Rational Expressions and Equations

BIG Ideas

- Solve problems involving inverse variation.
- Simplify, add, subtract, multiply, and divide rational expressions.
- Divide polynomials.
- Solve rational equations.

Key Vocabulary

complex fraction (p. 621)

extraneous solutions (p. 629)

inverse variation (p. 577)

rational expression (p. 583)

🌐 Real-World Link

Marching Band Performing operations on rational expressions is an important part of working with equations. For example, the uniform manager for The Ohio State Marching Band can use rational expressions to determine the number of uniforms that can be repaired in a certain time given the number of tailors available and the time needed to repair each uniform.

FOLDABLES™
Study Organizer

Rational Expressions and Equations Make this Foldable to help you organize information about rational expressions and equations. Begin with a sheet of $8\frac{1}{2}$″ by 11″ paper.
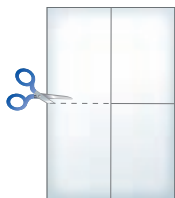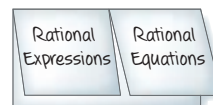
1. **Fold** in half lengthwise.

2. **Fold** the top to the bottom.

3. **Open.** Cut along the second fold to make two tabs.

4. **Label** each tab as shown.

Rational Expressions | Rational Equations

GET READY for Chapter 11

Diagnose Readiness You have two options for checking Prerequisite Skills.

Option 1

Take the Quick Check below. Refer to the Quick Review for help.

QUICK Check

Solve each proportion. (Lesson 2-6)

1. $\frac{y}{9} = -\frac{7}{16}$ **2.** $\frac{4}{x} = \frac{2}{10}$

3. $\frac{3}{15} = \frac{1}{n}$ **4.** $\frac{x}{8} = \frac{0.21}{2}$

5. $\frac{1.1}{0.6} = \frac{8.47}{n}$ **6.** $\frac{9}{8} = \frac{y}{6}$

7. $\frac{2.7}{3.6} = \frac{8.1}{a}$ **8.** $\frac{0.19}{2} = \frac{x}{24}$

Find the greatest common factor for each pair of monomials. (Lesson 8-1)

9. 30, 42   **10.** $60r^2, 45r^3$

11. GAMES There are 64 girls and 80 boys who attend an after-school program. For a game, the boys are going to split into groups and the girls are going to split into groups. The number of people in each group has to be the same. How large can the groups be?

Factor each polynomial. (Lessons 8-1 through 8-4)

12. $3c^2d - 6c^2d^2$ **13.** $6mn + 15m$

14. $x^2 + 11x + 24$ **15.** $x^2 + 4x - 45$

16. $2x^2 + x - 21$ **17.** $3x^2 - 12x + 9$

18. AREA The area of a rectangle can be represented by the expression $x^2 + 7x + 12$. What expressions represent the sides of the rectangle?

QUICK Review

EXAMPLE 1

Solve the proportion $\frac{4}{z} = \frac{13}{5}$.

$\frac{4}{z} = \frac{13}{5}$ Original equation

$4 \cdot 5 = 13 \cdot z$ Cross multiply.

$20 = 13z$ Simplify.

$\frac{20}{13} = \frac{13z}{13}$ Divide each side by 13.

$\frac{20}{13} = z$ Simplify.

EXAMPLE 2

Find the greatest common factor of 12 and 18.

3, 2, 2   Factors of 12

3, 3, 2   Factors of 18

3 · 2   The product of the common factors

The greatest common factor of 12 and 18 is 6.

EXAMPLE 3

Factor $12x^2y^3 - 3xy$.

3, 2, 2, x, x, y, y, y   Factors of $12x^2y^3$

−1, 3, x, y   Factors of $-3xy$

3 · x · y   The product of the common factors

Factor out the common factors from each term of the expression.

$3xy(4xy^2) - 3xy(1)$ Rewrite the terms using the GCF.

$= 3xy(4xy^2 - 1)$ Distributive Property

Graphing Calculator Lab
Investigating Inverse Variation

You can use a data collection device to investigate the relationship between volume and pressure.

▶ SET UP the Lab

• Connect a syringe to the gas pressure sensor. Then connect the data collection device to both the sensor and the calculator as shown.

• Start the data collection program and select the sensor.
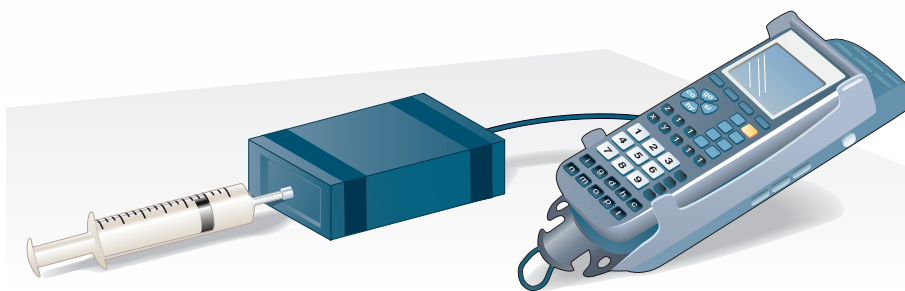
ACTIVITY

Step 1 Open the valve between the atmosphere and the syringe. Set the inside ring of the syringe to 20 mL and close the valve. This ensures that the amount of air inside the syringe will be constant throughout the experiment.

Step 2 Press the plunger of the syringe down to the 5 mL mark. Wait for the pressure gauge to stop changing, then take the data reading. Enter 5 as the volume on the calculator. The pressure will be measured in atmospheres (atm).

Step 3 Repeat step 2, pressing the plunger down to 7.5 mL, 10.0 mL, 12.5 mL, 15.0 mL, 17.5 mL, and 20.0 mL. Record the volume as you take each data reading.

Step 4 After taking the last data reading, use STAT PLOT to create a line graph of the data.

ANALYZE THE RESULTS

1. Does the pressure vary directly as the volume? Explain.

2. As the volume changes from 10 to 20 mL, what happens to the pressure?

3. Predict what the pressure of the gas in the syringe would be if the volume was increased to 40 mL.

4. Add a column to the data table to find the product of the volume and the pressure for each data reading. What pattern do you observe?

5. **MAKE A CONJECTURE** The relationship between the pressure and volume of a gas is called Boyle's Law. Write an equation relating the volume v in milliliters and pressure p in atmospheres in your experiment.

Inverse Variation

- Graph inverse variations.
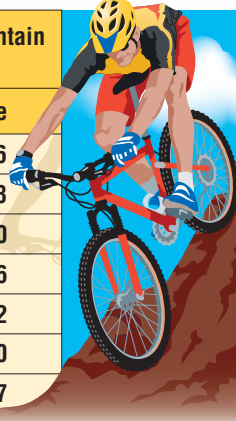- Solve problems involving inverse variation.

New Vocabulary

inverse variation
product rule

Study Tip

Look Back

To review **direct variation**, see Lesson 4-2.

GET READY for the Lesson

The number of revolutions of the pedals made when riding a bicycle at a constant speed varies inversely as the gear ratio of the bicycle. In other words, as the gear ratio *decreases*, the revolutions per minute (rpm) *increase*. This is why shifting to a lower gear allows you to pedal with less difficulty when riding uphill.

Pedaling Rates to Maintain Speed of 10 mph	
Gear Ratio	Rate
117.8	89.6
108.0	97.8
92.6	114.0
76.2	138.6
61.7	171.2
49.8	212.0
40.5	260.7

Graph Inverse Variation Recall that some situations in which y increases as x increases are *direct variations*. If y varies directly as x, we can represent this relationship with an equation of the form $y = kx$, where $k \neq 0$. However, in the application above, as one value increases the other value decreases. When the product of two variables remains constant, the relationship forms an **inverse variation**. We say y *varies inversely as x* or y *is inversely proportional to x*. Recall that the constant k is called the *constant of variation*.

KEY CONCEPT *Inverse Variation*

y varies inversely as x if there is some nonzero constant k such that $xy = k$.

Real-World EXAMPLE **Graph an Inverse Variation**

1 **DRIVING** The time t it takes to travel a certain distance varies inversely as the rate r at which you travel. The equation $rt = 250$ can be used to represent a person driving 250 miles. Complete the table and draw a graph of the relation.

r (mph)	5	10	15	20	25	30	35	40	45	50
t (hours)										

Solve for t when $r = 5$.

$rt = 250$	Original equation
$5t = 250$	Replace r with 5.
$t = \dfrac{250}{5}$	Divide each side by 5.
$t = 50$	Simplify.

Solve for t using the other values of r. Complete the table.

r (mph)	5	10	15	20	25	30	35	40	45	50
t (hours)	50	25	16.67	12.5	10	8.33	7.14	6.25	5.56	5

Next, graph the ordered pairs.

Because rate cannot be negative, it is only reasonable to use positive values for r.

The graph of an inverse variation is not a straight line like the graph of a direct variation. As the rate r increases, the time t that it takes to travel the same distance decreases.
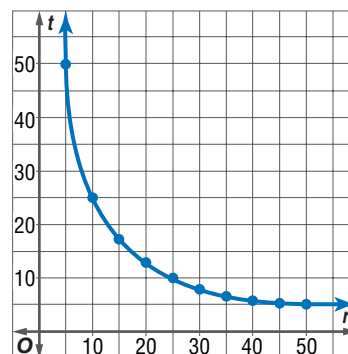
CHECK Your Progress

1. Graph $64 = xy$ using a table.

Graphs of inverse variations can also be drawn using negative values of x.

EXAMPLE Graph an Inverse Variation

2 Graph an inverse variation in which y varies inversely as x and $y = 15$ when $x = 6$.
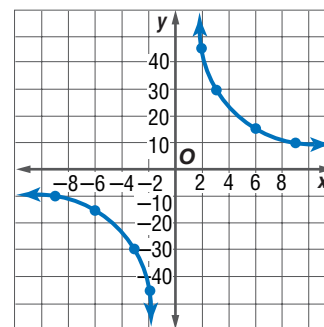
Solve for k.

$$xy = k \quad \text{Inverse variation equation}$$

$$(6)(15) = k \quad x = 6, y = 15$$

$$90 = k \quad \text{The constant of variation is 90.}$$

Choose values for x and y with a product of 90.

x	−9	−6	−3	−45	0	2	3	6	9
y	−10	−15	−30	−2	undefined	45	30	15	10

CHECK Your Progress

2. Graph an inverse variation equation in which y varies inversely as x when $y = 12$ and when $x = 4$.

Use Inverse Variation If (x_1, y_1) and (x_2, y_2) are solutions of an inverse variation, then $x_1, y_1 = k$ and $x_2 y_2 = k$.

$$x_1 y_1 = k \text{ and } x_2 y_2 = k$$

$$x_1 y_1 = x_2 y_2 \quad \text{Substitute } x_2 y_2 \text{ for } k.$$

The equation $x_1 y_1 = x_2 y_2$ is called the **product rule** for inverse variations. You can use this equation to form a proportion.

$$x_1 y_1 = x_2 y_2 \quad \text{Product rule for inverse variations}$$

$$\frac{x_1 y_1}{x_2 y_1} = \frac{x_2 y_2}{x_2 y_1} \quad \text{Divide each side by } x_2 y_1.$$

$$\frac{x_1}{x_2} = \frac{y_2}{y_1} \quad \text{Simplify.}$$

Study Tip

Proportions

Notice that the proportion for inverse variations is different from the proportion for direct variation, $\frac{x_1}{x_2} = \frac{y_1}{y_2}$.

③ If *y* varies inversely as *x* and $y = 4$ when $x = 7$, find *x* when $y = 14$.

