHALLENGE** The expression $2(\ell + w)$ can be used to find the perimeter of a rectangle with a length ℓ and width w . What are the length and width of a rectangle if the area is $13\frac{1}{2}$ square units and the length of one side is $\frac{1}{5}$ the measure of the perimeter? Explain your reasoning.
47. **Writing in Math** Use the data about video game prices on page 26 to explain how the Distributive Property can be used to calculate quickly. Also, compare and contrast the two methods of finding the total video game sales.

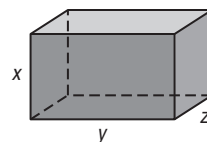
STANDARDIZED TEST PRACTICE

48. In three months, Mayuko had 108 minutes of incoming calls on her voice mail. What was the total cost of voice mail for those three months?

Voice Mail	
Item	Cost
service fee	\$4.95 per month
incoming calls	\$0.07 per minute

- A \$7.61
 B \$12.51
 C \$22.41
 D \$37.80

49. **REVIEW** If each dimension of the prism is tripled, which expression represents the new volume?



- F $x^3y^3z^3$
 G $3xyz$
 H $3(x + y + z)$
 J $27xyz$

Spiral Review

Name the property illustrated by each statement or equation. (Lesson 1-4)

50. If $7 \cdot 2 = 14$, then $14 = 7 \cdot 2$. 51. $mnp = 1mnp$
 52. $\frac{3}{4} \cdot \frac{4}{3} = 1$ 53. $32 + 21 = 32 + 21$

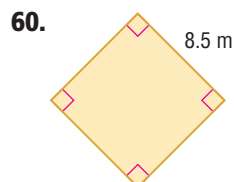
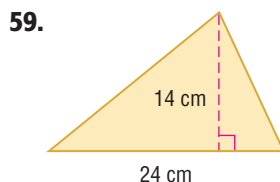
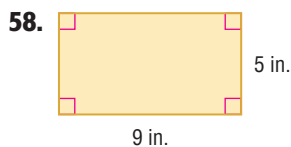
54. **PHYSICAL SCIENCE** Sound travels through air at an approximate speed of 344 meters per second. Write and solve an equation to find how far sound travels through air in 2 seconds. (Lesson 1-3)

Evaluate each expression if $a = 4$, $b = 6$, and $c = 3$. (Lesson 1-2)

55. $3b - c$ 56. $8(a - c)^2 + 3$ 57. $\frac{6ab}{2(c + 5)}$

GET READY for the Next Lesson

PREREQUISITE SKILL Find the area of each figure. (Pages 704–705)



Write an algebraic expression for each verbal expression. (Lesson 1-1)

- the quotient of y and 3
- 5 minus the product of 7 and t
- x squared increased by 2

Write a verbal expression for each algebraic expression. (Lesson 1-1)

- $5n + 2$
- a^3

- GOLF** At a driving range, a small bucket of golf balls costs \$6 and a large bucket costs \$8. Write an expression for the total cost of buying s small and t large buckets. (Lesson 1-1)

- MULTIPLE CHOICE** Jasmine bought a satellite radio receiver and a subscription to satellite radio. What was her total cost after 7 months? (Lesson 1-2)

Satellite Radio	
Item	Cost
receiver	\$78
subscription	\$12.95 per month

- A \$90.95 C \$558.95
 B \$168.65 D \$636.65

Evaluate each expression. (Lesson 1-2)

- $2 + 18 \div 9$
- $6 \cdot 9 - 2(8 + 5)$
- $9(3) - 4^2$
- $\frac{(5 - 2)^2}{4 \times 2 - 7}$
- Evaluate $\frac{5a^2 + c - 2}{6 + b}$ if $a = 4$, $b = 5$, and $c = 10$. (Lesson 1-2)

Find the solution of each equation if the replacement set is {10, 11, 12, 13}. (Lesson 1-3)

- $x - 3 = 10$
- $25 = 2r + 1$
- $\frac{t}{5} = 2$
- $4y - 9 = 35$
- Find the solution set for $2n^2 + 3 \leq 75$ if the replacement set is {4, 5, 6, 7, 8, 9}. (Lesson 1-3)

- MULTIPLE CHOICE** Dion bought 1 pound of dried greens, 3 pounds of sesame seeds, and 2 pounds of flax seed to feed his birds. According to the table, which expression best represents the total cost? (Lesson 1-3)

Bird Food	Cost (\$)
dried greens (0.5 lb)	4.95
sesame seeds (1 lb)	5.75
flax seed (2 lb)	2.75

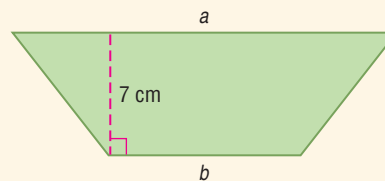
- F $4.95 + 5.75 + 2.75$
 G $4.95 + 3(5.75) + 2(2.75)$
 H $2(4.95) + 3(5.75) + 2.75$
 J $0.5(4.95) + 5.75 + 2(2.75)$

Find the value of n in each equation. Then name the property that is used. (Lesson 1-4)

- $n = 11 + 0$
- $\frac{1}{3} \cdot 3 = n$

- GEOMETRY** The expression $\frac{1}{2}(7)(a + b)$ represents the area of the trapezoid.

What is the area if $a = 22.4$ centimeters and $b = 10.8$ centimeters? (Lesson 1-5)



- MULTIPLE CHOICE** Which expression represents the second step of simplifying the algebraic expression? (Lesson 1-5)

Step 1 $9(x + 4y) + 5 + 2(x + 7)$

Step 2

Step 3 $11x + 36y + 19$

- A $9x + 36y + 7(x + 7)$
 B $9x + 36y + 5 + 2x + 14$
 C $9(x + 4y + 5) + 2x + 14$
 D $9(x + 5) + 4y + 2x + 14$

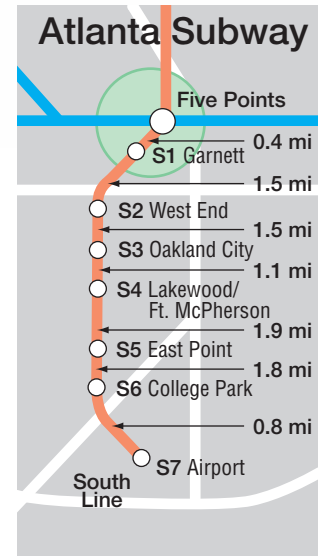
Commutative and Associative Properties

Main Ideas

- Recognize the Commutative and Associative Properties.
- Use the Commutative and Associative Properties to simplify algebraic expressions.

GET READY for the Lesson

The South Line of the Atlanta subway leaves Five Points and heads for Garnett and then West End. The distance from Five Points to West End can be found by evaluating $0.4 + 1.5$. Likewise, the distance from West End to Five Points can be found by evaluating $1.5 + 0.4$.



Commutative and Associative Properties In the situation above, the distance from Five Points to West End is the same as the distance from West End to Five Points.

$$\begin{array}{ccc} \text{The distance from Five} & & \text{the distance from West} \\ \text{Points to West End} & \text{equals} & \text{End to Five Points.} \\ 0.4 + 1.5 & = & 1.5 + 0.4 \end{array}$$



Vocabulary Link

Commute

Everyday Use to travel back and forth, as in commute to work

Commutative

Math Use property that allows you to change the order in which numbers are added or multiplied

This is an example of the **Commutative Property** for addition.

KEY CONCEPT

Commutative Property

Words The order in which you add or multiply numbers does not change their sum or product.

Symbols For any numbers a and b , $a + b = b + a$ and $a \cdot b = b \cdot a$.

Examples $5 + 6 = 6 + 5$, $3 \cdot 2 = 2 \cdot 3$



Vocabulary Link

Associate

Everyday Use to come together as in associate with friends

Associative

Math Use property that allows you to group three or more numbers when adding or multiplying

An easy way to find the sum or product of numbers is to group, or associate, the numbers using the **Associative Property**.

KEY CONCEPT

Associative Property

Words The way you group three or more numbers when adding or multiplying does not change their sum or product.

Symbols For any numbers a , b , and c , $(a + b) + c = a + (b + c)$ and $(ab)c = a(bc)$.

Examples $(2 + 4) + 6 = 2 + (4 + 6)$, $(3 \cdot 5) \cdot 4 = 3 \cdot (5 \cdot 4)$



Real-World EXAMPLE

Use Addition Properties



TRANSPORTATION Refer to the beginning of the lesson. Find the distance between Five Points and Lakewood/Ft. McPherson.

Five Points to Garnett	+	Garnett to West End	+	West End to Oakland City	+	Oakland City to Lakewood/Ft. McPherson
0.4		1.5		1.5		1.1

$$0.4 + 1.5 + 1.5 + 1.1 = 0.4 + 1.1 + 1.5 + 1.5$$

Commutative (+)

$$= (0.4 + 1.1) + (1.5 + 1.5)$$

Associative (+)

$$= 1.5 + 3.0 \text{ or } 4.5$$

Add mentally.

Lakewood/Ft. McPherson is 4.5 miles from Five Points.



CHECK Your Progress

Evaluate each expression using properties of numbers. Name the property used in each step.

1A. $35 + 17 + 5 + 3$

1B. $8\frac{3}{4} + 12 + 5\frac{1}{4}$



Real-World Link

New York City has the most extensive subway system, covering 842 miles of track and serving about 4.3 million passengers per day.

Source: *The Guinness Book of Records*

EXAMPLE

Use Multiplication Properties



Evaluate $8 \cdot 2 \cdot 3 \cdot 5$ using properties of numbers. Name the property used in each step.

$$8 \cdot 2 \cdot 3 \cdot 5 = 8 \cdot 3 \cdot 2 \cdot 5$$

Commutative (\times)

$$= (8 \cdot 3) \cdot (2 \cdot 5)$$

Associative (\times)

$$= 24 \cdot 10 \text{ or } 240$$

Multiply mentally.



CHECK Your Progress

Evaluate each expression using properties of numbers. Name the property used in each step.

2A. $2.9 \cdot 4 \cdot 10$

2B. $\frac{5}{3} \cdot 25 \cdot 3 \cdot 2$

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Simplify Expressions The Commutative and Associative Properties can be used with other properties when evaluating and simplifying expressions.

CONCEPT SUMMARY		Properties of Numbers
The following properties are true for any numbers a , b , and c .		
Properties	Addition	Multiplication
Commutative	$a + b = b + a$	$ab = ba$
Associative	$(a + b) + c = a + (b + c)$	$(ab)c = a(bc)$
Identity	0 is the identity. $a + 0 = 0 + a = a$	1 is the identity. $a \cdot 1 = 1 \cdot a = a$
Zero	—	$a \cdot 0 = 0 \cdot a = 0$
Distributive	$a(b + c) = ab + ac$ and $(b + c)a = ba + ca$	
Substitution	If $a = b$, then a may be substituted for b .	



EXAMPLE Write and Simplify an Expression

3 Use the expression *four times the sum of a and b increased by twice the sum of a and $2b$* .

a. Write an algebraic expression for the verbal expression.

Words	four times the sum of a and b increased by twice the sum of a and $2b$
Variables	Let a and b represent the numbers.
Expression	$4(a + b) + 2(a + 2b)$

b. Simplify the expression and indicate the properties used.

$$\begin{aligned}
 4(a + b) + 2(a + 2b) &= 4(a) + 4(b) + 2(a) + 2(2b) && \text{Distributive Property} \\
 &= 4a + 4b + 2a + 4b && \text{Multiply.} \\
 &= 4a + 2a + 4b + 4b && \text{Commutative (+)} \\
 &= (4a + 2a) + (4b + 4b) && \text{Associative (+)} \\
 &= (4 + 2)a + (4 + 4)b && \text{Distributive Property} \\
 &= 6a + 8b && \text{Substitution}
 \end{aligned}$$

CHECK Your Progress

- 3A. Write an algebraic expression for *5 times the difference of q squared and r plus 8 times the sum of $3q$ and $2r$* . Then simplify the expression and indicate the properties used.
- 3B. Simplify $6(x - 2y) + 4(-3x + y)$ and indicate the properties used.

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CHECK Your Understanding

Examples 1, 2
(p. 34)

Evaluate each expression using properties of numbers. Name the property used in each step.

1. $14 + 18 + 26$

2. $3\frac{1}{2} + 4 + 2\frac{1}{2}$

3. $5 \cdot 3 \cdot 6 \cdot 4$

4. $\frac{5}{6} \cdot 16 \cdot \frac{3}{4}$

Example 3
(p. 35)

Simplify each expression.

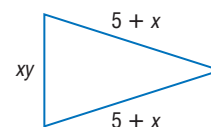
5. $4x + 5y + 6x$

6. $5a + 3b + 2a + 7b$

7. $3(4x + 2) + 2x$

8. $7(ac + 2b) + 2ac$

9. **GEOMETRY** Find the perimeter of the triangle.



10. Write an algebraic expression for *half the sum of p and $2q$ increased by three-fourths q* . Then simplify, indicating the properties used.

Exercises

HOMEWORK HELP

For Exercises	See Examples
11–16	1
17–22	2
23–34	3

Evaluate each expression using properties of numbers. Name the property used in each step.

11. $17 + 6 + 13 + 24$

12. $8 + 14 + 22 + 9$

13. $4.25 + 3.50 + 8.25$

14. $6.2 + 4.2 + 4.3 + 5.8$

15. $6\frac{1}{2} + 3 + \frac{1}{2} + 2$

16. $2\frac{3}{8} + 4 + 3\frac{3}{8}$

17. $5 \cdot 11 \cdot 4 \cdot 2$

18. $3 \cdot 10 \cdot 6 \cdot 3$

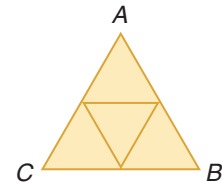
19. $0.5 \cdot 2.4 \cdot 4$

20. $8 \cdot 1.6 \cdot 2.5$

21. $3\frac{3}{7} \cdot 14 \cdot 1\frac{1}{4}$

22. $2\frac{5}{8} \cdot 24 \cdot 6\frac{2}{3}$

23. **GEOMETRY** Find the area of $\triangle ABC$ if each small triangle has a base of 5.2 inches and a height of 4.5 inches.



24. **GEOMETRY** A regular hexagon measures $(3x + 5)$ units on each side. What is the perimeter?

Simplify each expression.

25. $4a + 2b + a$

26. $2y + 2x + 8y$

27. $x^2 + 3x + 2x + 5x^2$

28. $4a^3 + 6a + 3a^3 + 8a$

29. $6x + 2(2x + 7)$

30. $4(3n + 9) + 5n$

Write an algebraic expression for each verbal expression. Then simplify, indicating the properties used.

31. twice the sum of s and t decreased by s

32. 5 times the product of x and y increased by $3xy$

33. the product of 6 and the square of z , increased by the sum of 7, z^2 , and 6

34. 6 times the sum of x and y squared minus 3 times the sum of x and y squared

Simplify each expression.

35. $\frac{1}{4}q + 2q + 2\frac{3}{4}q$

36. $3.2(x + y) + 2.3(x + y) + 4x$

37. $3(4m + n) + 2m(4 + n)$

38. $6(0.4f + 0.2g) + 0.5f$

39. $\frac{3}{4} + \frac{2}{3}(s + 2t) + s$

40. $2p + \frac{3}{5}\left(\frac{1}{2}p + 2q\right) + \frac{2}{3}$

41. **ANALYZE TABLES** A traveler checks into a hotel on Friday and checks out the following Tuesday morning. Use the table to find the total cost including tax.

Hotel Rates Per Day		
Day	Room Charge	Sales Tax
Monday–Friday	\$72	\$5.40
Saturday–Sunday	\$63	\$5.10

SCUBA DIVING For Exercises 42 and 43, use the following information.

A scuba diving store rents air tanks for \$7.50, dive flags for \$5.00, and wet suits for \$10.95. The store also sells disposable underwater cameras for \$18.99.

42. Write two expressions to represent the total sales after renting 2 wet suits, 3 air tanks, 2 dive flags, and selling 5 underwater cameras.

43. What are the total sales?



Real-World Link

In 1943, Jacques Cousteau and Emile Gagnan invented the *aqualung*, or SCUBA (Self-Contained Underwater Breathing Apparatus).

Source: howstuffworks.com

H.O.T. Problems

EXTRA PRACTICE
See pages 718, 744.

Math online
Self-Check Quiz at algebra1.com

- 44. CHALLENGE** Does the Commutative Property *sometimes, always, or never* hold for subtraction? Explain your reasoning.
- 45. Which One Doesn't Belong?** Identify the sentence that does not belong with the other three. Explain your reasoning.
- $x + 12 = 12 + x$

$7h = h \cdot 7$

$1 + a = a + 1$

$n \div 2 = 2 \div n$
- 46. OPEN ENDED** Write examples of the Commutative Property of Addition and the Associative Property of Multiplication using the numbers 1, 5, and 8 in each. Justify your examples.
- 47. Writing in Math** Use the information about subways on page 33 to explain how the Commutative and Associative Properties are useful in performing calculations. Include an expression using the properties that could help you find the distance from the airport to Five Points Station.

STANDARDIZED TEST PRACTICE

- 48.** Which expression is equivalent to $4 + 6(ac + 2b) + 2ac$?
- A $4 + 8ac + 12b$
 B $4 + 10ab + 2ac$
 C $4 + 12abc + 2ac$
 D $12ac + 20b$
- 49. REVIEW** Daniel is buying a jacket that is regularly \$59.99 and is on sale for $\frac{1}{3}$ off. Which expression can he use to estimate the discount on the jacket?
- F 0.0003×60
 G 0.003×60
 H 0.03×60
 J 0.33×60

Spiral Review

Simplify each expression. If not possible, write *simplified*. (Lesson 1-5)

- 50.** $5x - 8 + 7x$ **51.** $7m + 6n + 8$ **52.** $t^2 + 2t + 4t$
53. $3(5 + 2p)$ **54.** $(a + 2b)3 - 3a$ **55.** $(d + 5)8 + 2f$

56. Evaluate $3(5 - 5 \cdot 1^2) + 21 \div 7$. Name the property used in each step. (Lesson 1-4)

- 57. LAUNDRY** Jonathan is meeting friends for dinner in 3 hours and he wants to do laundry beforehand. If it takes 50 minutes to do each load of laundry, write and use an inequality to find the maximum number of loads that he can finish. (Lesson 1-3)

GET READY for the Next Lesson

PREREQUISITE SKILL Evaluate each expression. (Lesson 1-2)

- 58.** If $x = 4$, then $2x + 7 = ?$. **59.** If $x = 8$, then $6x + 12 = ?$.
60. If $n = 6$, then $5n - 14 = ?$. **61.** If $n = 7$, then $3n - 8 = ?$.

READING MATH

Arguments with Properties of Real Numbers

A lawyer who is presenting a case in court pays careful attention to make statements that are accurate, that follow a logical order, and that are justified.

In writing an argument that uses many steps, it is important to evaluate each step to check for errors. It is also important to provide the correct justification for each statement.

Example

Manuel has simplified the expression $3y + 5(x + y) - 3(y - x) + 2x$ and listed the properties used in each step. Evaluate each step. Determine whether the solution is accurate. If not, indicate the correct steps.

Step 1	$3y + 5(x + y) - 3(y - x) + 2x$	Original expression
Step 2	$3y + 5x + 5y - 3y + 3x + 2x$	Distributive Property
Step 3	$3y + 5y - 3y + 5x + 3x + 2x$	Commutative Property of Addition
Step 4	$(3y + 5y - 3y) + (5x + 3x + 2x)$	Commutative Property of Addition
Step 5	$(3 + 5 - 3)y + (5 + 3 + 2)x$	Distributive Property
Step 6	$5y + 10x$	Substitution

Manuel's solution is accurate until Step 4. The property used in this step is the Associative Property of Addition, not the Commutative Property of Addition. There are no other errors.

READING TO LEARN

Evaluate each step. Determine whether the solution is accurate. If not, indicate the correct steps.

1. Step 1	$3(2 + x) + 4(x - 8) - 3x$	Original expression
Step 2	$3(2) + 3(x) + 4(x) + 4(-8) - 3x$	Distributive Property
Step 3	$6 + 3x + 4x - 32 - 3x$	Multiply.
Step 4	$3x + 4x - 3x + 6 - 32$	Commutative Property of Addition
Step 5	$(3x + 4x - 3x) + 6 - 32$	Associative Property of Addition
Step 6	$(1 + 4 - 3)x + 6 - 32$	Distributive Property
Step 7	$2x - 26$	Substitution
2. Step 1	$4(3b - 2a) + 3(a + b) + b + 2(a - b)$	Original expression
Step 2	$12b - 2a + 3a + 3b + b + 2a - 2b$	Distributive Property
Step 3	$12b + 3b + b - 2b - 2a + 3a + 2a$	Commutative Property of Addition
Step 4	$(12b + 3b + b - 2b) + (-2a + 3a + 2a)$	Associative Property of Addition
Step 5	$(12 + 3 + 1 - 2)b + (-2 + 3 + 2)a$	Distributive Property
Step 6	$14b + 3a$	Substitution



Logical Reasoning and Counterexamples

Main Ideas

- Identify the hypothesis and conclusion in a conditional statement.
- Use a counterexample to show that an assertion is false.

New Vocabulary

conditional statement
if-then statement
hypothesis
conclusion
deductive reasoning
counterexample

Reading Math

If-Then Statements

Note that “if” is not part of the hypothesis and “then” is not part of the conclusion.

GET READY for the Lesson

The directions at the right can help you make perfect popcorn.

If the popcorn burns, then the heat was too high or the kernels heated unevenly.

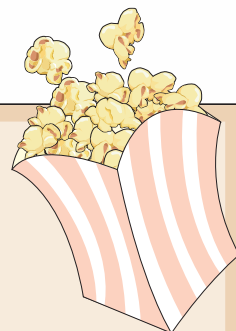
Stovetop Popping

To pop popcorn on a stovetop, you need:

- A 3- to 4-quart pan with a loose lid that allows steam to escape
- Enough popcorn to cover the bottom of the pan, one kernel deep
- 1/4 cup of oil for every cup of kernels

Heat the oil to 400–460°F (if the oil smokes, it is too hot). Test the oil on a couple of kernels. When they pop, add the rest of the popcorn, cover the pan, and shake to spread the oil. When the popping begins to slow, remove the pan from the stovetop.

Source: Popcorn Board



Conditional Statements The statement *If the popcorn burns, then the heat was too high or the kernels heated unevenly* is called a conditional statement.

Conditional statements can be written in the form *If A, then B*.

Statements in this form are called **if-then statements**.

If A , then B .
If $\underbrace{\hspace{2cm}}_{\text{the popcorn burns,}}$ then $\underbrace{\hspace{2cm}}_{\text{the heat was too high or the kernels heated unevenly.}}$

The part of the statement immediately following *if* is called the **hypothesis**.

The part of the statement immediately following *then* is called the **conclusion**.

EXAMPLE Identify Hypothesis and Conclusion

1 Identify the hypothesis and conclusion of each statement.

- a. **ENTERTAINMENT** If it is Friday, then Ofelia and Miguel are going to the movies.

The hypothesis follows the word *if* and the conclusion follows the word *then*.

Hypothesis: it is Friday

Conclusion: Ofelia and Miguel are going to the movies

- b. If $4x + 3 > 27$, then $x > 6$.

Hypothesis: $4x + 3 > 27$ Conclusion: $x > 6$

CHECK Your Progress

- 1A. If it is warm this afternoon, then we will have the party outside.
1B. If $8w - 5 = 11$, then $w = 2$.



Real-World Link

In 2005, more than 74 million people attended a Major League baseball game.

Source: ballparksbaseball.com

Sometimes a conditional statement is written without using the words *if* and *then*. But a conditional statement can always be rewritten in if-then form.

EXAMPLE Write a Conditional in If-Then Form

2 Identify the hypothesis and conclusion of each statement. Then write each statement in if-then form.

a. I will go to the ball game with you on Saturday.

Hypothesis: it is Saturday

Conclusion: I will go to the ball game with you

If it is Saturday, then I will go to the ball game with you.

b. For a number x such that $6x - 8 = 16$, $x = 4$.

Hypothesis: $6x - 8 = 16$

Conclusion: $x = 4$

If $6x - 8 = 16$, then $x = 4$.