Let $x_1 = 7$, $y_1 = 4$, and $y_2 = 14$. Solve for x_2.

Method 1 Use the product rule.

$$x_1 y_1 = x_2 y_2 \qquad \text{Product rule for inverse variations}$$

$$7 \cdot 4 = x_2 \cdot 14 \qquad x_1 = 7, y_1 = 4, \text{ and } y_2 = 14$$

$$\frac{28}{14} = x_2 \qquad \text{Divide each side by 14.}$$

$$2 = x_2 \qquad \text{Simplify.}$$

Method 2 Use a proportion.

$$\frac{x_1}{x_2} = \frac{y_2}{y_1} \qquad \text{Proportion for inverse variations}$$

$$\frac{7}{x_2} = \frac{14}{4} \qquad x_1 = 7, y_1 = 4, \text{ and } y_2 = 14$$

$$28 = 14 x_2 \qquad \text{Cross multiply.}$$

$$2 = x_2 \qquad \text{Divide each side by 14.}$$

✓ **CHECK Your Progress**

3. If *y* varies inversely as *x* and $y = 4$ when $x = -8$, find *y* when $x = -4$.

Real-World EXAMPLE **Use Inverse Variation**

④ **PHYSICAL SCIENCE** When two people are balanced on a seesaw, their distances from the center of the seesaw are inversely proportional to their weights. If a 118-pound person sits 1.8 meters from the center of the seesaw, how far should a 125-pound person sit from the center to balance the seesaw?

Let $w_1 = 118$, $d_1 = 1.8$, and $w_2 = 125$. Solve for d_2.

$$w_1 d_1 = w_2 d_2 \qquad \text{Product rule for inverse variations}$$

$$118 \cdot 1.8 = 125 \cdot d_2 \qquad \text{Substitution}$$

$$\frac{212.4}{125} = d_2 \qquad \text{Divide each side by 125.}$$

$$1.7 = d_2 \qquad \text{Simplify.}$$

To balance the seesaw, the second person should sit 1.7 meters from the center.

✓ **CHECK Your Progress**

4. **EARTH SCIENCE** As the temperature increases, the level of water in a river decreases. When the temperature was 90° Fahrenheit, the water level was 11 feet. If the temperature was 110° Fahrenheit, what was the level of water in the river?

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Examples 1, 2
(pp. 577–578)

Graph each variation if y varies inversely as x.

1. $y = 24$ when $x = 8$ **2.** $y = -6$ when $x = -2$

Example 3
(p. 579)

Write an inverse variation equation that relates x and y. Assume that y varies inversely as x. Then solve.

3. If $y = 2.7$ when $x = 8.1$, find x when $y = 5.4$.

4. If $x = \frac{1}{2}$ when $y = 16$, find x when $y = 32$.

5. If $y = 12$ when $x = 6$, find y when $x = 8$.

6. If $y = -8$ when $x = -3$, find y when $x = 6$.

Example 4
(p. 579)

7. MUSIC When under equal tension, the frequency of a vibrating string from a piano varies inversely with the string length. If a string that is 420 millimeters in length vibrates at a frequency of 523 cycles a second, at what frequency will a 707-millimeter string vibrate?

Exercises

Graph each variation if y varies inversely as x.

HOMEWORK HELP	
For Exercises	**See Examples**
8–13	1, 2
14, 15	4
16–23	3

8. $y = 24$ when $x = -8$ **9.** $y = 3$ when $x = 4$

10. $y = 5$ when $x = 15$ **11.** $y = -4$ when $x = -12$

12. $y = 9$ when $x = 8$ **13.** $y = 2.4$ when $x = 8.1$

14. MUSIC The pitch of a musical note varies inversely as its wavelength. If the tone has a pitch of 440 vibrations per second and a wavelength of 2.4 feet, find the pitch of a tone that has a wavelength of 1.6 feet.

15. COMMUNITY SERVICE Students at Roosevelt High School are collecting canned goods for a local food pantry. They plan to distribute flyers to homes in the community asking for donations. Last year, 12 students were able to distribute 1000 flyers in nine hours. How long would it take if 15 students hand out 1000 flyers this year?

Write an inverse variation equation that relates x and y. Assume that y varies inversely as x. Then solve.

16. If $y = 8.5$ when $x = -1$, find x when $y = -1$.

17. If $y = 8$ when $x = 1.55$, find x when $y = -0.62$.

18. If $y = 6.4$ when $x = 4.4$, find x when $y = 3.2$.

19. If $y = 1.6$ when $x = 0.5$, find x when $y = 3.2$.

20. If $y = 12$ when $x = 5$, find y when $x = 3$.

21. If $y = 7$ when $x = -2$, find y when $x = 7$.

22. If $y = 4$ when $x = 4$, find y when $x = 7$.

23. If $y = -6$ when $x = -2$, find y when $x = 5$.

24. TRAVEL The Zalinski family can drive the 220 miles to their cabin in 4 hours at 55 miles per hour. Son Jeff claims that they could save half an hour if they drove 65 miles per hour, the speed limit. Is Jeff's claim true? Explain.

Write an inverse variation equation that relates x and y. Assume that y varies inversely as x. Then solve.

25. Find the value of y when $x = 7$ if $y = -7$ when $x = \frac{2}{3}$.

26. Find the value of y when $x = 32$ if $y = -16$ when $x = \frac{1}{2}$.

27. If $x = 6.1$ when $y = 4.4$, find x when $y = 3.2$.

28. If $x = 0.5$ when $y = 2.5$, find x when $y = 20$.
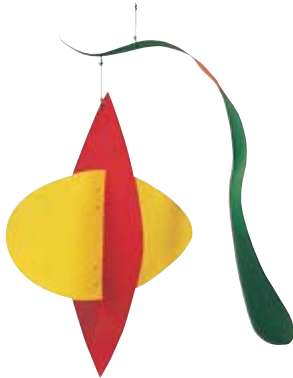
CHEMISTRY For Exercises 29–31, use the following information.
Boyle's Law states that the volume of a gas V varies inversely with applied pressure P.

29. Write an equation to show this relationship.

30. Pressure on 60 cubic meters of a gas is raised from 1 atmosphere to 3 atmospheres. What new volume does the gas occupy?

31. A helium-filled balloon has a volume of 22 cubic meters at sea level where the air pressure is 1 atmosphere. The balloon is released and rises to a point where the air pressure is 0.8 atmosphere. What is the volume of the balloon at this height?

32. GEOMETRY A rectangle is 36 inches wide and 20 inches long. How wide is a rectangle of equal area if its length is 90 inches?

33. ART Anna is designing a mobile to suspend from a gallery ceiling. A chain is attached eight inches from the end of a bar that is 20 inches long. On the shorter end of the bar is a sculpture weighing 36 kilograms. She plans to place another piece of artwork on the other end of the bar. How much should the second piece of art weigh if she wants the bar to be balanced?

Determine whether the data in each table represent an inverse variation or a direct variation. Explain.

34.

x	y
-6	3
-2	9
2	-9

35.

x	y
5	2.5
8	6
11	11.5

36.

x	y
-3	-7
-2	-10.5
4	5.25

H.O.T. Problems

37. OPEN ENDED Give a real-world situation or phenomena that can be modeled by an indirect variation equation. Use the correct terminology to describe your example and explain why this situation is an indirect variation.

38. REASONING Determine which situation is an example of inverse variation. Justify your answer.

a. Emily spends $2 each day for snacks on her way home from school. The total amount she spends each week depends on the number of days school was in session.

b. A business donates $200 to buy prizes for a school event. The number of prizes that can be purchased depends upon the price of each prize.

**CHALLENGE** For Exercises 39 and 40, assume that y varies inversely as x.

39. If the value of x is doubled, what happens to the value of y?

40. If the value of y is tripled, what happens to the value of x?

41. *Writing in Math* Use the data provided on page 577 to explain how the gears on a bicycle are related to inverse variation. Include an explanation of why the gear ratio affects the pedaling speed of the rider.

STANDARDIZED TEST PRACTICE

42. Which function *best* describes the graph?
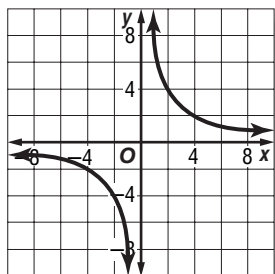
A $xy = 8$ **C** $x + y = 8$

B $xy = -8$ **D** $y = x + 8$

43. REVIEW A submarine is currently 200 feet below sea level. If the submarine begins to descend at a rate of 35 feet per minute, which equation could be used to determine t the time in minutes it will take the submarine to reach a depth of 2750 feet below sea level?

F $2750 = 200 + -35t$

G $-2750 = (-200 + 35)t$

H $2750 = (200 + 35)t$

J $-2750 = -200 + -35t$

Spiral Review

For each set of measures given, find the measures of the missing sides if $\triangle ABC \sim \triangle DEF$. (Lesson 10-6)

44. $a = 3, b = 10, c = 9, d = 12$

45. $b = 8, c = 4, d = 21, e = 28$

Find the possible values of a if the points with the given coordinates are the indicated distance apart. (Lesson 10-5)

46. $(3, 2), (a, 9); d = \sqrt{113}$ **47.** $(a, 6), (13, -6); d = 13$ **48.** $(-7, 1), (2, a); d = \sqrt{82}$

49. MUSIC Two musical notes played at the same time produce harmony. The closest harmony is produced by frequencies with the greatest GCF. A, C, and C sharp have frequencies of 220, 264, and 275, respectively. Which pair of these notes produce the closest harmony? (Lesson 8-1)

Solve each system of inequalities by graphing. (Lesson 6-8)

50. $y \le 3x - 5$
$y > -x + 1$

51. $y \ge 2x + 3$
$2y \ge -5x - 14$

52. $x + y \le 1$
$x - y \le -3$
$y \ge 0$

Solve each equation. (Lesson 2-5)

53. $7(2y - 7) = 5(4y + 1)$

54. $w(w + 2) = 2w(w - 3) + 16$

GET READY for the Next Lesson

PREREQUISITE SKILL Find the greatest common factor for each set of monomials. (Lesson 8-1)

55. $36, 15, 45$

56. $48, 60, 84$

57. $210, 330, 150$

58. $17a, 34a^2$

59. $12xy^2, 18x^2y^3$

60. $12pr^2, 40p^4$

Rational Expressions

Main Ideas

- Identify values excluded from the domain of a rational expression.
- Simplify rational expressions.

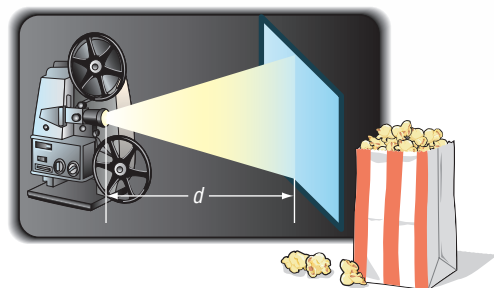## New Vocabulary

rational expression
excluded values

The intensity I of an image on a movie screen is inversely proportional to the square of the distance d between the projector and the screen. Recall from Lesson 11-1 that this can be represented by the equation $I = \dfrac{k}{d^2}$, where k is a constant.

EXCLUDED VALUES OF RATIONAL EXPRESSIONS The expression $\dfrac{k}{d^2}$ is an example of a **rational expression**. A **rational expression** is an algebraic fraction whose numerator and denominator are polynomials.

Because a rational expression involves division, the denominator may not equal zero. Any values of a variable that result in a denominator of zero must be excluded from the domain of that variable. These are called **excluded values** of the rational expression.