CHECK Your Progress

2A. Brianna wears goggles when she is swimming.

2B. A rhombus with side lengths of $(x - y)$ units has a perimeter of $(4x - 4y)$ units.

Deductive Reasoning and Counterexamples **Deductive reasoning** is the process of using facts, rules, definitions, or properties to reach a valid conclusion. Suppose you have a true conditional and you know that the hypothesis is true for a given case. Deductive reasoning allows you to say that the conclusion is true for that case.

EXAMPLE Deductive Reasoning

3 Determine a valid conclusion that follows from the statement below for each condition. If a valid conclusion does not follow, write *no valid conclusion* and explain why.

"If two numbers are odd, then their sum is even."

a. The two numbers are 7 and 3.

7 and 3 are odd, so the hypothesis is true.

Conclusion: The sum of 7 and 3 is even.

CHECK $7 + 3 = 10$ ✓ The sum, 10, is even.

b. The sum of two numbers is 14.

The conclusion is true. If the numbers are 11 and 3, the hypothesis is true also. However, if the numbers are 8 and 6, the hypothesis is false. There is no way to determine the two numbers. Therefore, there is no valid conclusion.

CHECK Your Progress

Determine a valid conclusion that follows from the statement *"There will be a quiz every Wednesday."*

3A. It is Wednesday.

3B. It is Tuesday.

Study Tip

Common Misconception

Suppose the conclusion of a conditional is true. This does not mean that the hypothesis is true. Consider the conditional "If it rains, Annie will stay home." If Annie stays home, it does not necessarily mean that it is raining.

To show that a conditional is false, we can use a counterexample. A **counterexample** is a specific case in which the hypothesis is true and the conclusion is false. For example, consider the conditional *if a triangle has a perimeter of 3 centimeters, then each side measures 1 centimeter*. A counterexample is a triangle with perimeter 3 and sides 0.9, 0.9, and 1.2 centimeters long. It takes only one counterexample to show that a statement is false.

STANDARDIZED TEST EXAMPLE

- 4 Rachel believes that if $x \div y = 1$, then x and y are whole numbers. José states that this theory is not always true. Which pair of values for x and y could José use to disprove Rachel's theory?
- A $x = 2, y = 2$ C $x = 0.25, y = 0.25$
 B $x = 1.2, y = 0.6$ D $x = 6, y = 3$

Read the Test Item

The question is asking for a counterexample. Find the values of x and y that make the statement false.

Solve the Test Item

Replace x and y in the equation $x \div y = 1$ with the given values.

A $x = 2, y = 2$
 $2 \div 2 \stackrel{?}{=} 1$
 $1 = 1 \checkmark$

The hypothesis is true, and both values are whole numbers. The statement is true.

B $x = 1.2, y = 0.6$
 $1.2 \div 0.6 \stackrel{?}{=} 1$
 $2 \neq 1$

The hypothesis is false, and the conclusion is false. This is not a counterexample.

C $x = 0.25, y = 0.25$
 $0.25 \div 0.25 \stackrel{?}{=} 1$
 $1 = 1 \checkmark$

The hypothesis is true, but 0.25 is not a whole number. Thus, the statement is false.

D $x = 6, y = 3$
 $6 \div 3 \stackrel{?}{=} 1$
 $2 \neq 1$

The hypothesis is false. Therefore, this is not a counterexample even though the conclusions are true.

The only values that prove the statement false are $x = 0.25$ and $y = 0.25$. So these numbers are counterexamples. The answer is C.

Test-Taking Tip

Checking Results
 Since choice C is the correct answer, you can check your results by testing the other values.

CHECK Your Progress

4. Which numbers disprove the statement below?

If $x + y > xy$, then $x > y$.

- F $x = 1, y = 2$ G $x = 2, y = 3$ H $x = 4, y = 1$ J $x = 4, y = 2$

Example 1
(p. 39)

Identify the hypothesis and conclusion of each statement.

- If it is April, then it might rain.
- If you play tennis, then you run fast.
- If $34 - 3x = 16$, then $x = 6$.

Example 2
(p. 40)

Identify the hypothesis and conclusion of each statement. Then write each statement in if-then form.

- Colin watches television when he does not have homework.
- A number that is divisible by 10 is also divisible by 5.
- A rectangle is a quadrilateral with four right angles.

Example 3
(p. 40)

Determine a valid conclusion that follows from the statement below for each given condition. If a valid conclusion does not follow, write *no valid conclusion* and explain why.

If the last digit of a number is 2, then the number is divisible by 2.

- The number is 10,452.
- The number is divisible by 2.
- The number is 946.

Example 4
(p. 41)

Find a counterexample for each conditional statement.

- If Anna is in school, then she has a science class.
- If you can read 8 pages in 30 minutes, then you can read a book in a day.
- If a number x is squared, then $x^2 > x$.
- If $3x + 7 \geq 52$, then $x > 15$.
- STANDARDIZED TEST PRACTICE** Which number disproves the statement $x < 2x$?
A 0 B 1 C 2 D 4

Exercises

HOMEWORK HELP	
For Exercises	See Examples
15–18	1
19–24	2
25–28	3
29–38	4

Identify the hypothesis and conclusion of each statement.

- If both parents have red hair, then their children have red hair.
- If you are in Hawaii, then you are in the tropics.
- If $2n - 7 > 25$, then $n > 16$.
- If $a = b$ and $b = c$, then $a = c$.

Identify the hypothesis and conclusion of each statement. Then write each statement in if-then form.

- The trash is picked up on Monday.
- Vito will call after school.
- For $x = 8$, $x^2 - 3x = 40$.
- $4s + 6 > 42$ when $s > 9$.
- A triangle with all sides congruent is an equilateral triangle.
- The sum of the digits of a number is a multiple of 9 when the number is divisible by 9.

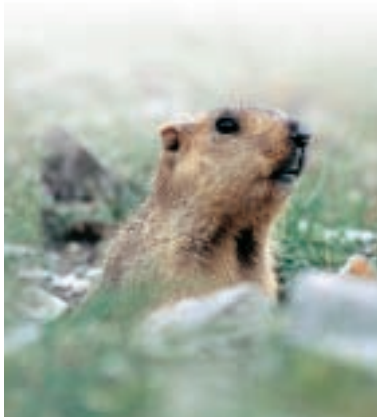
Determine whether a valid conclusion follows from the statement below for each given condition. If a valid conclusion does not follow, write *no valid conclusion* and explain why.

If a DVD box set costs less than \$70, then Ian will buy one.

25. A DVD box set costs \$59. 26. A DVD box set costs \$89.
 27. Ian will not buy a DVD box set. 28. Ian bought 2 DVD box sets.

Find a counterexample for each conditional statement.

29. If you were born in North Carolina, then you live in North Carolina.
 30. If you are a professional basketball player, then you play in the United States.
 31. If the product of two numbers is even, then both numbers must be even.
 32. If two times a number is greater than 16, then the number must be greater than 7.
 33. If $4n - 8 \geq 52$, then $n > 15$.
 34. If $x \cdot y = 1$, then x or y must equal 1.



Real-World Link

Groundhog Day has been celebrated in the United States since 1897. The most famous groundhog, Punxsutawney Phil, has seen his shadow about 85% of the time.

Source: www.infoplease.com

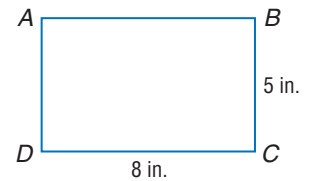
GEOMETRY For Exercises 35 and 36, use the following information. If points P , Q , and R lie on the same line, then Q is between P and R .



35. Copy the diagram. Label the points so that the conditional is true.
 36. Copy the diagram. Provide a counterexample for the conditional.

Determine whether a valid conclusion follows from the statement below for each given condition. If a valid conclusion does not follow, write *no valid conclusion* and explain why.

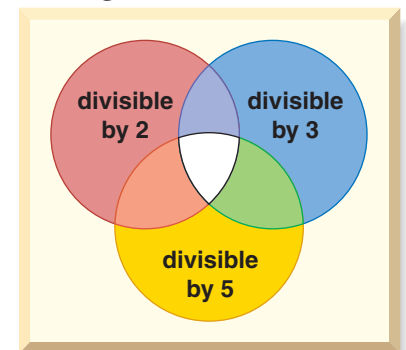
If the dimensions of rectangle ABCD are doubled, then the perimeter is doubled.



37. The new rectangle measures 16 inches by 10 inches.
 38. The perimeter of the new rectangle is 52 inches.
 39. **RESEARCH** On Groundhog Day (February 2) of each year, some people say that if a groundhog comes out of its hole and sees its shadow, then there will be 6 more weeks of winter weather. If it does not see its shadow, then there will be an early spring. Use the Internet or another resource to research the weather on Groundhog Day for your city for the past 10 years. Summarize your data as examples or counterexamples for this belief.

NUMBER THEORY For Exercises 40–42, use the following information.

Copy the Venn diagram and place the numbers 1 to 25 in the appropriate places on the diagram.



40. What conclusions can you make about the numbers and where they appear on the diagram?
 41. What conclusions can you form about numbers that are divisible by both 2 and 3?
 42. Provide a counterexample for the data you have collected if possible.

EXTRA PRACTICE
 See pages 719, 744.
Math online
 Self-Check Quiz at algebra1.com

H.O.T. Problems

43. CHALLENGE Determine whether the following statement is always true. If it is not, provide a counterexample.

If the mathematical operation $$ is defined for all numbers a and b as $a * b = a + 2b$, then the operation $*$ is commutative.*

44. OPEN ENDED Write a conditional statement and label the hypothesis and conclusion. Describe how conditional statements are used to solve problems.

45. REASONING Explain how deductive reasoning is used to show that a conditional is true or false.

46. Writing in Math Use the information about popcorn found on page 39 to explain how logical reasoning is helpful in cooking. Include in your answer the hypothesis and conclusion of the statement *If you have small, underpopped kernels, then you have not used enough oil in your pan.*

STANDARDIZED TEST PRACTICE

47. Which number serves as a counterexample to the statement below?

$$2x < 3x$$

- A -2 B $\frac{1}{4}$ C $\frac{1}{2}$ D 2

48. REVIEW If $4a = a$, which of the following is true?

- F $a > 4$ H $a = 1$
G $a = 4$ J $a = 0$

49. What value of n makes the following statement true?

If $14n - 12 \geq 100$, then $n \geq$ _____.

- A 8
B 10
C 12
D 24

Spiral Review

Simplify each expression. (Lesson 1-6)

50. $2x + 5y + 9x$

51. $4(5mn + 6) + 3mn$

52. $2(3a + b) + 3b + 4$

53. ENVIRONMENT A typical family of four uses the water shown in the table. Write two expressions that represent the amount of water a typical family of four uses for these activities in d days. (Lesson 1-5)

Average Water Usage Per Day	
Activity	Gallons Used
flushing toilet	100
showering/bathing	80
using bathroom sink	8

Source: U.S. Environmental Protection Agency

Find the value of n in each expression. Then name the property used. (Lesson 1-4)

54. $1(n) = 64$

55. $12 + 7 = 12 + n$

56. $(9 - 7)5 = 2n$

57. $4n = 1$

58. $n + 18 = 18$

59. $36n = 0$

GET READY for the Next Lesson

PREREQUISITE SKILL Evaluate each expression. (Lesson 1-2)

60. 6^2

61. $(-8)^2$

62. 1.6^2

63. $(-11.5)^2$

Algebra Lab

Logic and Properties of Numbers

You can apply what you have learned about the properties of numbers to determine whether a mathematical statement is *always*, *sometimes*, or *never* true.

ACTIVITY 1

Determine whether $2y \leq 6$ is *always*, *sometimes*, or *never* true.

First determine the greatest value of y that satisfies the inequality.

$$2(3) \leq 6 \quad \text{Substitute 3 for } y.$$

$$6 \leq 6 \quad \text{Simplify.}$$

Next, substitute a value less than 3 for y and a value greater than 3.

$$2(2) \leq 6 \quad \text{Substitute 2 for } y. \qquad 2(4) \leq 6 \quad \text{Substitute 4 for } y.$$

$$4 \leq 6 \quad \text{Simplify.} \qquad 8 \not\leq 6 \quad \text{Simplify.}$$

Substituting 2 for y yields a true inequality. Therefore, the inequality is sometimes true, when $y \leq 3$. When $y > 3$, the inequality is not true.