EXAMPLE Excluded Values

1 State the excluded value for each rational expression.

a. $\dfrac{5m + 3}{m - 6}$

Exclude the values for which $m - 6 = 0$, because the denominator cannot equal 0.

$m - 6 = 0 \rightarrow m = 6$ Therefore, m cannot equal 6.

b. $\dfrac{x^2 - 5}{x^2 - 5x + 6}$

Exclude the values for which $x^2 - 5x + 6 = 0$.

$$x^2 - 5x + 6 = 0$$
$$(x - 2)(x - 3) = 0 \quad \text{Factor.}$$

$x - 2 = 0 \quad \text{or} \quad x - 3 = 0$ Zero Product Property
$\quad\quad x = 2 \quad\quad\quad\quad x = 3$ Therefore, x cannot equal 2 or 3.

Study Tip

Look Back
You can review the Zero Product Property in Lesson 8-2.

✔ **CHECK Your Progress** State the excluded values.

1A. $\dfrac{16x + 5}{3x}$

1B. $\dfrac{2x + 1}{3x^2 + 14x - 5}$

EXAMPLE **Use Rational Expressions**

2 **HISTORY** The ancient Egyptians probably used levers to help them maneuver the giant blocks they used to build the pyramids. The diagram shows how the devices may have worked using a 10-foot lever.

a. The mechanical advantage of a lever is $\dfrac{L_E}{L_R}$, where L_E is the length of the effort arm and L_R is the length of the resistance arm. Calculate the mechanical advantage of the lever the Egyptian worker is using.
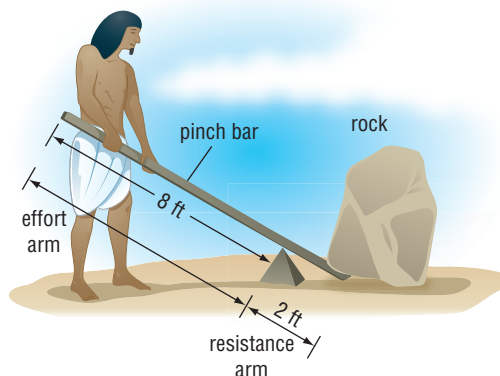
Let b represent the length of the bar and e represent the length of the effort arm. Then $b - e$ is the length of the resistance arm.

Use the expression for mechanical advantage to write an expression for the mechanical advantage in this situation.

$$\frac{L_E}{L_R} = \frac{e}{b - e} \qquad L_E = e, L_R = b - e$$

$$= \frac{8}{10 - 8} \qquad e = 8, b = 10$$

$$= 4 \qquad \text{Simplify.}$$

The mechanical advantage is 4.

b. The force placed on the rock is the product of the mechanical advantage and the force applied to the end of the lever. If the Egyptian worker can apply a force of 180 pounds, what is the greatest weight he can lift with the lever?

Since the mechanical advantage is 4, the Egyptian worker can lift 4 · 180 or 720 pounds with this lever.

CHECK Your Progress

2. Kelli is going to lift a 535-pound rock using a 7-foot lever. If she places the fulcrum 2 feet from the rock, how much force will she have to use to lift the rock?

Simplify Rational Expressions Simplifying rational expressions is similar to simplifying fractions with numbers. To simplify a rational expression, you must eliminate any common factors in the numerator and denominator. To do this, use their greatest common factor (GCF). Remember that $\dfrac{ab}{ac} = \dfrac{a}{a} \cdot \dfrac{b}{c}$ and $\dfrac{a}{a} = 1$. So, $\dfrac{ab}{ac} = 1 \cdot \dfrac{b}{c}$ or $\dfrac{b}{c}$.

When a rational expression is in simplest form, the numerator and denominator have no common factors other than 1 or −1.

STANDARDIZED TEST EXAMPLE Expression Involving Monomials

3 Which expression is equivalent to $\dfrac{(-3x^2)(4x^5)}{9x^6}$?

A $\dfrac{4}{3}x$ **B** $\dfrac{4}{3x}$ **C** $\dfrac{-4}{3x}$ **D** $\dfrac{-4}{3}x$

Read the Test Item

The expression $\dfrac{(-3x^2)(4x^5)}{9x^6}$ represents the product of two monomials and the division of that product by another monomial.

Solve the Test Item

Step 1 Find the product in the numerator. $(-3x^2)(4x^5) = -12x^7$

Step 2 Find the GCF of the numerator and denominator. $\dfrac{(3x^6)(-4x)}{(3x^6)(3)}$

Step 3 Simplify. The correct answer is D. $\dfrac{(3x^6)(-4x)}{(3x^6)(3)}$ or $\dfrac{-4x}{3}$

✓ CHECK Your Progress

3. Which expression is equivalent to $\dfrac{16c^2b^4}{8c^3b}$?

F $\dfrac{2b^3}{c}$ **G** $\dfrac{b^3}{2c}$ **H** $\dfrac{1}{2b^3c}$ **J** $2b^3c$

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You can use the same procedure to simplify a rational expression in which the numerator and denominator are polynomials. Determine the excluded values using the *original* expression rather than the simplified expression.

EXAMPLE Excluded Values

4 Simplify $\dfrac{3x - 15}{x^2 - 7x + 10}$. State the excluded values of x.

$\dfrac{3x - 15}{x^2 - 7x + 10} = \dfrac{3(x - 5)}{(x - 2)(x - 5)}$ Factor.

$= \dfrac{3(x \overset{1}{\cancel{- 5}})}{(x - 2)(x \underset{1}{\cancel{- 5}})}$ Divide the numerator and denominator by the GCF, $x - 5$.

$= \dfrac{3}{x - 2}$ Simplify.

Exclude the values for which $x^2 - 7x + 10$ equals 0.

$x^2 - 7x + 10 = 0$ The denominator cannot equal zero.

$(x - 5)(x - 2) = 0$ Factor.

$x = 5$ or $x = 2$ Zero Product Property   Therefore, $x \neq 5$ and $x \neq 2$.

✓ CHECK Your Progress

Simplify each expression. State the excluded values.

4A. $\dfrac{12x + 36}{x^2 - x - 12}$ **4B.** $\dfrac{x^2 - 2x - 35}{x^2 - 9x + 14}$

Extra Examples at algebra1.com

✓ CHECK Your Understanding

Example 1
(p. 583)

State the excluded values for each rational expression.

1. $\dfrac{4a}{3 + a}$

2. $\dfrac{x^2 - 9}{2x + 6}$

3. $\dfrac{n + 5}{n^2 + n - 20}$

Examples 3, 4
(pp. 584–585)

Simplify each expression. State the excluded values of the variables.

4. $\dfrac{56x^2y}{70x^3y^2}$

5. $\dfrac{x^2 - 49}{x + 7}$

6. $\dfrac{x + 4}{x^2 + 8x + 16}$

7. $\dfrac{3x - 9}{x^2 - 7x + 12}$

8. $\dfrac{a^2 + 4a - 12}{a^2 + 2a - 8}$

9. $\dfrac{2x^2 - x - 21}{2x^2 - 15x + 28}$

Example 2
(p. 584)

AQUARIUMS For Exercises 10 and 11, use the following information.
Jenna has guppies in her aquarium. One week later, she adds four neon fish.

10. Define a variable. Then write an expression that represents the fraction of neon fish in the aquarium.

11. Suppose that two months later the guppy population doubles. Jenna still has four neons, and she buys 5 different tropical fish. Write an expression that shows the fraction of neons in the aquarium after the other fish have been added.

Exercises

State the excluded values for each rational expression.

12. $\dfrac{m + 3}{m - 2}$

13. $\dfrac{3b}{b + 5}$
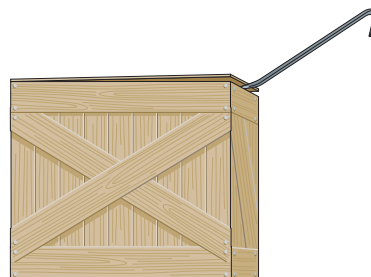
14. $\dfrac{3n + 18}{n^2 - 36}$

15. $\dfrac{2x - 10}{x^2 - 25}$

16. $\dfrac{n^2 - 36}{n^2 + n - 30}$

17. $\dfrac{25 - x^2}{x^2 + 12x + 35}$

PHYSICAL SCIENCE For Exercises 18 and 19, use the following information.
To pry the lid off a crate, a crowbar that is 18.2 inches long is used as a lever. It is placed so that 1.5 inches of its length extends inward from the edge of the crate.

18. Write an equation that can be used to calculate the mechanical advantage. Then find the mechanical advantage.

19. If a force of 16 pounds is applied to the end of the crowbar, what is the force placed on the lid?

COOKING For Exercises 20 and 21, use the following information.
The formula $t = \dfrac{40(25 + 1.85a)}{50 - 1.85a}$ relates the time t in minutes that it takes to cook an average-size potato in an oven that is at an altitude of a thousands of feet.

20. What is the value of a for an altitude of 4500 feet?

21. Calculate the time it takes to cook a potato at an altitude of 3500 feet and at 7000 feet. How do your cooking times compare at these two altitudes?

Simplify each expression. State the excluded values of the variables.

22. $\dfrac{35yz^2}{14y^2z}$

23. $\dfrac{14a^3b^2}{42ab^3}$

24. $\dfrac{64qr^2s}{16q^2rs}$

25. $\dfrac{9x^2yz}{24xyz^2}$

26. $\dfrac{7a^3b^2}{21a^2b + 49ab^3}$

27. $\dfrac{3m^2n^3}{36mn^3 - 12m^2n^2}$

28. $\dfrac{x^2 + x - 20}{x + 5}$

29. $\dfrac{z^2 + 10z + 16}{z + 2}$

30. $\dfrac{4x + 8}{x^2 + 6x + 8}$

31. $\dfrac{2y - 4}{y^2 + 3y - 10}$

32. $\dfrac{m^2 - 36}{m^2 - 5m - 6}$

33. $\dfrac{a^2 - 9}{a^2 + 6a - 27}$

34. $\dfrac{x^2 + x - 2}{x^2 - 3x + 2}$

35. $\dfrac{b^2 + 2b - 8}{b^2 - 20b + 64}$

36. $\dfrac{x^2 - x - 20}{x^3 + 10x^2 + 24x}$

37. $\dfrac{n^2 - 8n + 12}{n^3 - 12n^2 + 36n}$

38. $\dfrac{4x^2 - 6x - 4}{2x^2 - 8x + 8}$

39. $\dfrac{3m^2 + 9m + 6}{4m^2 + 12m + 8}$

FIELD TRIPS For Exercises 40–43, use the following information.

Mrs. Hoffman's art class is taking a trip to the museum. A bus that can seat up to 56 people costs $450 for the day, and group rate tickets at the museum cost $4 each.

40. If there are no more than 56 students going on the field trip, write an expression for the total cost for n students to go to the museum.

41. Write a rational expression that could be used to calculate the cost of the trip per student.

42. How many students must attend in order to keep the cost under $15 per student?

43. How would you change the expression for cost per student if the school were to cover the cost of two adult chaperones?

44. AGRICULTURE Some farmers use an irrigation system that waters a circular region in a field. Suppose a square field with sides of length $2x$ is irrigated from the center of the square. The irrigation system can reach a radius of x. What percent of the field is irrigated to the nearest whole percent?
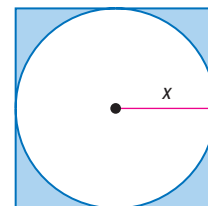
H.O.T. Problems

45. OPEN ENDED Write a rational expression involving one variable for which the excluded values are -4 and -7. Explain how you found the expression.

46. REASONING Explain why -2 may not be the only excluded value of a rational expression that simplifies to $\dfrac{x - 3}{x + 2}$.

47. CHALLENGE Two students graphed the following equations on their calculators.

$$y = \frac{x^2 - 16}{x - 4} \qquad\qquad y = x + 4$$

They were surprised to see that the graphs appeared to be identical. Explain how the graphs are different.

48. *Writing in Math* Use the information on page 583 to explain how rational expressions can be used in a movie theater. Include a description of how you determine the excluded values of the rational expression that is given.

49. The area of each wall in LaTisha's room is $x^2 + 3x + 2$ square feet. A gallon of paint will cover an area of $x^2 - 2x - 3$ square feet. Which expression gives the number of gallons of paint that LaTisha will need to buy to paint her room?

A $\dfrac{4x}{x-3}$

B $\dfrac{x+2}{x-3}$

C $\dfrac{4x+4}{x-3}$

D $\dfrac{4x+8}{x-3}$

50. REVIEW What is the volume of the triangular prism shown below?
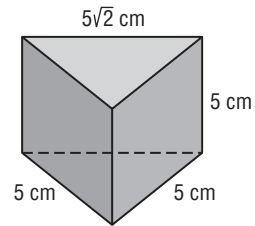
5√2 cm
5 cm
5 cm 5 cm

F 12.5 cm^3

G $25\sqrt{2} \text{ cm}^3$

H 62.5 cm^3

J $125\sqrt{2} \text{ cm}^3$

Spiral Review

Write an inverse variation equation that relates x and y. Assume that y varies inversely as x. Then solve. (Lesson 11-1)

51. If $y = 6$ when $x = 10$, find y when $x = -12$.
52. If $y = 16$ when $x = \frac{1}{2}$, find x when $y = 32$.

53. If $y = -9$ when $x = 6$, find x when $y = 3$.
54. If $y = -2.5$ when $x = 3$, find y when $x = -8$.

For each set of measures given, find the measures of the missing sides if $\triangle KLM \sim \triangle NOP$. (Lesson 10-6)

55. $k = 5, \ell = 3, m = 2, n = 10$

56. $\ell = 9, m = 3, n = 12, p = 4.5$
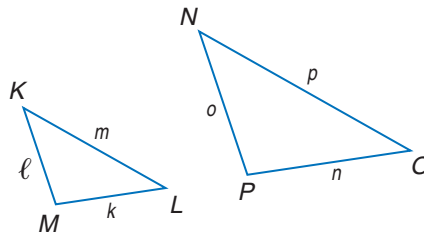
Solve each equation. Check your solution(s). (Lesson 10-3)

57. $\sqrt{a+3} = 2$

58. $\sqrt{2z+2} = z - 3$

59. $\sqrt{13-4p} - p = 8$

60. $\sqrt{3r^2+61} = 2r+1$

61. GROCERIES The Ricardos drink about 5 gallons of milk every 2.5 weeks. At this rate, how much money will they spend on milk in a year if it costs $3.58 a gallon? (Lesson 2-6)

GET READY for the Next Lesson

PREREQUISITE SKILL Complete.

62. 84 in. = ___ ft

63. 4.5 m = ___ cm

64. 4 h 15 min = ___ s

65. 18 mi = ___ ft

66. 3 days = ___ h

67. 220 mL = ___ L

Graphing Calculator Lab
Rational Expressions

When simplifying rational expressions, you can use a TI-83/84 Plus graphing calculator to support your answer. If the graphs of the original expression and the simplified expression coincide, they are equivalent.

ACTIVITY

Simplify $\dfrac{x^2 + 5x}{x^2 + 10x + 25}$.
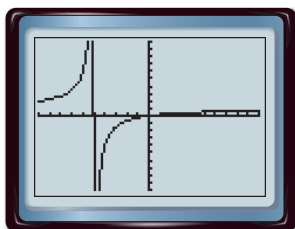
Step 1 Factor the numerator and denominator.

$$\frac{x^2 + 5x}{x^2 + 10x + 25} = \frac{x(x + 5)}{(x + 5)(x + 5)} = \frac{x}{(x + 5)}$$

When $x = -5$, $x + 5 = 0$. Therefore, x cannot equal -5 because you cannot divide by zero.

Step 2 Graph the original expression.

- Set the calculator to **Dot** mode.

- Enter $\dfrac{x^2 + 5x}{x^2 + 10x + 25}$ as **Y1** and graph.

KEYSTROKES: MODE ▼ ▼ ▼ ▼ ▶
ENTER Y= (X,T,θ,n x^2
+ 5 X,T,θ,n) ÷ (X,T,θ,n
x^2 + 10 X,T,θ,n + 25)
ZOOM 6

[−10, 10] scl: 1 by [−10, 10] scl: 1

Step 3 Graph the simplified expression.

- Enter $\dfrac{x}{(x + 5)}$ as **Y2** and graph.

KEYSTROKES: Y= ▼ X,T,θ,n ÷
(X,T,θ,n +
5) GRAPH
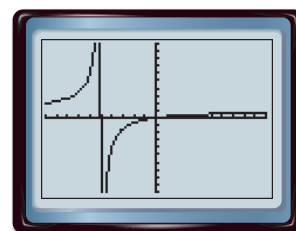
[−10, 10] scl: 1 by [−10, 10] scl: 1

Since the graphs overlap, the two expressions are equivalent.

EXERCISES

Simplify each expression. Then verify your answer graphically. Name the excluded values.

1. $\dfrac{3x + 6}{x^2 + 7x + 10}$

2. $\dfrac{2x + 8}{x^2 + 6x + 8}$

3. $\dfrac{5x^2 + 10x + 5}{3x^2 + 6x + 3}$

4. Simplify $\dfrac{2x - 9}{4x^2 - 18x}$ and answer each question using the **TABLE** menu.

   a. How can you use the **TABLE** function to verify that the original expression and the simplified expression are equivalent?

   b. How does the **TABLE** function show you that an x-value is excluded?

Main Ideas

- Multiply rational expressions.
- Use dimensional analysis with multiplication.

There are 25 lights around a patio. Each light is 40 watts, and the cost of electricity is 15 cents per kilowatt-hour. You can use the expression below to calculate the cost of using the lights for h hours.

$$25 \text{ lights} \cdot \frac{40 \text{ watts}}{\text{light}} \cdot \frac{1 \text{ kilowatt}}{1000 \text{ watts}} \cdot \frac{15 \text{ cents}}{1 \text{ kilowatt} \cdot \text{hour}} \cdot \frac{1 \text{ dollar}}{100 \text{ cents}} \cdot h \text{ hours}$$

MULTIPLY RATIONAL EXPRESSIONS The multiplication expression above is similar to the multiplication of rational expressions. Recall that to multiply fractions, you multiply numerators and multiply denominators. You can use the same method to multiply rational expressions.

Study Tip

Rational Expressions

From this point on, you may assume that no denominator of a rational expression in this text has a value of zero.

EXAMPLE Expressions Involving Monomials

1 Find $\dfrac{5ab^3}{8c^2} \cdot \dfrac{16c^3}{15a^2b}$.

Method 1 Divide by the greatest common factor after multiplying.

$$\frac{5ab^3}{8c^2} \cdot \frac{16c^3}{15a^2b} = \frac{80ab^3c^3}{120a^2bc^2} \qquad \begin{array}{l} \leftarrow \text{Multiply the numerators.} \\ \leftarrow \text{Multiply the denominators.} \end{array}$$

$$= \frac{\overset{1}{\cancel{40abc^2}}(2b^2c)}{\underset{1}{\cancel{40abc^2}}(3a)} \qquad \text{The GCF is } 40abc^2.$$

$$= \frac{2b^2c}{3a} \qquad \text{Simplify.}$$

Method 2 Divide by the common factors before multiplying.

$$\frac{5ab^3}{8c^2} \cdot \frac{16c^3}{15a^2b} = \frac{\overset{1}{\cancel{5}}a\overset{b^2}{\cancel{b^3}}}{\underset{1}{\cancel{8}}\underset{1}{\cancel{c^2}}} \cdot \frac{\overset{2}{\cancel{16}}\overset{c}{\cancel{c^3}}}{\underset{3}{\cancel{15}}\underset{a}{\cancel{a^2}}\underset{1}{\cancel{b}}} \qquad \text{Divide by common factors 5, 8, } a, b, \text{ and } c^2.$$

$$= \frac{2b^2c}{3a} \qquad \text{Multiply.}$$

✓ **CHECK Your Progress** Find each product.

1A. $\dfrac{5c^3d}{c^4d} \cdot \dfrac{f^2d^3c}{10cf^4}$

1B. $\dfrac{16g^2h^3}{8gh^2} \cdot \dfrac{3g^2h}{32hj^2}$

Sometimes you must factor a quadratic expression before you can simplify a product of rational expressions.

EXAMPLE Expressions Involving Polynomials

2 Find $\dfrac{x-5}{x} \cdot \dfrac{x^2}{x^2 - 2x - 15}$.

$$\dfrac{x-5}{x} \cdot \dfrac{x^2}{x^2 - 2x - 15} = \dfrac{x-5}{x} \cdot \dfrac{x^2}{(x-5)(x+3)} \qquad \text{Factor the denominator.}$$

$$= \dfrac{\overset{x}{\cancel{x^2}}(\overset{1}{\cancel{x-5}})}{\underset{1}{\cancel{x}}(\underset{1}{\cancel{x-5}})(x+3)} \qquad \text{The GCF is } x(x-5).$$

$$= \dfrac{x}{x+3} \qquad \text{Simplify.}$$

CHECK Your Progress Find each product.

2A. $\dfrac{x+3}{x} \cdot \dfrac{5}{x^2 + 7x + 12}$

2B. $\dfrac{y^2 - 3y - 4}{y + 5} \cdot \dfrac{y + 5}{y^2 - 4y}$

DIMENSIONAL ANALYSIS When you multiply fractions that involve units of measure, you can divide by the units in the same way that you divide by variables. Recall that this process is called *dimensional analysis.*

Real-World EXAMPLE Dimensional Analysis

3 **OLYMPICS** In the 2004 Summer Olympics in Athens, Greece, Justin Gatlin of the United States won the gold medal for the 100-meter sprint. His winning time was 9.85 seconds. What was his speed in kilometers per hour? Round to the nearest hundredth.

$$\dfrac{100 \text{ m}}{9.85 \text{ s}} \cdot \dfrac{1 \text{ k}}{1000 \text{ m}} \cdot \dfrac{60 \text{ s}}{1 \text{ min}} \cdot \dfrac{60 \text{ min}}{1 \text{ h}} = \dfrac{100 \cancel{\text{ m}}}{9.85 \cancel{\text{ s}}} \cdot \dfrac{1 \text{ k}}{1000 \cancel{\text{ m}}} \cdot \dfrac{60 \cancel{\text{ s}}}{1 \cancel{\text{ min}}} \cdot \dfrac{60 \cancel{\text{ min}}}{1 \text{ h}}$$

$$= \dfrac{\overset{1}{\cancel{100}} \cdot 1 \cdot 60 \cdot 60 \cdot \text{k}}{9.85 \cdot \underset{10}{\cancel{1000}} \cdot 1 \cdot 1 \text{ h}} \qquad \text{Simplify.}$$

$$= \dfrac{60 \cdot 60 \text{ k}}{9.85 \cdot 10 \text{ h}} \qquad \text{Multiply.}$$

$$= \dfrac{3600 \text{ k}}{98.5 \text{ h}} \qquad \text{Multiply.}$$

$$= \dfrac{36.54 \text{ k}}{1 \text{ h}} \qquad \text{Divide numerator and denominator by 98.5.}$$

His speed was 36.54 kilometers per hour.