ACTIVITY 2

Determine whether the following statement is *true* or *false*.

Use the properties of numbers to justify your answer.

The equation $y = 2(x + 4) - 3$ is negative when $x < 0$.

Substitute a negative value for x in the equation.

$$y = 2(x + 4) - 3 \quad \text{Original equation}$$

$$y = 2(-1 + 4) - 3 \quad \text{Substitute } -1 \text{ for } x.$$

$$y = -2 + 8 - 3 \quad \text{Distributive Property}$$

$$y = 3 \quad \text{Add.}$$

Since substituting -1 for x into the equation yields a positive value for y , the statement is false.

EXERCISES

Determine whether each statement or inequality is *always*, *sometimes*, or *never* true.

1. $3t > -6$

2. $-2v < 4$

3. $3w + 4 > 0$

Determine whether each statement is *true* or *false*. Use the properties of numbers to justify your answer.

4. In the equation $y = 3(x + 2)$, y is positive when $x > 0$.

5. In the linear equation $y = -2x$, y is always positive.

GET READY for the Lesson

Main Ideas

- Classify and graph real numbers.
- Find square roots and order real numbers.

New Vocabulary

positive number
negative number
natural number
whole number
integers
rational number
square root
perfect square
irrational numbers
real numbers
Closure Property
graph
coordinate
radical sign
principal square root
rational approximation

In the 2000 Summer Olympics, Australian sprinter Cathy Freeman wore a special running suit that covered most of her body. The surface area of the human body may be found using the expression

$\sqrt{\frac{\text{height} \times \text{weight}}{3600}}$, where height is in centimeters, weight is in kilograms, and surface area is in square meters. The symbol $\sqrt{\quad}$ designates a square root.

Classify and Graph Real Numbers A number line can be used to show the sets of natural numbers, whole numbers, and integers. Values greater than 0, or **positive numbers**, are listed to the right of 0, and values less than 0, or **negative numbers**, are listed to the left of 0.

natural numbers: 1, 2, 3, ...

whole numbers: 0, 1, 2, 3, ...

integers:
..., -3, -2, -1, 0, 1, 2, 3, ...

rational numbers: numbers that can be expressed in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$.

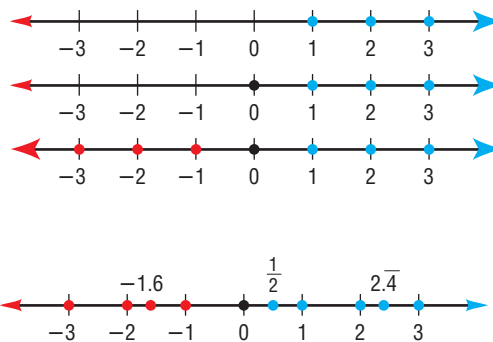
A rational number can also be expressed as a decimal that terminates, or as a decimal that repeats indefinitely.

A **square root** is one of two equal factors of a number. For example, one square root of 64, written as $\sqrt{64}$, is 8 since $8 \cdot 8$ or 8^2 is 64. Another square root of 64 is -8 since $(-8) \cdot (-8)$ or $(-8)^2$ is also 64. A number like 64, with a square root that is a rational number, is called a **perfect square**. The square roots of a perfect square are rational numbers.

A number such as $\sqrt{3}$ is the square root of a number that is *not* a perfect square. It cannot be expressed as a terminating or repeating decimal.

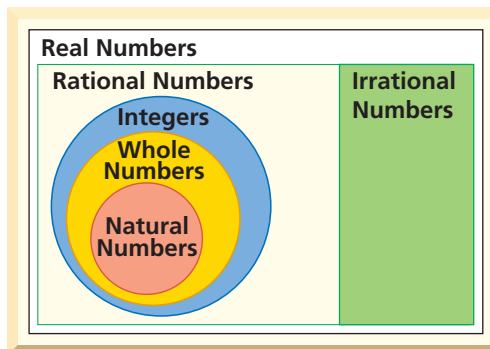
$$\sqrt{3} = 1.73205080\dots$$

Numbers that cannot be expressed as terminating or repeating decimals, or in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$, are called **irrational numbers**. Irrational numbers and rational numbers together form the set of **real numbers**.



Concepts in Motion

Animation at algebra1.com



Study Tip

Common Misconception

Pay close attention to the placement of a negative sign when working with square roots. $\sqrt{-81}$ is undefined for real numbers since no real number multiplied by itself can result in a negative product.

EXAMPLE Classify Real Numbers

1 Name the set or sets of numbers to which each real number belongs.

a. $\frac{5}{22}$

Because 5 and 22 are integers and $5 \div 22 = 0.2272727\dots$, which is a repeating decimal, this number is a rational number.

b. $\sqrt{81}$

Because $\sqrt{81} = 9$, this number is a natural number, a whole number, an integer, and a rational number.

c. $\sqrt{56}$

Because $\sqrt{56} = 7.48331477\dots$, which is not a repeating or terminating decimal, this number is irrational.

CHECK Your Progress

1A. $\frac{6}{11}$

1B. $-\sqrt{9.16}$

In Lesson 1-4, you learned about properties of real numbers. Another property of real numbers is the **Closure Property**. For example, the sum of any two whole numbers is a whole number. So, the set of whole numbers is said to be *closed* under addition.

EXAMPLE Closure Property

2 Determine whether each set of numbers is closed under the indicated operation.

a. whole numbers, multiplication

Select two different whole numbers and then determine whether the product is a whole number.

$$0 \times 4 = 0 \qquad 5 \times 2 = 10 \qquad 1 \times 6 = 6$$

Since the products of each pair of whole numbers are whole numbers, the set of whole numbers is closed under multiplication.

b. whole numbers, subtraction

We need to determine whether the difference of any two whole numbers is a whole number.

$$3 - 4 = -1$$

This is not a whole number, so the set of whole numbers is not closed under subtraction.

CHECK Your Progress

2A. integers, division

2B. integers, addition

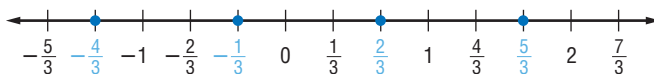
To **graph** a set of numbers means to draw, or plot, the points named by those numbers on a number line. The number that corresponds to a point on a number line is called the **coordinate** of that point. The rational numbers alone do not complete the number line. By including irrational numbers, the number line is complete.



EXAMPLE Graph Real Numbers

3 Graph each set of numbers.

a. $\left\{-\frac{4}{3}, -\frac{1}{3}, \frac{2}{3}, \frac{5}{3}\right\}$



b. $x > -2$



The heavy arrow indicates that all numbers to the right of -2 are included in the graph. Not only does this set include integers like 3 and -1 , but it also includes rational numbers like $\frac{3}{8}$ and $-\frac{12}{13}$ and irrational numbers like $\sqrt{40}$ and π . The circle at -2 indicates -2 is *not* included in the graph.

c. $a \leq 4.5$



The heavy arrow indicates that all points to the left of 4.5 are included in the graph. The dot at 4.5 indicates that 4.5 is included in the graph.

CHECK Your Progress

3A. $\{-5, -4, -3, -2, \dots\}$

3B. $x \leq 8$

Reading Math

Square Roots

$\pm\sqrt{64}$ is read *plus or minus the square root of 64*.

Exponents can also be used to indicate the square root. $9^{\frac{1}{2}}$ means the same thing as $\sqrt{9}$.

$9^{\frac{1}{2}}$ is read *nine to the one-half power*.

$9^{\frac{1}{2}} = 3$.

Square Roots and Ordering Real Numbers The symbol $\sqrt{\quad}$, called a **radical sign**, is used to indicate a nonnegative or **principal square root** of the expression under the radical sign.

$\sqrt{64} = 8$ ← $\sqrt{64}$ indicates the *principal* square root of 64.

$-\sqrt{64} = -8$ ← $-\sqrt{64}$ indicates the *negative* square root of 64.

$\pm\sqrt{64} = \pm 8$ ← $\pm\sqrt{64}$ indicates *both* square roots of 64.

EXAMPLE Find Square Roots

4 Find $-\sqrt{\frac{49}{256}}$.

$-\sqrt{\frac{49}{256}}$ represents the negative square root of $\frac{49}{256}$.

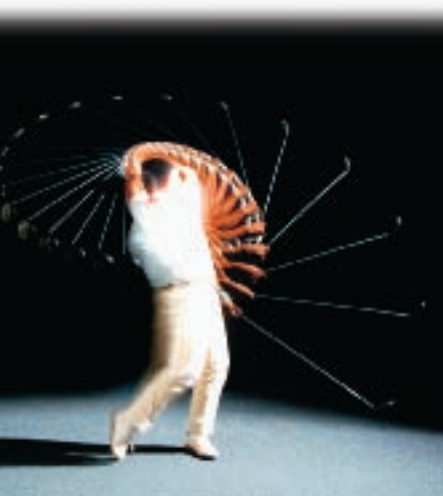
$$\sqrt{\frac{49}{256}} = \left(\frac{7}{16}\right)^2 \rightarrow -\sqrt{\frac{49}{256}} = -\frac{7}{16}$$

CHECK Your Progress

Find each square root.

4A. $\sqrt{\frac{4}{121}}$

4B. $\pm\sqrt{1.69}$



Real-World Career...

Scientist

Sports scientists use math to conduct experiments and interpret data. One of their jobs is to help athletes enhance their performance.



For more information, go to algebra1.com.

Real-World EXAMPLE

- 5 SPORTS SCIENCE** Refer to the application at the beginning of the lesson. Find the surface area of an athlete whose height is 192 centimeters and whose weight is 48 kilograms.

$$\begin{aligned} \text{surface area} &= \sqrt{\frac{\text{height} \times \text{weight}}{3600}} && \text{Write the formula.} \\ &= \sqrt{\frac{192 \times 48}{3600}} && \text{Replace height with 192 and weight with 48.} \\ &= \sqrt{\frac{9216}{3600}} && \text{Simplify.} \\ &= \sqrt{\left(\frac{96}{60}\right)^2} && \frac{9216}{3600} = \left(\frac{96}{60}\right)^2 \\ &= \frac{96}{60} \text{ or } 1.6 && \text{Simplify.} \end{aligned}$$

The surface area of the athlete is 1.6 square meters.

CHECK Your Progress

- 5.** Find the surface area of the athlete whose height is 200 centimeters and whose weight is 50 kilograms.

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To express irrational numbers as decimals, you need to use a rational approximation. A **rational approximation** of an irrational number is a rational number that is close to, but not equal to, the value of the irrational number. For example, a rational approximation of $\sqrt{2}$ is 1.41.

EXAMPLE Compare Real Numbers

- 6** Replace each \bullet with $<$, $>$, or $=$ to make each sentence true.

a. $\sqrt{19} \bullet 3.\bar{8}$

Find two perfect squares closest to $\sqrt{19}$, and write an inequality.

$$16 < 19 < 25 \quad \text{19 is between 16 and 25.}$$

$$\sqrt{16} < \sqrt{19} < \sqrt{25} \quad \text{Find the square root of each number.}$$

$$4 < \sqrt{19} < 5 \quad \sqrt{19} \text{ is between 4 and 5.}$$

Since $\sqrt{19}$ is between 4 and 5, it must be greater than $3.\bar{8}$.

$$\text{So, } \sqrt{19} > 3.\bar{8}.$$

b. $7.\bar{2} \bullet \sqrt{52}$

You can use a calculator to find an approximation for $\sqrt{52}$.

$$\sqrt{52} = 7.211102551\dots$$

$$7.\bar{2} = 7.222\dots \quad \text{Therefore, } 7.\bar{2} > \sqrt{52}.$$

CHECK Your Progress

6A. $2\frac{2}{3} \bullet \sqrt{5}$

6B. $0.\bar{8} \bullet \frac{8}{9}$

Study Tip

Ordering Real Numbers

When ordering real numbers, it is helpful to first simplify any square roots of perfect squares. For example,

$$\sqrt{\frac{25}{64}} = \frac{5}{8}.$$

To order a set of real numbers from greatest to least or from least to greatest, find a decimal approximation for each number in the set and compare.