CHECK Your Progress

3. SPEED Todd is driving to his grandparents' house at 65 miles per hour. How fast is he going in feet per second?

Online Personal Tutor at algebra1.com

✓ CHECK Your Understanding

Find each product.

Example 1
(p. 590)

1. $\dfrac{64y^2}{5y} \cdot \dfrac{5y}{8y}$

2. $\dfrac{15s^2t^3}{12st} \cdot \dfrac{16st^2}{10s^3t^3}$

Example 2
(p. 591)

3. $\dfrac{m+4}{3m} \cdot \dfrac{4m^2}{(m+4)(m+5)}$

4. $\dfrac{x^2-4}{2} \cdot \dfrac{4}{x-2}$

5. $\dfrac{n^2-16}{n+4} \cdot \dfrac{n+2}{n^2+-8n+16}$

6. $\dfrac{x-5}{x^2-7x+10} \cdot \dfrac{x^2+x-6}{5}$

Example 3
(p. 591)

7. Find $\dfrac{24 \text{ feet}}{1 \text{ second}} \cdot \dfrac{60 \text{ seconds}}{1 \text{ minute}} \cdot \dfrac{60 \text{ minutes}}{1 \text{ hour}} \cdot \dfrac{1 \text{ mile}}{5280 \text{ feet}}$.

8. SPACE The Moon is about 240,000 miles from Earth. How many days would it take a spacecraft to reach the Moon if it travels at an average speed of 100 miles per minute?

Exercises

Find each product.

9. $\dfrac{8}{x^2} \cdot \dfrac{x^4}{4x}$

10. $\dfrac{10r^3}{6n^3} \cdot \dfrac{42n^2}{35r^3}$

11. $\dfrac{10y^3z^2}{6wx^3} \cdot \dfrac{12w^2x^2}{25y^2z^4}$

12. $\dfrac{3a^2b}{2gh} \cdot \dfrac{24g^2h}{15ab^2}$

13. $\dfrac{(x-8)}{(x+8)(x-3)} \cdot \dfrac{(x+4)(x-3)}{(x-8)}$

14. $\dfrac{(n-1)(n+1)}{(n+1)} \cdot \dfrac{(n-4)}{(n-1)(n+4)}$

15. $\dfrac{(z+4)(z+6)}{(z-6)(z+1)} \cdot \dfrac{(z+1)(z-5)}{(z+3)(z+4)}$

16. $\dfrac{(x-1)(x+7)}{(x-7)(x-4)} \cdot \dfrac{(x-4)(x+10)}{(x+1)(x+10)}$

17. $\dfrac{x^2-25}{9} \cdot \dfrac{x+5}{x-5}$

18. $\dfrac{y^2-4}{y^2-1} \cdot \dfrac{y+1}{y+2}$

19. $\dfrac{x+3}{x+4} \cdot \dfrac{x}{x^2+7x+12}$

20. $\dfrac{n}{n^2+8n+15} \cdot \dfrac{2n+10}{n^2}$

21. $\dfrac{b^2+12b+11}{b^2-9} \cdot \dfrac{b+9}{b^2+20b+99}$

22. $\dfrac{a^2-a-6}{a^2-16} \cdot \dfrac{a^2+7a+12}{a^2+4a+4}$

23. DECORATING Alani's bedroom is 12 feet wide and 14 feet long. What will it cost to carpet her room if the carpet costs $18 per square yard? Is this a reasonable answer?

24. EXCHANGE RATES While traveling in Canada, Johanna bought some gifts to bring home. She bought 2 T-shirts that cost a total of $21.95 (Canadian). If the exchange rate at the time was 1 U.S. dollar for 1.21 Canadian dollars, how much did Johanna spend in U.S. dollars?

25. RESEARCH Use the Internet or other sources to research exchange rates for the U.S. dollar against a foreign currency of your choosing over the last six months. What has been the average rate of exchange? What has been the overall trend of the exchange rate? What current events have affected the change in rates?

Find each product.

26. $\dfrac{2.54 \text{ centimeters}}{1 \text{ inch}} \cdot \dfrac{12 \text{ inches}}{1 \text{ foot}} \cdot \dfrac{3 \text{ feet}}{1 \text{ yard}}$

27. $\dfrac{60 \text{ kilometers}}{1 \text{ hour}} \cdot \dfrac{1000 \text{ meters}}{1 \text{ kilometer}} \cdot \dfrac{1 \text{ hour}}{60 \text{ minutes}} \cdot \dfrac{1 \text{ minute}}{60 \text{ seconds}}$

28. $\dfrac{32 \text{ feet}}{1 \text{ second}} \cdot \dfrac{60 \text{ seconds}}{1 \text{ minute}} \cdot \dfrac{60 \text{ minutes}}{1 \text{ hour}} \cdot \dfrac{1 \text{ mile}}{5280 \text{ feet}}$

29. $10 \text{ feet} \cdot 18 \text{ feet} \cdot 3 \text{ feet} \cdot \dfrac{1 \text{ yard}^3}{27 \text{ feet}^3}$

30. CITY MAINTENANCE Street sweepers can clean 3 miles of streets per hour. A city owns 2 street sweepers, and each sweeper can be used for three hours before it comes in for an hour to refuel. During an 18 hour shift, how many miles of street can be cleaned?

TRAINS For Exercises 31–33, use the following information.

Trying to get into a train yard one evening, all of the trains are backed up for 2 miles along a system of tracks. Assume that each car occupies an average of 75 feet of space on a track and that the train yard has 5 tracks.

31. Write and solve an expression that could be used to determine the number of train cars involved in the backup.

32. How many train cars are involved in the backup?

33. Suppose that there are 8 attendants doing safety checks on each car, and it takes each vehicle an average of 45 seconds for each check. Approximately how many hours will it take for all the vehicles in the backup to exit?

H.O.T. Problems

34. OPEN ENDED Write two rational expressions with a product of $\dfrac{2}{x}$.

35. CHALLENGE Identify the expressions that are equivalent to $\dfrac{x}{y}$. Explain why the expressions are equivalent.

a. $\dfrac{x+3}{y+3}$ **b.** $\dfrac{3-x}{3-y}$ **c.** $\dfrac{3x}{3y}$ **d.** $\dfrac{x^3}{y^3}$ **e.** $\dfrac{n^3 x}{n^3 y}$

36. FIND THE ERROR Amiri and Hoshi multiplied $\dfrac{x-3}{x+3}$ and $\dfrac{4x}{x^2-4x+3}$. Who is correct? Explain your reasoning.

Amiri

$\dfrac{x-3}{x+3} \cdot \dfrac{4x}{x^2-4x+3}$

$= \dfrac{(x-3)4x}{(x+3)(x-3)(x-1)}$

$= \dfrac{4x}{(x+3)(x-1)}$

Hoshi

$\dfrac{x-3}{x+3} \cdot \dfrac{4x}{x^2-4x+3}$

$= \dfrac{4x}{x^2-4x+3}$

$= \dfrac{1}{x^2+3}$

37. CHALLENGE Explain why $-\dfrac{x+6}{x-5}$ is not equivalent to $\dfrac{-x+6}{x-5}$. What property of mathematics was used to reach this conclusion?

38. *Writing in Math* Use the information provided on page 590 to explain how multiplying rational expressions can determine the cost of electricity. Include an expression that you could use to determine the cost of using 60-watt light bulbs instead of 40-watt bulbs.

39. In order to stay in a low-Earth orbit, an object must reach a speed of about 17,500 miles per hour. How fast is this in meters per second? (1609.34 meters $\approx$ 1 mile)

 A 10.9 m/s

 B 7823.18 m/s

 C 469,390.8 m/s

 D 2.8×10^7 m/s

40. REVIEW Stanley used toothpicks to make the shapes below. If x is a shape's order in the pattern (for the first shape $x = 1$, for the second shape $x = 2$, and so on), which expression can be used to find the number of toothpicks needed to make any shape in the pattern?

 F $3x - 3$

 G $4x$

 H $3x + 1$

 J $4x + 3$

Spiral Review

State the excluded values for each rational expression. (Lesson 11-2)

41. $\dfrac{s + 6}{s^2 - 36}$

42. $\dfrac{a^2 - 25}{a^2 + 3a - 10}$

43. $\dfrac{x + 3}{x^2 + 6x + 9}$

Write an inverse variation equation that relates x and y. Assume that y varies inversely as x. Then solve. (Lesson 11-1)

44. If $y = 9$ when $x = 8$, find x when $y = 6$.

45. If $y = 2.4$ when $x = 8.1$, find y when $x = 3.6$.

46. If $y = 24$ when $x = -8$, find y when $x = 4$.

47. If $y = 6.4$ when $x = 4.4$, find x when $y = 3.2$.

Solve each inequality. Then check your solution. (Lesson 6-2)

48. $\dfrac{g}{8} < \dfrac{7}{2}$

49. $3.5r \geq 7.35$

50. $\dfrac{9k}{4} > \dfrac{3}{5}$

Simplify. Assume that no denominator is equal to zero. (Lesson 7-2)

51. $\dfrac{-7^{12}}{7^9}$

52. $\dfrac{20p^6}{8p^8}$

53. $\dfrac{24a^3b^4c^7}{6a^6c^2}$

54. FINANCE The total amount of money Antonio earns mowing lawns and doing yard work varies directly with the number of days he works. At one point, he earned $340 in 4 days. At this rate, how long will it take him to earn $935? (Lesson 4-2)

GET READY for the Next Lesson

Factor each polynomial (Lessons 8-3 and 8-4)

55. $x^2 - 3x - 40$

56. $n^2 - 64$

57. $x^2 - 12x + 36$

**58.** $a^2 + 2a - 35$

59. $2x^2 - 5x - 3$

60. $3x^3 - 24x^2 + 36x$

Main Ideas

- Divide rational expressions.
- Use dimensional analysis with division.

▶ **GET READY for the Lesson**

Most soft drinks come in aluminum cans. Although more cans are used today than in the 1970s, the demand for new aluminum has declined. This is due in large part to the great number of cans that are recycled. In recent years, approximately 63.9 billion cans were recycled annually. This represents $\frac{5}{8}$ of all cans produced.
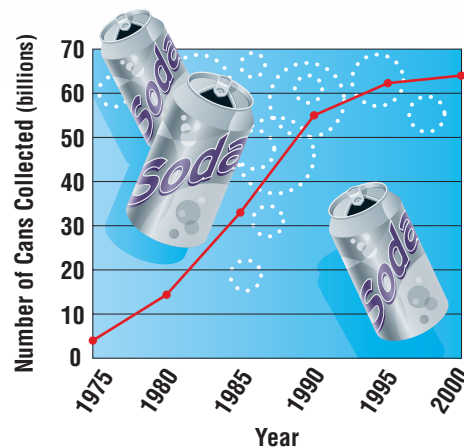
DIVIDE RATIONAL EXPRESSIONS Recall that to divide fractions, you multiply by the reciprocal of the divisor. You can use this same method to divide rational expressions.

EXAMPLE Divide by Fractions

1 Find each quotient.

a. $\dfrac{5x^2}{7} \div \dfrac{10x^3}{21}$

$\dfrac{5x^2}{7} \div \dfrac{10x^3}{21} = \dfrac{5x^2}{7} \cdot \dfrac{21}{10x^3}$ Multiply by $\dfrac{21}{10x^3}$, the reciprocal of $\dfrac{10x^3}{21}$.

$= \dfrac{\overset{1}{5x^2}}{\underset{1}{7}} \cdot \dfrac{\overset{3}{21}}{\underset{2x}{10x^3}}$ Divide by common factors 5, 7, and x^2.

$= \dfrac{3}{2x}$ Simplify.

b. $\dfrac{n+1}{n+3} \div \dfrac{2n+2}{n+4}$

$\dfrac{n+1}{n+3} \div \dfrac{2n+2}{n+4} = \dfrac{n+1}{n+3} \cdot \dfrac{n+4}{2n+2}$ Multiply by $\dfrac{n+4}{2n+2}$, the reciprocal of $\dfrac{2n+2}{n+4}$.

$= \dfrac{n+1}{n+3} \cdot \dfrac{n+4}{2(n+1)}$ Factor $2n+2$.

$= \dfrac{\overset{1}{n+1}}{n+3} \cdot \dfrac{n+4}{2\underset{1}{(n+1)}}$ The GCF is $n+1$.

$= \dfrac{n+4}{2(n+3)}$ or $\dfrac{n+4}{2n+6}$ Simplify.

1A. $\dfrac{15y^2}{4x} \div \dfrac{5y}{8x^3}$

1B. $\dfrac{27c^3d^2}{11d} \div \dfrac{2c^3}{9d^3e}$

1C. $\dfrac{b+4}{3b+2} \div \dfrac{3b+12}{b+1}$

1D. $\dfrac{6b-12}{3b+15} \div \dfrac{12b+18}{b+5}$

Sometimes you must factor a quadratic expression before you can simplify the quotient of rational expressions.

EXAMPLE **Expression Involving Polynomials**

2 Find $\dfrac{m^2+3m+2}{4} \div \dfrac{m+2}{m+1}$.

$$\dfrac{m^2+3m+2}{4} \div \dfrac{m+2}{m+1} = \dfrac{m^2+3m+2}{4} \cdot \dfrac{m+1}{m+2} \qquad \text{Multiply by the reciprocal, } \dfrac{m+1}{m+2}.$$

$$= \dfrac{(m+1)(m+2)}{4} \cdot \dfrac{m+1}{m+2} \qquad \text{Factor } m^2+3m+2.$$

$$= \dfrac{(m+1)(\overset{1}{\cancel{m+2}})}{4} \cdot \dfrac{m+1}{\underset{1}{\cancel{m+2}}} \qquad \text{The GCF is } m+2.$$

$$= \dfrac{(m+1)^2}{4} \qquad \text{Simplify.}$$

✔ **CHECK Your Progress**

Find each quotient.

2A. $\dfrac{p^2-4}{5p} \div \dfrac{p-2}{p+q}$

2B. $\dfrac{q^2+3q+2}{12} \div \dfrac{q+1}{q^2+4}$

DIMENSIONAL ANALYSIS You can divide rational expressions that involve units of measure by using dimensional analysis.

Real-World EXAMPLE

3 **SPACE** In April, 2001, NASA launched the *Mars Odyssey* spacecraft. It took 200 days for the spacecraft to travel 466,000,000 miles from Earth to Mars. What was the average speed of the spacecraft in miles per hour? Round to the nearest mile per hour.

$$rt = d \qquad\qquad\qquad \text{rate} \cdot \text{time} = \text{distance}$$

$$r \cdot 200 \text{ days} = 466{,}000{,}000 \text{ mi} \qquad t = 200 \text{ days, } d = 466{,}000{,}000$$

$$r = \dfrac{466{,}000{,}000 \text{ mi}}{200 \text{ days}} \qquad \text{Divide each side by 200 days.}$$

$$= \dfrac{466{,}000{,}000 \text{ miles}}{200 \text{ days}} \cdot \dfrac{1 \text{ day}}{24 \text{ hours}} \qquad \text{Convert days to hours.}$$

$$= \dfrac{466{,}000{,}000 \text{ miles}}{4800 \text{ hours}} \text{ or about } \dfrac{97{,}083 \text{ miles}}{1 \text{ hour}}$$

Thus, the spacecraft traveled at a rate of about 97,083 miles per hour.

Real-World Link

The first successful Mars probe was the Mariner 4, which arrived at Mars on July 14, 1965.

Source: NASA

3. On July 7, 2003, the rover *Opportunity* was launched. It landed on Mars on January 25, 2004. Assuming that *Opportunity* traveled the same distance as the *Mars Odyssey Spacecraft*, how fast did *Opportunity* travel? Round to the nearest mile per hour.

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✓ CHECK Your Understanding

Find each quotient.

Example 1
(p. 595)

1. $\dfrac{10n^3}{7} \div \dfrac{5n^2}{21}$

2. $\dfrac{2a}{3} \div \dfrac{a^7}{b^3}$

3. $\dfrac{3m+15}{m+4} \div \dfrac{m+5}{6m+24}$

4. $\dfrac{3n^2-12}{n-6} \div \dfrac{(n-6)(n-2)}{n+4}$

Example 2
(p. 596)

5. $\dfrac{k+3}{k^2+4k+4} \div \dfrac{2k+6}{k+2}$

6. $\dfrac{2x+4}{x^2+11x+18} \div \dfrac{x+1}{x^2+5x+6}$

Example 3
(p. 596)

7. Express 85 kilometers per hour in meters per second.

8. Express 32 pounds per square foot as pounds per square inch.

9. COOKING Latisha was making candy using a two-quart pan. As she stirred the mixture, she noticed that the pan was about $\frac{2}{3}$ full. If each piece of candy has a volume of about $\frac{3}{4}$ ounce, approximately how many pieces of candy will Latisha make? (*Hint*: There are 32 ounces in a quart.)

Exercises

Find each quotient.

HOMEWORK HELP	
For Exercises	**See Examples**
10–15	1
16–21	2
28, 29	3

10. $\dfrac{a^2}{b^2} \div \dfrac{a}{b^3}$

11. $\dfrac{n^4}{p^2} \div \dfrac{n^2}{p^3}$

12. $\dfrac{10m^2}{7n^2} \div \dfrac{25m^4}{14n^3}$

13. $\dfrac{a^4bc^3}{g^2h^3} \div \dfrac{ab^2c^2}{g^3h^3}$

14. $\dfrac{3x+12}{4x-18} \div \dfrac{2x+8}{x+4}$

15. $\dfrac{4a-8}{2a-6} \div \dfrac{2a-4}{a-4}$

16. $\dfrac{x^2+2x+1}{2} \div \dfrac{x+1}{x-1}$

17. $\dfrac{n^2+3n+2}{4} \div \dfrac{n+1}{n+2}$

18. $\dfrac{a^2+8a+16}{a^2-6a+9} \div \dfrac{2a+8}{3a-9}$

19. $\dfrac{b+2}{b^2+4b+4} \div \dfrac{2b+4}{b+4}$

20. $\dfrac{x^2+x-2}{x^2+5x+6} \div \dfrac{x^2+2x-3}{x^2+7x+12}$

21. $\dfrac{x^2+2x-15}{x^2-x-30} \div \dfrac{x^2-3x-18}{x^2-2x-24}$

22. What is the quotient when $\dfrac{2x+6}{x+5}$ is divided by $\dfrac{2}{x+5}$?

23. Find the quotient when $\dfrac{m-8}{m+7}$ is divided by m^2-7m-8.

Complete.

24. $24 \text{ yd}^3 = \underline{\quad} \text{ ft}^3$

25. $0.35 \text{ m}^3 = \underline{\quad} \text{ cm}^3$

26. $330 \text{ ft/s} = \underline{\quad} \text{ mi/h}$

27. $1730 \text{ plants/km}^2 = \underline{\quad} \text{ plants/m}^2$

28. TRIATHLONS Sadie is training for an upcoming triathlon and plans to run the full length of the running section today. Jorge offered to ride his bicycle to help her maintain her pace. If Sadie wants to finish her run in about 4 hours and 28 minutes, how fast should Jorge ride in miles per hour?

29. VOLUNTEERING Tyrell is passing out orange drink from a 3.5-gallon cooler. If each cup of orange drink is 4.25 ounces, about how many cups can he hand out? (*Hint*: There are 128 ounces in a gallon.)

LANDSCAPING For Exercises 30 and 31, use the following information.

A landscaping supervisor needs to determine how many truckloads of dirt must be removed from a site before a brick patio can be completed. The truck bed has the shape shown at the right.
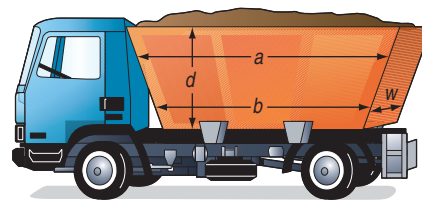
30. Write an equation involving units that represents the volume of the truck bed in cubic yards. Use the formula $V = \dfrac{d(a + b)}{2} \cdot w$ with $a = 10$ feet, $b = 17$ feet, $w = 4$ feet, and $d = 3.5$ feet.

31. The supervisor found that there are 45 cubic yards of dirt that must be removed from the site. Write an equation involving units that represents the number of truckloads that will be required to remove all of the dirt. How many trips will they have to take to remove all the dirt?

TRUCKS For Exercises 32 and 33, use the following information.

The speedometer of John's truck uses the revolutions of his tires to calculate the speed of the truck.

32. How many times per minute do the tires revolve when the truck is traveling at 55 miles per hour?

33. Suppose John buys tires with a diameter of 30 inches. When the speedometer reads 55 miles per hour, the tires would still revolve at the same rate as before. However, with the new tires, the truck travels a different distance in each revolution. Calculate the actual speed when the speedometer reads 55 miles per hour.

26 in.
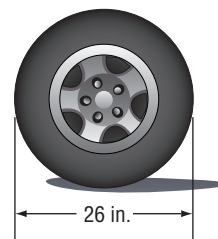
SCULPTURE For Exercises 34 and 35, use the following information.

A sculptor had a block of marble in the shape of a cube with sides x feet long. A piece that was $\frac{1}{2}$-foot thick was chiseled from the bottom of the block. Later, the sculptor removed a piece $\frac{3}{4}$-foot wide from the side of the marble block.

34. Write a rational expression that represents the volume of the block of marble.

35. If the remaining marble was cut into pieces weighing 85 pounds each, write an expression that represents the weight of the original block of marble.

H.O.T. Problems

36. OPEN ENDED Give an example of a real-world situation that could be modeled by the quotient of two rational expressions. Provide an example of this quotient.

37. REASONING Tell whether the following statement is *always, sometimes,* or *never* true. Explain your reasoning. *For a real number x, there is a reciprocal y.*

38. CHALLENGE Which expression is *not* equivalent to the reciprocal of $\dfrac{x^2 - 4y^2}{x + 2y}$? Justify your answer.

a. $\dfrac{1}{\dfrac{x^2 - 4y^2}{x + 2y}}$

b. $\dfrac{-1}{2y - x}$

c. $\dfrac{1}{x - 2y}$

d. $\dfrac{1}{x} - \dfrac{1}{2y}$

39. *Writing in Math* Use the information about soft drinks and aluminum on page 595 to explain how you can determine the number of aluminum soft drink cans made each year. Include a rational expression that will give the amount of new aluminum needed to produce x aluminum cans today when $\dfrac{5}{8}$ of the cans are recycled and 33 cans are produced from a pound of aluminum.

STANDARDIZED TEST PRACTICE

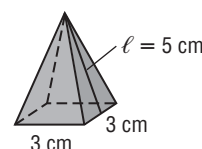**40.** Which expression could be used to represent the width of the rectangle?