EXAMPLE Order Real Numbers

1 Order $2.\overline{63}$, $-\sqrt{7}$, $\frac{8}{3}$, and $\frac{53}{-20}$ from least to greatest.

$$2.\overline{63} = 2.6363636\ldots \text{ or about } 2.636$$

$$-\sqrt{7} = -2.64575131\ldots \text{ or about } -2.646$$

$$\frac{8}{3} = 2.6666666\ldots \text{ or about } 2.667$$

$$\frac{53}{-20} = -2.65$$

$$-2.65 < -2.646 < 2.636 < 2.667$$

The numbers arranged in order from least to greatest are $\frac{53}{-20}$, $-\sqrt{7}$, $2.\overline{63}$, $\frac{8}{3}$.

CHECK Your Progress

Order each set of numbers from greatest to least.

7A. $\sqrt{0.42}$, $0.\overline{63}$, $\sqrt{\frac{4}{9}}$

7B. $-1.\overline{46}$, 0.2 , $\sqrt{2}$, $-\frac{1}{6}$

CHECK Your Understanding

Example 1
(p. 47)

Name the set or sets of numbers to which each real number belongs.

1. $-\sqrt{64}$

2. $\frac{8}{3}$

3. $\sqrt{28}$

4. $\frac{56}{7}$

Example 2
(p. 47)

Determine whether each set of numbers is closed under the indicated operation.

5. whole, division

6. rational, addition

7. rational, division

8. natural, subtraction

Example 3
(p. 48)

Graph each set of numbers.

9. $\{-4, -2, 1, 5, 7\}$

10. $x < -3.5$

11. $x \geq -7$

Example 4
(p. 48)

Find each square root.

12. $-\sqrt{25}$

13. $\sqrt{1.44}$

14. $\pm\sqrt{\frac{16}{49}}$

15. $\sqrt{361}$

Example 5
(p. 49)

16. PHYSICAL SCIENCE The time it takes a falling object to travel a distance d is

given by $t = \sqrt{\frac{d}{16}}$, where t is in seconds and d is in feet. If Isabel drops a

ball from 29.16 feet, how long will it take for it to reach the ground?

Example 6
(p. 49)

Replace each \bullet with $<$, $>$, or $=$ to make each sentence true.

17. $\sqrt{17} \bullet 4\frac{1}{10}$

18. $\frac{2}{9} \bullet 0.\overline{2}$

19. $\frac{1}{6} \bullet \sqrt{6}$

Example 7
(p. 50)

Order each set of numbers from least to greatest.

20. $\frac{1}{8}$, $\sqrt{\frac{1}{4}}$, $0.\overline{15}$, -15

21. $\sqrt{30}$, $5\frac{4}{9}$, 13 , $\sqrt{\frac{1}{30}}$

Exercises

HOMEWORK HELP	
For Exercises	See Examples
22–27	1
28–37	2
38–41	3
42–47	4, 5
48–51	6
52–55	7

Name the set or sets of numbers to which each real number belongs.

22. $-\sqrt{22}$ 23. $\frac{36}{6}$ 24. $-\frac{5}{12}$
25. $\sqrt{10.24}$ 26. $\frac{-54}{19}$ 27. $\sqrt{\frac{82}{20}}$

Determine whether each set of numbers is closed under the indicated operation.

28. irrational, addition 29. irrational, subtraction
30. natural, addition 31. natural, multiplication
32. irrational, multiplication 33. irrational, division
34. integers, subtraction 35. rational, subtraction
36. rational, multiplication 37. integers, multiplication

Graph each set of numbers.

38. $\{-4, -2, -1, 1, 3\}$ 39. $\{\dots -2, 0, 2, 4, 6\}$
40. $x > -12$ 41. $x \geq -10.2$

Find each square root.

42. $\sqrt{49}$ 43. $\pm\sqrt{0.64}$ 44. $\pm\sqrt{5.29}$
45. $-\sqrt{6.25}$ 46. $\sqrt{\frac{169}{196}}$ 47. $\sqrt{\frac{25}{324}}$

Replace each \bullet with $<$, $>$, or $=$ to make each sentence true.

48. $5.\overline{72} \bullet \sqrt{5}$ 49. $\sqrt{22} \bullet 4.7$ 50. $\sqrt{\frac{2}{3}} \bullet \frac{2}{3}$ 51. $8 \bullet \sqrt{67}$

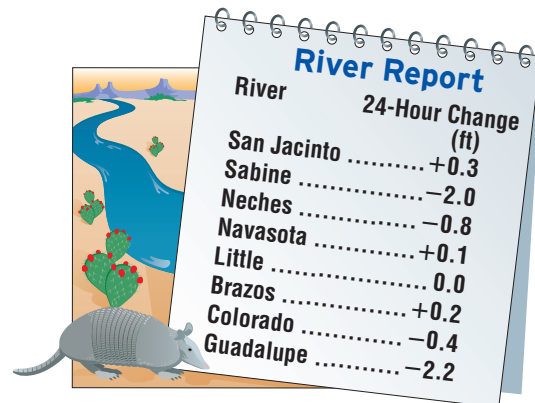
Order each set of numbers from least to greatest.

52. $\sqrt{0.06}, 0.24, \sqrt{\frac{9}{144}}$ 53. $0.6, \sqrt{\frac{16}{49}}, \frac{5}{9}$
54. $-4.\overline{83}, 0.4, \sqrt{8}, -\frac{3}{8}$ 55. $-0.25, 0.\overline{14}, -\sqrt{\frac{5}{8}}, \sqrt{0.5}$

Evaluate each expression if $a = 4$, $b = 9$, and $c = 100$.

56. $2 \cdot \sqrt{a}$ 57. $\sqrt{a} \cdot \sqrt{b}$ 58. $\sqrt{a \cdot b}$ 59. $\sqrt{a} + \sqrt{c}$

60. **RIVERS** The table shows the change in river depths for various rivers over a 24-hour period. Use a number line to graph these values and compare the changes in each river.



GEOMETRY For Exercises 61–63, consider squares with the following areas: 1 unit², 4 units², 9 units², 16 units², and 25 units².

61. Find the side length and perimeter of each square.
62. Describe the relationship between the lengths of the sides and the areas.
63. Write an expression to find the perimeter of a square with a area of a units².

EXTRA PRACTICE
See pages 719, 744.

Math online
Self-Check Quiz at algebra1.com

H.O.T. Problems

64. **REASONING** Determine whether the following statement is *true* or *false*. Justify your answer. *Since the natural numbers are a subset of the whole numbers then for each operation that the whole numbers are closed, the natural numbers are closed also.*
65. **CHALLENGE** Determine whether the following statement is *true* or *false*. Include an example or a counterexample in your answer. *The average of two irrational numbers is an irrational number.*
66. **CHALLENGE** Determine when the following statements are all true for real numbers q and r .
- a. $q^2 > r^2$ b. $\frac{1}{q} < \frac{1}{r}$ c. $\sqrt{q} > \sqrt{r}$ d. $\frac{1}{\sqrt{q}} < \frac{1}{\sqrt{r}}$
67. **OPEN-ENDED** Give a real-life example in which numbers are ordered.
68. *Writing in Math* Use the information on page 46 to explain how square roots can be used to find the surface area of the human body.

STANDARDIZED TEST PRACTICE

69. **REVIEW** Which is an irrational number?
- A -6
B $\frac{3}{2}$
C $-\sqrt{8}$
D $-\sqrt{4}$
70. For what value of a is $-\sqrt{a} < -\frac{1}{\sqrt{a}}$ true?
- F 2
G $\frac{1}{3}$
H 1
J -4

Spiral Review

Find a counterexample for each statement. (Lesson 1-7)

71. If the sum of two numbers is even, then both numbers must be even.
72. If $x^2 < 1$, then $x = 0$.

Simplify each expression. (Lesson 1-6)

73. $8x + 2y + x$ 74. $7(5a + 3b) - 4a$ 75. $4[1 + 4(5x + 2y)]$

MOVIE THEATERS For Exercises 76 and 77, use the following information. (Lesson 1-2)

One adult ticket at a movie theater costs \$8.50. One small popcorn costs \$3.50.

76. Write an algebraic expression to represent how much could be spent at the movie theater.
77. Evaluate the expression to find the total cost if Julio and his three friends each bought a ticket and popcorn.

GET READY for the Next Lesson

PREREQUISITE SKILL

78. Refer to the table. If the pattern continues, what are the values for y if $x = 6$ and $x = 7$?

x	0	1	2	3	4	5
y	2	5	8	11	14	17

Main Ideas

- Interpret graphs of functions.
- Draw graphs of functions.

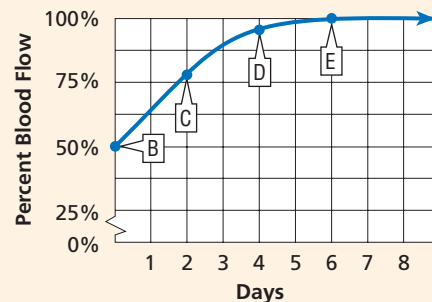
New Vocabulary

function
 coordinate system
 y-axis
 origin
 x-axis
 ordered pair
 x-coordinate
 y-coordinate
 independent variable
 dependent variable
 relation
 domain
 range
 discrete function
 continuous function

GET READY for the Lesson

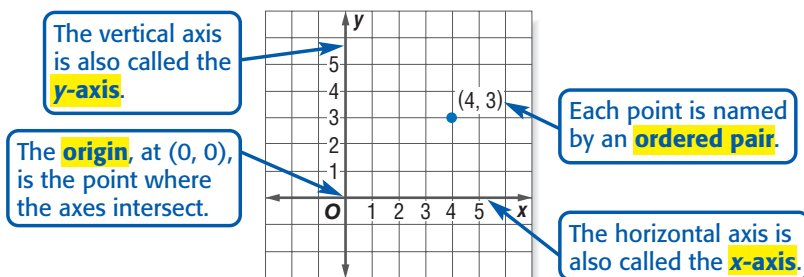
The graph shows that as the number of days after a concussion increases, the percent of blood flow increases.

The return of normal blood flow is said to be a function of the number of days since the concussion.

Blood Flow After Concussion

Source: *Scientific American*

Interpret Graphs A **function** is a relationship between input and output. In a function, the output depends on the input. There is exactly one output for each input. A function is graphed using a **coordinate system**, or *coordinate plane*. It is formed by the intersection of two number lines, the *horizontal axis* and the *vertical axis*.



Each input x and its corresponding output y is graphed using an ordered pair in the form (x, y) . The x -value, called the **x -coordinate** corresponds to the x -axis and the y -value, or **y -coordinate**, corresponds to the y -axis.

**Real-World EXAMPLE****Identify Coordinates**

- MEDICINE** Refer to the application above. Name the ordered pair at point C and explain what it represents.

Point C is at 2 along the x -axis and about 80 along the y -axis. So, its ordered pair is $(2, 80)$. This represents 80% normal blood flow 2 days after the injury.

**CHECK Your Progress**

1. Name the ordered pair at point E and explain what it represents.

Study Tip

Graphs of Functions

Usually the independent variable is graphed on the horizontal axis, and the dependent variable is graphed on the vertical axis.

In Example 1, the blood flow depends on the number of days from the injury. Therefore, the number of days from the injury is called the **independent variable** and the percent of normal blood flow is called the **dependent variable**.

EXAMPLE Independent and Dependent Variables

2 Identify the independent and dependent variables for each function.

a. In general, the average price of gasoline slowly and steadily increases throughout the year.

Time is the independent variable as it is unaffected by the price of gasoline, and the price is the dependent quantity as it is affected by time.

b. Art club members are drawing caricatures of students to raise money for their trip to New York City. The profit that they make increases as the price of their drawings increases.

In this case, price is the independent quantity. Profit is the dependent quantity as it is affected by the price.

CHECK Your Progress

2A. The distance a person runs increases with time.