$A = x^2 - 4$

$\dfrac{x^2 - x - 2}{x + 1}$
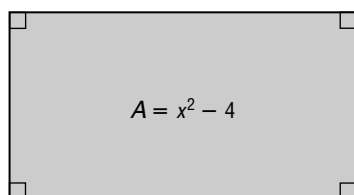
A $x - 2$

C $x + 2$

B $(x + 2)(x - 2)^2$

D $(x + 2)(x - 2)$

41. REVIEW What is the surface area of the regular square pyramid?

$\ell = 5$ cm
3 cm
3 cm

F 39 cm^2

H 83 cm^2

G 65 cm^2

J 117 cm^2

Spiral Review

Find each product. (Lesson 11-3)

42. $\dfrac{x - 5}{x^2 - 7x + 10} \cdot \dfrac{x - 2}{1}$

43. $\dfrac{x^2 + 3x - 10}{x^2 + 8x + 15} \cdot \dfrac{x^2 + 5x + 6}{x^2 + 4x + 4}$

44. $\dfrac{x + 4}{4y} \cdot \dfrac{16y}{x^2 + 7x + 12}$

Simplify each expression. (Lesson 11-2)

45. $\dfrac{c - 6}{c^2 - 12c + 36}$

46. $\dfrac{25 - x^2}{x^2 + x - 30}$

47. $\dfrac{a + 3}{a^2 + 4a + 3}$

48. $\dfrac{n^2 - 16}{n^2 - 8n + 16}$

49. MANUFACTURING Global Sporting Equipment sells tennis racket covers for $2.35 each. It costs the company $0.68 in materials and labor for each cover and $1300 each month for equipment and building rental. Write an equation that gives the net profit the company makes, if x is the number of racket covers they can make in a month. (Lesson 4-4)

GET READY for the Next Lesson

PREREQUISITE SKILL Simplify. (Lesson 7-2)

50. $\dfrac{6x^2}{x^4}$

51. $\dfrac{5m^4}{25m}$

52. $\dfrac{b^6c^3}{b^3c^6}$

53. $\dfrac{12x^3y^2}{28x^4y}$

54. $\dfrac{7x^4z^2}{z^3}$

READING MATH

Rational Expressions

Several concepts need to be applied when reading rational expressions.

A fraction bar acts as a grouping symbol, where the entire numerator is divided by the entire denominator.

EXAMPLE

1 **Read the expression** $\frac{6x + 4}{10}$.

It is <u>correct</u> to read the expression as *the quantity six x plus four divided by ten.*

It is <u>incorrect</u> to read the expression as *six x divided by ten plus four, or six x plus four divided by ten.*

If a fraction consists of two or more terms divided by a one-term denominator, the denominator divides each term.

EXAMPLE

2 **Simplify** $\frac{6x + 4}{10}$.

It is <u>correct</u> to write $\frac{6x + 4}{10} = \frac{6x}{10} + \frac{4}{10}$.

$$= \frac{3x}{5} + \frac{2}{5} \quad \text{or} \quad \frac{3x + 2}{5}$$

It is also <u>correct</u> to write $\frac{6x + 4}{10} = \frac{2(3x + 2)}{2 \cdot 5}$.

$$= \frac{\cancel{2}(3x + 2)}{\cancel{2} \cdot 5} \quad \text{or} \quad \frac{3x + 2}{5}$$

It is <u>incorrect</u> to write $\frac{6x + 4}{10} = \frac{\overset{3x}{\cancel{6x}} + 4}{\underset{5}{\cancel{10}}} = \frac{3x + 4}{5}$.

Reading to Learn

Write the verbal translation of each rational expression.

1. $\frac{m + 2}{4}$

2. $\frac{3x}{x - 1}$

3. $\frac{a + 2}{a^2 + 8}$

4. $\frac{x^2 - 25}{x + 5}$

5. $\frac{x^2 - 3x + 18}{x - 2}$

6. $\frac{x^2 + 2x - 35}{x^2 - x - 20}$

Simplify each expression.

7. $\frac{3x + 6}{9}$

8. $\frac{2n - 12}{8}$

9. $\frac{5x^2 - 25x}{10x}$

10. $\frac{x + 3}{x^2 + 7x + 12}$

11. $\frac{x + y}{x^2 + 2xy + y^2}$

12. $\frac{x^2 - 16}{x^2 - 8x + 16}$

Dividing Polynomials

Main Ideas

- Divide a polynomial by a monomial.
- Divide a polynomial by a binomial.

Suppose a partial bolt of fabric is used to make marching band flags. The original bolt was 36 yards long, and $7\frac{1}{2}$ yards of the fabric were used to make a banner for the band. Each flag requires $1\frac{1}{2}$ yards of fabric. The expression $\dfrac{36 \text{ yards} - 7\frac{1}{2} \text{ yards}}{1\frac{1}{2} \text{ yards}}$ can be used to represent the number of flags that can be made using the bolt of fabric.

Divide Polynomials By Monomials To divide a polynomial by a monomial, divide each term of the polynomial by the monomial.

EXAMPLE — Divide Polynomials by Monomials

1 a. Find $(3r^2 - 15r) \div 3r$.

$$(3r^2 - 15r) \div 3r = \frac{3r^2 - 15r}{3r} \qquad \text{Write as a rational expression.}$$

$$= \frac{3r^2}{3r} - \frac{15r}{3r} \qquad \text{Divide each term by } 3r.$$

$$= \frac{\overset{r}{\cancel{3r^2}}}{\underset{1}{\cancel{3r}}} - \frac{\overset{5}{\cancel{15r}}}{\underset{1}{\cancel{3r}}} \qquad \text{Simplify each term.}$$

$$= r - 5 \qquad \text{Simplify.}$$

b. Find $(n^2 + 10n + 12) \div 5n$.

$$(n^2 + 10n + 12) \div 5n = \frac{n^2 + 10n + 12}{5n} \qquad \text{Write as a rational expression.}$$

$$= \frac{n^2}{5n} + \frac{10n}{5n} + \frac{12}{5n} \qquad \text{Divide each term by } 5n.$$

$$= \frac{\overset{n}{\cancel{n^2}}}{\underset{5}{\cancel{5n}}} + \frac{\overset{2}{\cancel{10n}}}{\underset{1}{\cancel{5n}}} + \frac{12}{5n} \qquad \text{Simplify each term.}$$

$$= \frac{n}{5} + 2 + \frac{12}{5n} \qquad \text{Simplify.}$$

Study Tip

Alternative Method

You could also solve Example 1a as

$\frac{3r^2 - 15r}{3r} =$

$\frac{3r(r - 5)}{3r}$ or $r - 5$.

✔ CHECK Your Progress **Find each quotient.**

1A. $(3q^3 - 6q) \div 3q$

1B. $(4t^5 - 5t^2 - 12) \div 2t^2$

Divide Polynomials by Binomials You can use algebra tiles to model some quotients of polynomials.
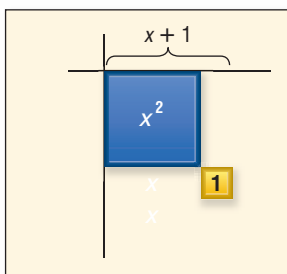
Dividing Polynomials

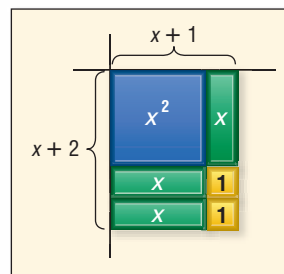**Use algebra tiles to find $(x^2 + 3x + 2) \div (x + 1)$.**

Step 1   Model the polynomial $x^2 + 3x + 2$.

Step 2   Place the x^2 tile at the corner of the product mat. Place one of the 1 tiles as shown to make a length of $x + 1$ because $x = 1$ is the divisor.

Step 3   Use the remaining tiles to make a rectangular array. Make sure the length of the rectangle, $x + 1$, does not change.

The width of the array, $x + 2$, is the quotient. This is because the dividend now fills the entire rectangular area.

MODEL AND ANALYZE

Use algebra tiles to find each quotient.

1. $(x^2 + 3x - 4) \div (x - 1)$

2. $(x^2 - 5x + 6) \div (x - 2)$

3. $(x^2 - 16) \div (x + 4)$

4. $(2x^2 - 4x - 6) \div (x - 3)$

5. Describe what happens when you try to model $(3x^2 - 4x + 3) \div (x + 2)$. What do you think the result means?

EXAMPLE   **Divide a Polynomial by a Binomial**

2 Find $(s^2 + 6s - 7) \div (s + 7)$.

$(s^2 + 6s - 7) \div (s + 7) = \dfrac{s^2 + 6s - 7}{(s + 7)}$ Write as a rational expression.

$= \dfrac{(s + 7)(s - 1)}{(s + 7)}$ Factor the numerator.

$= \dfrac{(\overset{1}{\cancel{s + 7}})(s - 1)}{\underset{1}{\cancel{(s + 7)}}}$ Divide by the GCF.

$= s - 1$ Simplify.

CHECK Your Progress   **Find each quotient.**

2A. $(b^2 - 2b - 15) \div (3 + b)$ **2B.** $(x^2 + 3x - 28) \div (x + 7)$

If you cannot factor and divide by a common factor, you can use a long division process similar to the one you use to divide numbers.

EXAMPLE Long Division

3 Find $(x^2 + 3x - 24) \div (x - 4)$.

Step 1 Divide the first term of the dividend, x^2, by the first term of the divisor, x.

$$
\begin{array}{r}
x \phantom{{}+ 3x - 24} \\
x - 4 \overline{)\, x^2 + 3x - 24} \\
(-)\; \underline{x^2 - 4x} \\
7x \phantom{{}- 24}
\end{array}
$$

$x^2 \div x = x$

Multiply x and $x - 4$.

Subtract.

Step 2 Divide the first term of the partial dividend, $7x - 24$, by the first term of the divisor, x.

$$
\begin{array}{r}
x + 7 \\
x - 4 \overline{)\, x^2 + 3x - 24} \\
(-)\; \underline{x^2 - 4x} \\
7x - 24 \\
(-)\; \underline{7x - 28} \\
4
\end{array}
$$

$7x \div x = 7$

Subtract and bring down the -24.

Multiply 7 and $x - 4$.

Subtract.

So, $(x^2 + 3x - 24) \div (x - 4)$ is $x + 7$ with a remainder of 4. This answer can be written as $x + 7 + \dfrac{4}{x - 4}$.

✓**CHECK Your Progress** Find each quotient.

3A. $(y^2 + 3y + 12) \div (y + 3)$ **3B.** $(3x^2 + 9x - 15) \div (x + 5)$

Some dividends have missing terms. These are terms that have zero as their coefficient. In this situation, you must rewrite the dividend, including the missing term with a coefficient of zero.

EXAMPLE Polynomial with Missing Terms

4 Find $(a^3 + 8a - 24) \div (a - 2)$.

$$
\begin{array}{r}
a^2 + 2a + 12 \\
a - 2 \overline{)\, a^3 + 0a^2 + 8a - 24} \\
(-)\; \underline{a^3 - 2a^2} \\
2a^2 + 8a \\
(-)\; \underline{2a^2 - 4a} \\
12a - 24 \\
(-)\; \underline{12a - 24} \\
0
\end{array}
$$

Insert an a^2 term that has a coefficient of 0.

Multiply a^2 and $a - 2$.

Subtract and bring down $8a$.

Multiply $2a$ and $a - 2$.

Subtract and bring down 24.

Multiply 12 and $a - 2$.

Subtract.

Therefore, $(a^3 + 8a - 24) \div (a - 2) = a^2 + 2a + 12$.