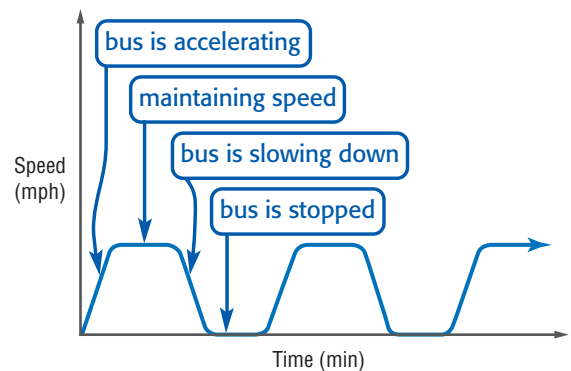
2B. As the dimensions of a square decrease, so does the area.

Functions can be graphed without using a scale on either axis to show the general shape of the graph.

EXAMPLE Analyze Graphs

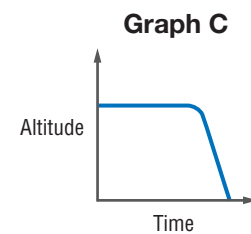
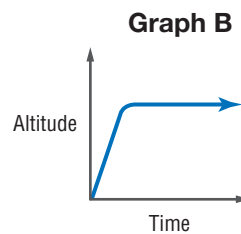
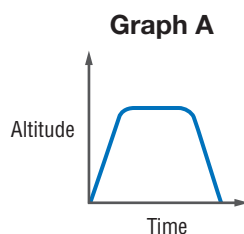
3 The graph at the right represents the speed of a school bus traveling along its morning route. Describe what is happening in the graph.

At the origin, the bus is stopped. It accelerates and maintains a constant speed. Then it begins to slow down, eventually stopping. After being stopped for a short time, the bus accelerates again. The process repeats continually.



CHECK Your Progress

3. Identify the graph that represents the altitude of a space shuttle above Earth, from the moment it is launched until the moment it lands.



Draw Graphs Graphs can be used to represent many real-world situations.

Real-World EXAMPLE Draw Graphs

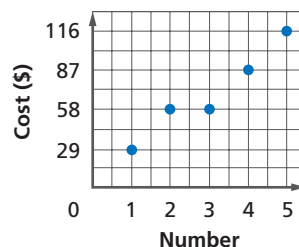
4 SHOPPING For every two pairs of earrings you buy at the regular price of \$29 each, you get a third pair free.

a. Make a table showing the cost of buying 1 to 5 pairs of earrings.

Pairs of Earrings	1	2	3	4	5
Total Cost (\$)	29	58	58	87	116

b. Write the data as a set of ordered pairs. Then graph the data.

Use the table. The number of pairs of earrings is the independent variable, and the cost is the dependent variable. So, the ordered pairs are (1, 29), (2, 58), (3, 58), (4, 87), and (5, 116).



Study Tip

Different Representations

Example 4 illustrates some of the ways data can be represented—tables, ordered pairs, and graphs.

CHECK Your Progress

- 4A.** Suppose you earn \$0.25 for each book that you sell. Make a table showing how much you earn when you sell 1 to 5 books.
- 4B.** Write the data as a set of ordered pairs. Then graph the data.

A set of ordered pairs, like those in Example 4, is called a **relation**. The set of the first numbers of the ordered pairs is the **domain**. The set of second numbers of the ordered pairs is the **range**. The function in Example 4 is a **discrete function** because its graph consists of points that are not connected. A function graphed with a line or smooth curve is a **continuous function**.

Study Tip

Domain and Range

The domain contains all values of the independent variable. The range contains all values of the dependent variable.

Real-World EXAMPLE Domain and Range

5 EXERCISE Rasha rides her bike an average of 0.25 mile per minute up to 36 miles each week. The distance that she travels each week is a function of the number of minutes that she rides.

a. Identify a reasonable domain and range for this situation.

The domain is the number of minutes that Rasha rides her bike. Since she rides up to 36 miles each week, she rides up to $36 \div 0.25$ or 144 minutes a week. A reasonable domain would be values from 0 to 144 minutes. The range is the weekly distance traveled. A reasonable range is 0 to 36 miles.

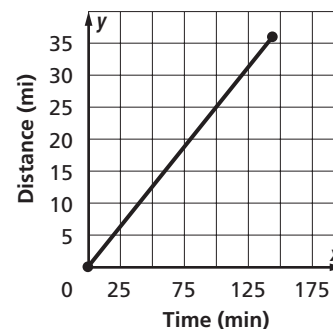
b. Draw a graph that shows the relationship.

Graph the ordered pairs (0, 0) and (36, 144). Since she rides any number of miles up to 36 miles, connect the two points with a line to include those points.

c. State whether the function is discrete or continuous. Explain.

Since the points are connected with a line, this function is continuous.

Rasha's Bike Riding



Cross-Curricular Project

Math Online A graph of the number of people over 65 in the U.S. for the years since 1900 will help you predict trends. Visit algebra1.com to continue work on your project.

CHECK Your Progress

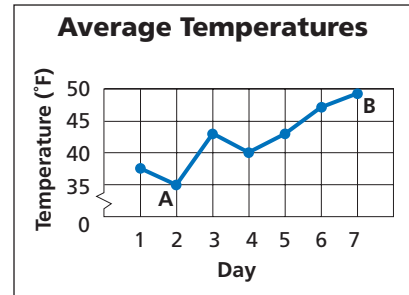
- 5A.** At Go-Cart World, each go-cart ride costs \$4. For a birthday party, a group of 7 friends are each allowed to take 1 go-cart ride. The total cost is a function of the number of rides. Identify a reasonable domain and range for this situation.
- 5B.** Draw a graph that shows the relationship between the number of rides and the total cost.
- 5C.** State whether the function is discrete or continuous. Explain.

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CHECK Your Understanding

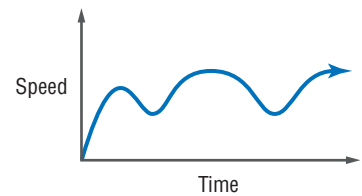
Example 1
(p. 53)

- 1.** Name the ordered pair at point *A* and explain what it represents.
- 2.** Name the ordered pair at point *B* and explain what it represents.
- 3.** Identify the independent and dependent variables for the function.



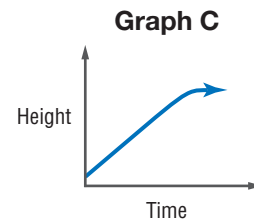
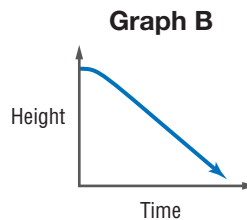
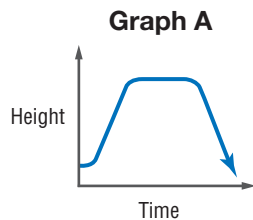
Example 2
(p. 54)

- 4.** The graph at the right represents Alexi's speed as he rides his bike. Describe what is happening in the graph.



Example 3
(p. 54)

- 5.** Identify the graph that represents the altitude of a skydiver just before she jumps from a plane until she lands.



PHYSICAL SCIENCE For Exercises 6–9, use the table and the information.

Ms. Blackwell's students recorded the height of an object above the ground at several intervals after it was dropped from a height of 5 meters.

Time (s)	0	0.2	0.4	0.6	0.8	1
Height (cm)	500	480	422	324	186	10

Example 4
(p. 55)

- 6.** Write a set of ordered pairs representing the data in the table.
- 7.** Draw a graph showing the relationship between the height of the falling object and time.

Example 5
(p. 55)

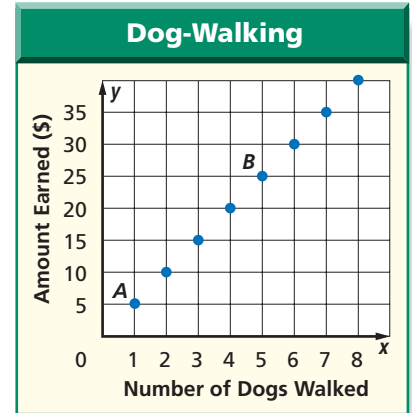
- 8.** Identify the domain and range for this situation.
- 9.** State whether the function is discrete or continuous. Explain.

Exercises

HOMEWORK HELP	
For Exercises	See Examples
10, 11, 13, 14	1
12, 15	2
16–19	3
20, 21	4
22–25	5

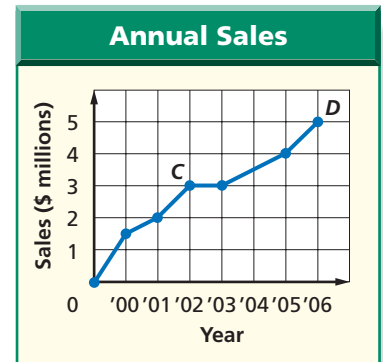
For Exercises 10–12, use the graph at the right.

- Name the ordered pair at point *A* and explain what it represents.
- Name the ordered pair at point *B* and explain what it represents.
- Identify the independent and dependent variables for the function.

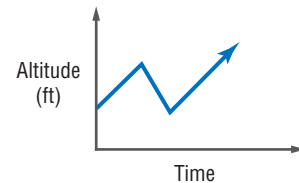
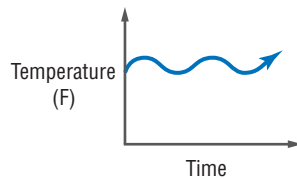


For Exercises 13–15, use the graph at the right.

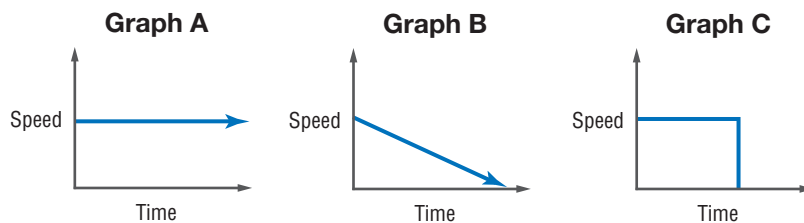
- Name the ordered pair at point *C* and explain what it represents.
- Name the ordered pair at point *D* and explain what it represents.
- Identify the independent and dependent variables.



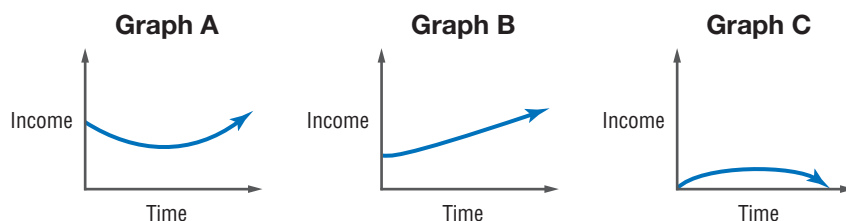
- The graph below represents Teresa's temperature when she was sick. Describe what is happening in the graph.
- The graph below represents the altitude of a group of hikers. Describe what is happening in the graph.



- TOYS** Identify the graph that displays the speed of a radio-controlled car as it moves along and then hits a wall.



- INCOME** In general, as people get older, their incomes increase until they retire. Which of the graphs below represents this?



EXTRA PRACTICE
See pages 719, 744.
Math online
Self-Check Quiz at algebra1.com



Real-World Link

Most new cars lose 15 to 30 percent of their value in the first year. After about 12 years, more popular cars tend to increase in value.

Source: *Consumer Guide*

20. **CARS** Refer to the information at the left. A car was purchased new in 1990. The owner has taken excellent care of the car, and it has relatively low mileage. Draw a reasonable graph to show the value of the car from the time it was purchased to the present.

21. **CHEMISTRY** When ice is exposed to temperatures above 32°F, it begins to melt. Draw a reasonable graph showing the relationship between the temperature of a block of ice and the time after it is removed from a freezer.

GEOMETRY For Exercises 22–25, use the table below.

Polygon	triangle	quadrilateral	pentagon	hexagon	heptagon
Number of Sides	3	4	5	6	7
Interior Angle Sum	180	360	540	720	900

22. Identify the independent and dependent variables. Then graph the data.

23. Identify the domain and range for this situation.

24. State whether the function is discrete or continuous. Explain.

25. Predict the sum of the interior angles for an octagon, nonagon, and decagon.

H.O.T. Problems

26. **REASONING** Compare and contrast dependent and independent variables.

27. **OPEN ENDED** Give an example of a relation. Identify the domain and range.

28. **CHALLENGE** Eva is 23 years older than Lisa. Draw a graph showing Eva's age as a function of Lisa's age for the first 40 years of Lisa's life. Then find the point on the graph when Eva is twice as old as Lisa.

29. *Writing in Math* Use the data about concussions on page 53 to explain how real-world situations can be modeled using graphs and functions.

STANDARDIZED TEST PRACTICE

30. What is the range for the function $\{(1, 3), (5, 7), (9, 11)\}$?

- A $\{1, 5, 9\}$ C $\{3\}$
 B $\{3, 7, 11\}$ D \emptyset

31. **REVIEW** If $3x - 2y = 5$ and $x = 2$, what value of y makes the equation true?

- F 0.5 H 2
 G 1 J 5

Spiral Review

32. What is $\pm\sqrt{121}$? (Lesson 1-8)

Identify the hypothesis and conclusion of each statement. (Lesson 1-7)

33. You can send e-mail with a computer.

34. The express lane is for shoppers with 9 or fewer items.

35. Evaluate $ab(a + b)$ and name the property used in each step. (Lesson 1-6)

Write an algebraic expression for each verbal expression. (Lesson 1-1)

36. the product of 8 and a number x raised to the fourth power

37. three times a number decreased by 10

Algebra Lab

Investigating Real-World Functions

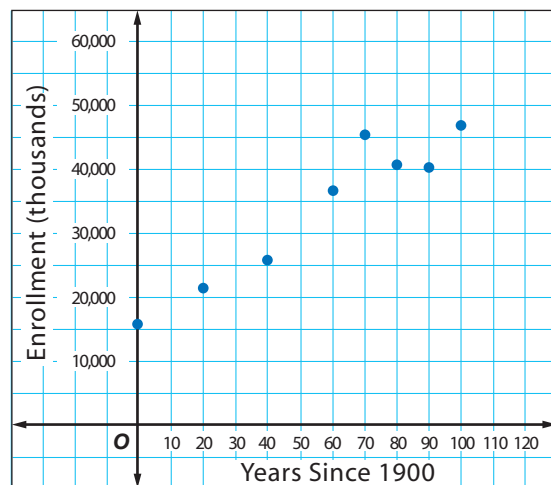
ACTIVITY

The table shows the number of students enrolled in elementary and secondary schools for the given years.

Year	1900	1920	1940	1960	1970	1980	1990	2000
Enrollment (thousands)	15,503	21,578	25,434	36,807	45,550	41,651	40,543	46,857

Source: *The World Almanac*

- On **grid paper**, draw a vertical and horizontal axis as shown. Make your graph large enough to fill most of the sheet. Label the horizontal axis 0 to 120 and the vertical axis 1 to 50,000.
- To make graphing easier, let x represent the number of years since 1900. Write the eight ordered pairs using this method. The first will be $(0, 15,503)$.
- Graph the ordered pairs on your grid paper.



ANALYZE THE RESULTS

1. Describe the graph you made.
2. Use your graph to estimate the number of students in elementary and secondary school in 1910, 1975, and 2020.
3. Describe the method you used to make estimates for Exercise 2.
4. Do you think your prediction for 2020 will be accurate? Explain your reasoning.
5. Graph this set of data, which shows the number of students per computer in U.S. schools. Predict the number of students per computer in 2010. Explain how you made your prediction.

Year	Students per Computer	Year	Students per Computer	Year	Students per Computer
1984	125	1990	22	1996	10
1985	75	1991	20	1997	7.8
1986	50	1992	18	1998	6.1
1987	37	1993	16	1999	5.7
1988	32	1994	14		
1989	25	1995	10.5		

Source: *The World Almanac*

FOLDABLES™
Study Organizer

GET READY to Study

1-1	Variables and Equations
1-2	Order of Operations
1-3	Open Sentences
1-4	Identifying and Solving Inequalities
1-5	Distributive Property
1-6	Commutative & Associative Properties
1-7	Logic
1-8	Number Systems
1-9	Functions & Graphs

Be sure the following
Key Concepts are noted
in your Foldable.

Key Concepts

Order of Operations (Lesson 1-2)

- Simplify the expression inside grouping symbols.
- Evaluate all powers.
- Multiply and divide in order from left to right.
- Add and subtract in order from left to right.

Properties (Lessons 1-4, 1-5, and 1-6)

- Additive Identity: For any number a , the sum of a and 0 is a .

Multiplication Properties

Identity	For any number a , $a \cdot 1 = a$
Property of Zero	For any number a , $a \cdot 0 = 0$
Inverse	For every number $\frac{a}{b}$, where a , $b \neq 0$, there is exactly one number $\frac{b}{a}$ such that $\frac{a}{b} \cdot \frac{b}{a} = 1$.

Properties of Equality

Reflexive	For any numbers a , b , and c
Symmetric	$a = a$
Transitive	If $a = b$, then $b = a$.
Substitution	If $a = b$, then a may be replaced by b in any expression.
Distributive	$a(b + c) = ab + ac$ and $a(b - c) = ab - ac$
Commutative	$a + b = b + a$ and $ab = ba$
Associative	$(a + b) + c = a + (b + c)$ and $(ab)c = a(bc)$

Number Systems (Lesson 1-8)

- Real Numbers:
 - Natural Numbers $\{1, 2, 3, \dots\}$
 - Whole Numbers $\{0, 1, 2, 3, \dots\}$
 - Integers $\{\dots, -2, -1, 0, 1, 2, \dots\}$
 - Rational Numbers: numbers that can be expressed in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$.

Key Vocabulary

algebraic expression (p. 6)	irrational numbers (p. 46)
coefficient (p. 29)	like terms (p. 28)
conditional statement (p. 39)	multiplicative inverses (p. 21)
continuous function (p. 55)	open sentence (p. 15)
coordinate system (p. 53)	order of operations (p. 10)
counterexample (p. 41)	perfect square (p. 46)
deductive reasoning (p. 40)	power (p. 6)
dependent variable (p. 54)	principal square root (p. 48)
discrete function (p. 55)	range (p. 55)
domain (p. 55)	rational approximation (p. 49)
exponent (p. 6)	rational number (p. 46)
factors (p. 6)	real numbers (p. 46)
function (p. 53)	reciprocal (p. 21)
independent variable (p. 54)	solution set (p. 15)
inequality (p. 16)	variable (p. 6)
integers (p. 46)	

Vocabulary Check

State whether each sentence is *true* or *false*.
If *false*, replace the underlined word or number to make a true sentence.

- The vertical axis is also called the y -axis.
- Two numbers with a product of 1 are called elements.
- A collection of objects or numbers is called a function.
- A nonnegative square root is called a principal square root.
- Irrational numbers and rational numbers together form the set of negative numbers.
- The exponent indicates the number of times the base is used as a factor.
- To evaluate an expression means to find its value.

Lesson-by-Lesson Review

1-1 Variables and Expressions (pp. 6-9)

Write an algebraic expression for each verbal expression.

8. a number to the fifth power
9. five times a number y squared
10. the sum of a number p and twenty-one
11. the difference of twice a number k and 8

Evaluate each expression.

12. 4^6 13. 2^5

Write a verbal expression for each algebraic expression.

14. $\frac{1}{2} + 7y$ 15. $6p^2$ 16. $3m^4 - 5$

17. **FROGS** A frog can jump twenty times the length of its body. If a frog's body length is b , write an algebraic expression to describe the length the frog could jump.

Example 1 Write an algebraic expression for the sum of twice a number and five.

Variable Let n represent the number.

Expression $2n + 5$

Example 2 Evaluate 7^5 .

$$\begin{aligned} 7^5 &= 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 && \text{Use 7 as a factor 5 times.} \\ &= 16,807 && \text{Multiply.} \end{aligned}$$

Example 3 Write a verbal expression for $4x^2 - 11$.

four times a number x squared minus eleven

1-2 Order of Operations (pp. 10-14)

Evaluate each expression.

18. $3 + 16 \div 8 \cdot 5$ 19. $4^2 \cdot 3 - 5(6 + 3)$

20. $288 \div [3(2^3 + 4)]$ 21. $\frac{6(4^3 + 2^2)}{9 + 3}$

Evaluate each expression if $x = 3$, $t = 4$, and $y = 2$.

22. $t^2 + 3y$ 23. $xt y^3$
24. $\frac{6ty}{x}$ 25. $8(x - y)^2 + 3t$

26. **RUNNING** Alan ran twice as many miles on Tuesday as he did on Monday and five more miles on Wednesday than he did on Monday. Write and evaluate an expression to find the total number of miles he ran if he ran 5 miles on Monday.

Example 4 Evaluate $x^2 - (y + 2)$ if $x = 4$ and $y = 3$.

$$\begin{aligned} x^2 - (y + 2) &= 4^2 - (3 + 2) && \text{Replace } x \text{ with 4} \\ & && \text{and } y \text{ with 3.} \\ &= 4^2 - 5 && \text{Add 3 and 2.} \\ &= 16 - 5 && \text{Evaluate } 4^2. \\ &= 11 && \text{Subtract 5 from 16.} \end{aligned}$$

Study Guide and Review

1-3

Open Sentences (pp. 15-20)

Find the solution of each equation or the solution set of each inequality if the replacement set is {4, 5, 6, 7, 8}.

27. $g + 9 = 16$

28. $10 - p < 7$

29. $\frac{x+1}{3} = 2$

30. $2a + 5 \geq 15$

Solve each equation.

31. $w = 4 + 3^2$

32. $d = \frac{7(4 \cdot 3)}{18 \div 3}$

33. $k = 5[2(4) - 1^3]$

34. $\frac{6(7) - 2(3)}{4^2 - 6(2)} = n$

35. **HOMECOMING** Tickets to the homecoming dance are \$9 for one person and \$15 for two people. If a group of seven students wish to go to the dance, write and solve an equation that would represent the least expensive price p of their tickets.

Example 5 Find the solution set for $14 + 2h < 24$ if the replacement set is {1, 3, 5, 7}.

Replace h in $14 + 2h < 24$ with each value in the replacement set.

h	$14 + 2h < 24$	True or False?
1	$14 + 2(1) < 24$	True
3	$14 + 2(3) < 24$	True
5	$14 + 2(5) < 24$	False
7	$14 + 2(7) < 24$	False

The solution set is {1, 3}.

Example 6 Solve $5^2 - 3 = y$.

$$5^2 - 3 = y \quad \text{Original equation}$$

$$25 - 3 = y \quad \text{Evaluate } 5^2.$$

$$22 = y \quad \text{Subtract 3 from 25.}$$

The solution is 22.

1-4

Identity and Equality Properties (pp. 21-25)

Find the value of n in each equation. Name the property that is used.

36. $4 + 0 = n$

37. $5n = 1$

38. $0 = 17n$

Evaluate each expression. Name the property used in each step.

39. $3(4 \div 4)^2 - \frac{1}{4}(4)$

40. $(7 - 7)(5) + 3 \cdot 1$

41. $\frac{1}{2} \cdot 2 + 2[2 \cdot 3 - 1]$

42. **COOKIES** Emilia promised her brother one half of the cookies that she made. If she did not make any cookies, determine how many she owes her brother and identify the property represented.

Example 7 Find the value of n in $n \cdot 9 = 9$. Name the property that is used.

$n = 1$, since $1 \cdot 9 = 9$. This is the Multiplicative Identity Property.

Example 8 Evaluate $7 \cdot 1 + 5(2 - 2)$. Name the property used in each step.

$$7 \cdot 1 + 5(2 - 2) = 7 \cdot 1 + 5(0) \quad \text{Substitution}$$

$$= 7 + 5(0) \quad \text{Multiplicative Identity}$$

$$= 7 + 0 \quad \text{Multiplicative Property of Zero}$$

$$= 7 \quad \text{Substitution}$$

1-5 The Distributive Property (pp. 26-31)

Rewrite each expression using the Distributive Property. Then evaluate.

43. $8(15 - 6)$ 44. $4(x + 1)$

Simplify each expression. If not possible, write *simplified*.

45. $3w - w + 4v + 3v$

46. $4np + 7mp$

47. **EXPENSES** Nikki's monthly expenses are \$550 for rent, \$225 for groceries, \$110 for transportation, and \$150 for utilities. Use the Distributive Property to write and evaluate an expression for her total expenses for nine months.

Example 9 Rewrite $5(t + 3)$ using the Distributive Property. Then evaluate.

$$\begin{aligned} 5(t + 3) &= 5(t) + 5(3) && \text{Distributive Property} \\ &= 5t + 15 && \text{Multiply.} \end{aligned}$$

Example 10 Simplify $2x^2 + 4x^2 + 7x$. If not possible, write *simplified*.

$$\begin{aligned} 2x^2 + 4x^2 + 7x &= (2 + 4)x^2 + 7x && \text{Distributive Property} \\ &= 6x^2 + 7x && \text{Substitution} \end{aligned}$$

1-6 Commutative and Associative Properties (pp. 33-37)

Simplify each expression.

48. $7w^2 + w + 2w^2$

49. $3(2 + 3x) + 21x$

50. **ZOO** At a zoo, each adult admission costs \$9.75, and each child costs \$7.25. Find the cost of admission for two adults and four children.

Example 11 Write an algebraic expression for *five times the sum of x and y increased by $2x$* . Then simplify.

$$\begin{aligned} 5(x + y) + 2x &= 5x + 5y + 2x \\ &= 5x + 2x + 5y \\ &= (5 + 2)x + 5y \\ &= 7x + 5y \end{aligned}$$

1-7 Logical Reasoning (pp. 39-44)

51. Identify the hypothesis and conclusion of the statement. Then write the statement in if-then form.
School begins at 7:30 A.M.
52. Find a counterexample for *if you have no umbrella, you will get wet*.
53. **LIGHTNING** It is said that lightning never strikes twice in the same place. Identify the hypothesis and conclusion of this statement and write it in if-then form.

Example 12 Identify the hypothesis and conclusion of the statement *The trumpet player must audition to be in the band*. Then write the statement in if-then form.

Hypothesis: a person is a trumpet player
Conclusion: the person must audition to be in the band

If a person is a trumpet player, then the person must audition to be in the band.

Study Guide and Review

1-8

Number Systems (pp. 46-52)

Name the set or sets of numbers to which each real number belongs.

54. $\frac{7}{15}$

55. $\sqrt{45}$

Graph each set of numbers.

56. $\{-3, -1, 0, 2, 5\}$ 57. $x \leq 6.5$

Order each set of numbers from least to greatest.

58. $-\sqrt{4}, 3.5, \sqrt{11}, 3\frac{11}{20}$

59. $\sqrt{27}, 5\frac{1}{5}, -\sqrt{34}, -\frac{47}{9}$

60. **ROOMS** Belinda's square bedroom is $10\frac{41}{50}$ feet long. The area of Jarrod's bedroom is 115 square feet. Whose bedroom is larger? Explain.

Example 13 Name the set or sets of numbers to which $\frac{48}{6}$ belongs.

$\frac{48}{6} = 8$, so $\frac{48}{6}$ is a natural number, a whole number, an integer, and a rational number.

Example 14 Graph $x > -1$.



Example 15 Order $-\frac{29}{4}$, $\sqrt{7}$, and -7.85 from least to greatest.

$-\frac{29}{4} = -7.25$ Write each number as a decimal and compare.

$$\sqrt{7} \approx 2.646$$

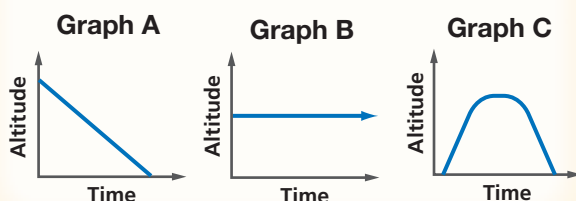
So, the order is $-7.85, -\frac{29}{4}$, and $\sqrt{7}$.

1-9

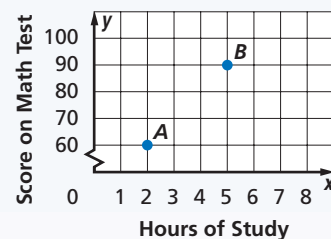
Functions and Graphs (pp. 53-58)

For Exercises 61 and 62, use the graph in Example 16.

61. Name the ordered pair at point A and explain what it represents.
62. Identify the independent and dependent variables for the function. Explain.
63. **ALTITUDE** Identify the graph that represents the altitude of an airplane taking off, flying for a while, then landing.



Example 16 Name the ordered pair at point B and explain what it represents.



Point B is at 5 along the x -axis and 90 along the y -axis. So, its ordered pair is $(5, 90)$. This represents a score of 90 on the math test with 5 hours of study.

Write an algebraic expression for each verbal expression.

- the sum of a number x and 13
- 25 increased by the product of 2 and a number
- 7 less a number y squared

Simplify each expression.

- $5(9 + 3) - 3 \cdot 4$
- $12 \cdot 6 \div 3 \cdot 2 \div 8$

Evaluate each expression if $a = 2$, $b = 5$, $c = 3$, and $d = 1$.

- $a^2b + c$
- $(cd)^3$
- $(a + d)c$

- MULTIPLE CHOICE** Tia owns a dog grooming business. How much did she earn in one week if she groomed 14 small dogs, 11 medium-size dogs, and 3 large dogs?

Dog Grooming	
Size of Dog	Cost (\$)
small	25.00
medium	27.50
large	36.50

- \$117.00
- \$636.65
- \$700.00
- \$762.00

Solve each equation.

- $y = (4.5 + 0.8) - 3.2$
- $4^2 - 3(4 - 2) = x$
- Evaluate $3^2 - 2 + (2 - 2)$. Name the property used in each step.

Rewrite each expression in simplest form.

- $4x + 2y - 2x + y$
- $3(2a + b) - 5a + 4b$

Find a counterexample for each conditional statement.

- EXERCISE** If you run 15 minutes today, then you will be able to run a marathon.
- If $x \leq 6$, then $2x - 3 < 9$.

Replace each \bullet with $<$, $>$, or $=$ to make each sentence true.

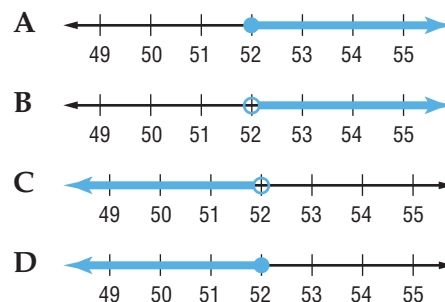
- $\sqrt{43} \bullet 6\bar{5}$
- $\frac{1}{10} \bullet \sqrt{10}$
- $-\sqrt{7} \bullet -\frac{5}{2}$
- $0.\overline{36} \bullet \frac{4}{11}$

- MULTIPLE CHOICE** Which number serves as a counterexample to the statement below?

If n is a prime number, then n is odd.

- 5
- 4
- 3
- 2

- MULTIPLE CHOICE** Riders must be at least 52 inches tall to ride the newest ride at an amusement park. Which graph represents the heights of these riders?



Sketch a reasonable graph for each situation.

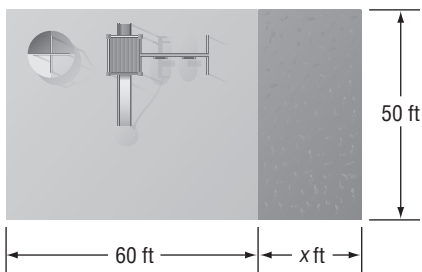
- A basketball is shot from the free throw line and falls through the net.
- A nickel is dropped on a stack of pennies and bounces off.

Standardized Test Practice

Chapter 1

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. Karla has a backyard play area that has the dimensions shown below. She is expanding the play area by adding x feet to the 60 foot side. The area of the addition is $50x$. How can the area of the new play area be expressed in terms of x ?

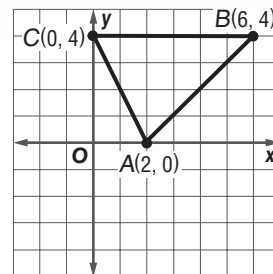


- A $3000 + 50x$ C $60 + 50x$
 B $3000 - 50x$ D $60 - 50x$
2. **GRIDDABLE** When $-3(4m^2 - 14)$ is simplified, what is the value of the constant term?
3. Laura makes \$8 an hour working at a day care center. 20% of her income is deducted for taxes. She needs to make more than \$288 each month to make her car payment. Solve the inequality $8x - 0.20(8x) > 288$ to determine how many hours per month she must work.
- F more than 45 hours
 G less than 45 hours
 H more than 36 hours
 J less than 36 hours

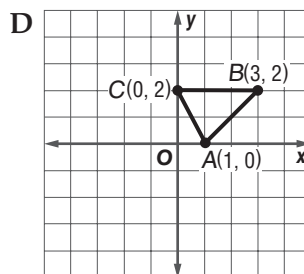
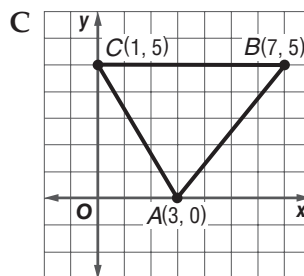
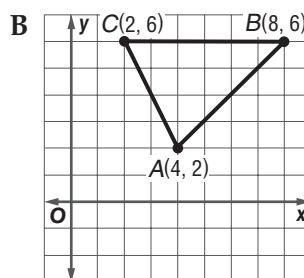
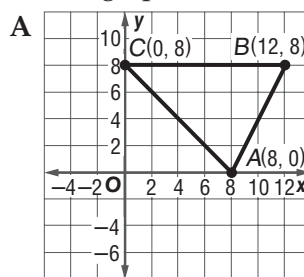
TEST-TAKING TIP

Question 3 Some multiple-choice questions ask you to solve an equation or inequality. You can check your solution by replacing the variable in the equation or inequality with your answer. The answer choice that results in a true statement is the correct answer.

4. The triangle in the graph is to be dilated by a scale factor of $\frac{1}{2}$.



Which graph shows this transformation?



5. Connor is redecorating his living room. He has budgeted \$1200 for the project, and has already spent \$420 on paint. If he wants to spend the rest of the money on curtains to cover 5 windows, which expression represents the maximum amount w he can spend on each window?

F $1200 = 420 - 5w$

G $420 = 1200 + 5w$

H $1200 = 420 + 5w$

J $420 = 1200 + 5 - w$

6. Which expression below illustrates the Commutative Property?

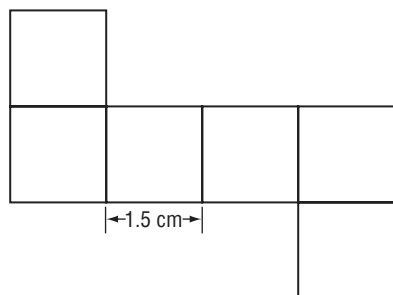
A $(x + y) + 2 = x + (y + 2)$

B $-3(a + b) = -3a + -3b$

C $qrs = rqs$

D $2 + 0 = 2$

7. The net of a cube is shown below.



Which best represents the volume of this cube to the nearest cubic centimeter?

F 3 cm^3

G 8 cm^3

H 14 cm^3

J 42 cm^3

8. Millie's Market sells peanuts in two different containers. The first container is a rectangular prism that has a height of 6 inches and a square base with a side length of 3 inches. The other container is a cylinder with a radius of 1.5 inches and a height of 6 inches. Which best describes the relationship between the two containers?

A The prism has the greater volume.

B The cylinder has the greater volume.

C The volumes are equivalent.

D The volumes cannot be determined.

9. **GRIDDABLE** Hugo's deck is shaped like a rectangle with a width of 15 feet and a length of 20 feet. Staining the deck will cost \$1.25 per square foot. How much money will it cost to stain the deck?

Pre-AP

Record your answers on a sheet of paper.
Show your work.

10. The Lee family is going to play miniature golf. The family is composed of two adults and four children.

	Greens Fees	
	Before 6 P.M.	After 6 P.M.
Adult (a)	\$5.00	\$6.50
Children (c)	\$3.00	\$4.50

- a. Write an inequality to show the cost for the family to play miniature golf if they don't want to spend more than \$30.
- b. How much will it cost the family to play after 6 P.M.?
- c. How much will it cost the family to play before 6 P.M.?

NEED EXTRA HELP?

If You Missed Question...

Go to Lesson or Page...

1	2	3	4	5	6	7	8	9	10
1-1	1-5	1-5	10-6	1-3	1-2	708	708	704	1-3