Concepts in MOtion

Animation
algebra1.com

✓**CHECK Your Progress** Find each quotient.

4A. $(c^4 + 2c^3 + 6c - 10) \div (c + 2)$ **4B.** $(6x^3 + 16x^2 - 60x + 39) \div (2x + 10)$

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Find each quotient.

Example 1
(p. 601)

1. $(5q^2 + q) \div q$

2. $(4z^3 + 1) \div 2z$

3. $(4x^3 + 2x^2 - 5) \div 2x$

4. $\dfrac{14a^2b^2 + 35ab^2 + 2a^2}{7a^2b^2}$

Example 2
(p. 602)

5. $(n^2 + 7n + 12) \div (n + 3)$

6. $(r^2 + 12r + 36) \div (r + 9)$

Example 3
(p. 603)

Example 4
(p. 603)

7. $(2b^2 + 3b - 5) \div (2b - 1)$

8. $(x^2 + x + 12) \div (x - 3)$

9. $\dfrac{4m^3 + 5m - 21}{2m - 3}$

10. $\dfrac{2n^4 + 2n^2 - 4}{n^2 - 1}$

11. ENVIRONMENT The equation $C = \dfrac{120{,}000p}{1 - p}$ models the cost C in dollars for a manufacturer to reduce pollutants by p percent. How much will the company have to pay to remove 75% of the pollutants it emits?

Exercises

Find each quotient.

12. $(9m^2 + 5m) \div 6m$

13. $(8k^2 - 6) \div 2k$

14. $(x^2 + 9x - 7) \div 3x$

15. $(a^2 + 7a - 28) \div 7a$

16. $\dfrac{9s^3t^2 - 15s^2t + 24t^3}{3s^2t^2}$

17. $\dfrac{12a^3b + 16ab^3 - 8ab}{4ab}$

18. $(x^2 + 9x + 20) \div (x + 5)$

19. $(x^2 + 6x - 16) \div (x - 2)$

20. $(n^2 - 2n - 35) \div (n + 5)$

21. $(s^2 + 11s + 18) \div (s + 9)$

22. $(z^2 - 2z - 30) \div (z + 7)$

23. $(a^2 + 4a - 22) \div (a - 3)$

24. $(3p^2 + 20p + 11) \div (p + 6)$

25. $(3x^3 + 8x^2 + x - 7) \div (x + 2)$

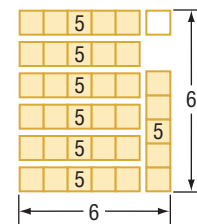**26.** $(6x^3 - 9x^2 + 6) \div (2x - 3)$

27. $(9g^3 + 5g - 8) \div (3g - 2)$

28. Determine the quotient when $6n^3 + 5n^2 + 12$ is divided by $2n + 3$.

29. What is the quotient when $4t^3 + 17t^2 - 1$ is divided by $4t + 1$?

GEOMETRY For Exercises 30–34, refer to the diagrams at the right.

30. The first picture models $6^2 \div 7$. Notice that the square is divided into seven equal parts. What are the quotient and the remainder?
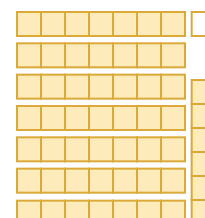
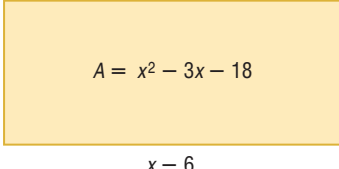**31.** What division problem does the second picture model?

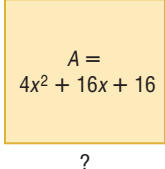**32.** Draw diagrams for $3^2 \div 4$ and $2^2 \div 3$.

33. Do you observe a pattern in the previous exercises? Express this pattern algebraically.

34. Use long division to find $x^2 \div (x + 1)$. Does this result match your expression from the previous exercise?

Use long division to find the expression that represents the missing side.

35.

$A = x^2 - 3x - 18$ | ?

$x - 6$

36.

$A = 4x^2 + 16x + 16$ | $2x + 4$

?

**ROAD TRIP** For Exercises 37–40, use the following information.
The Ski Club is taking two vans to Colorado. The first van has been on the road for 20 minutes, and the second van has been on the road for 35 minutes.

37. Write an expression for the amount of time that each van has spent on the road after an additional t minutes.

38. Write a ratio for the first van's time on the road to the second van's time on the road. Then use long division to rewrite this ratio as an expression.

39. Use the expression you wrote to find the ratio of the first van's time on the road to the second van's time on the road after 15 minutes, 60 minutes, 200 minutes, and 500 minutes.

40. As t increases, the ratio of the vans' times approaches 1. If t continues to increase, will this ratio ever be equal to 1?

Find each quotient.

41. $(21d^2 - 29d - 12) \div \left(\frac{7}{3}d - 4\right)$

42. $(x^2 - 4x + 4) \div (x + 3)$

43. $(3x^3 + 15x^2 - 12x) - 44 \div (x + 5)$

44. $\left(\frac{3}{2}x^2 - 8x - 32\right) \div (3x + 8)$

45. $\left(-5x^5 - \frac{5}{2}x^4 - 20x^3 + 5x\right) \div \left(5x + \frac{5}{2}\right)$

46. $(14y^5 + 21y^4 - 6y^3 - 9y^2 + 32y + 48) \div (2y + 3)$

47. GEOMETRY The volume of a prism with a triangular base is $10w^3 + 23w^2 + 5w - 2$. The height of the prism is $2w + 1$, and the height of the triangle is $5w - 1$. What is the measure of the base of the triangle? $\left(Hint: V = \frac{1}{2}Bh\right)$
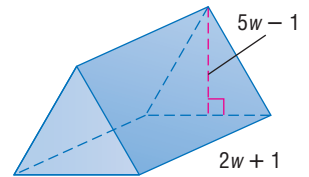
$5w - 1$

$2w + 1$

FUNCTIONS For Exercises 48–51, consider the function $f(x) = \dfrac{3x + 4}{x - 1}$.

48. Rewrite the function as a quotient plus a remainder. Then graph the quotient, ignoring the remainder.

49. Graph the original function using a graphing calculator.

50. How are the graphs of the function and quotient related?

51. What happens to the graph near the excluded value of x?

BOILING POINT For Exercises 52–54, use the following information.
The temperature at which water boils decreases by approximately 0.9°F for every 500 feet you are above sea level. The boiling point of water at sea level is 212°F.

52. Write an equation that gives the temperature at which water boils for x every foot you are above sea level.

53. Mount Whitney, the tallest point in California, is 14,494 feet above sea level. At approximately what temperature does water boil on Mount Whitney?

54. Write an expression for the quotient of the boiling point of water at any height x and the boiling point of water at half that height.

55. OPEN ENDED Write a third-degree polynomial that has a missing term. Rewrite the polynomial so that it can be divided by $x + 5$ using long division.

56. Which One Doesn't Belong? Select the divisor of $2x^2 - 9x + 9$ that does not belong with the other three. Explain your reasoning.

$x + 3$	$x - 2$	$2x - 3$	$2x + 3$

CHALLENGE Find the value of k in each situation.

57. k is an integer and $x + k$ is a factor of $x^2 + 7x + 12$.

58. When $x^2 + 7x + k$ is divided by $x + 2$, there is a remainder of 2.

59. $x + 7$ is a factor of $x^2 - 2x - k$.

60. *Writing in Math* Use the information about sewing on page 601 to describe how division can be used in sewing. Include a convincing argument to show that $\frac{a - b}{c} = \frac{a}{c} - \frac{b}{c}$.

STANDARDIZED TEST PRACTICE

61. Which expression represents the length of the rectangle?

$A = m^2 + 4m - 32$ $m - 4$
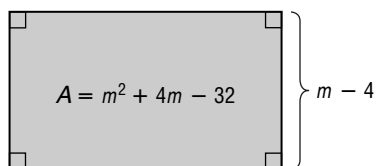
A $m + 7$ **C** $m - 7$

B $m - 8$ **D** $m + 8$

62. REVIEW Paul and Lupe are building a shelter for their dog. The length of the shelter is 4.5 feet and the width is 2.5 feet. If each corner is a right angle, what is the length of each diagonal?

F 26.5 feet   **H** 5.15 feet

G 20.25 feet   **J** 3.25 feet

Spiral Review

Find each quotient. (Lesson 11-4)

63. $\dfrac{x^2 + 5x + 6}{x^2 - x - 12} \div \dfrac{x + 2}{x^2 + x - 20}$

64. $\dfrac{m^2 + m - 6}{m^2 + 8m + 15} \div \dfrac{m^2 - m - 2}{m^2 + 9m + 20}$

Find each product. (Lesson 11-3)

65. $\dfrac{b^2 + 19b + 84}{b - 3} \cdot \dfrac{b^2 - 9}{b^2 + 15b + 36}$

66. $\dfrac{z^2 + 16z + 39}{z^2 + 9z + 18} \cdot \dfrac{z + 5}{z^2 + 18z + 65}$

67. BUSINESS Jorge Martinez has budgeted $150 to have business cards printed. A card printer charges $11 to set up each job and an additional $6 per box of 100 cards printed. What is the greatest number of cards Mr. Martinez can have printed? (Lesson 6-3)

GET READY for the Next Lesson

PREREQUISITE SKILL Find each sum. (Lesson 7-4)

68. $(6n^2 - 6n + 10m^3) + (5n - 6m^3)$

69. $(3x^2 + 4xy - 2y^2) + (x^2 + 9xy + 4y^2)$

70. $(a^3 - b^3) + (-3a^3 - 2a^2b + b^2 - 2b^3)$

71. $(2g^3 + 6h) + (-4g^2 - 8h)$

Graph each variation if y varies inversely as x. (Lesson 11-1)

1. $y = 28$ when $x = 7$

2. $y = -6$ when $x = 9$

3. If y varies inversely as x and $y = 3$ when $x = 6$, find x when $y = -14$.

4. If y varies inversely as x and $y = -6$ when $x = 9$, find y when $x = 6$.

5. **DESIGN** The height of a rectangular tank varies inversely with the area of the base. If the tank has a height of 2 feet when the area of the base is 9 square feet, how tall will the tank be if the area of the base is 6 square feet? (Lesson 11-1)

State the excluded value(s). (Lesson 11-2)

6. $\dfrac{16x + 5}{3x}$

7. $\dfrac{12y + 4}{3y + 6}$

8. $\dfrac{x^2 + 1}{x^2 - 1}$

9. $\dfrac{15x}{3x^2 - x - 2}$
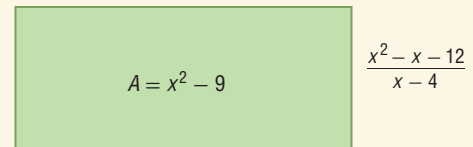
Simplify each expression. (Lesson 11-2)

10. $\dfrac{28a^2}{49ab}$

11. $\dfrac{y + 3y^2}{3y + 1}$

12. $\dfrac{b^2 - 3b - 4}{b^2 - 13b + 36}$

13. $\dfrac{3n^2 + 5n - 2}{3n^2 - 13n + 4}$

14. **LANDSCAPING** Kenyi is helping his parents landscape their yard and needs to move some large rocks. He plans to use a 6-foot bar as a lever. He positions the fulcrum 1 foot from the end of the bar touching the rock. If the rock weighs 200 pounds, how much force does he need to apply to the bar to lift the rock? (Lesson 11-2)

Find each product. (Lesson 11-3)

15. $\dfrac{3m^2}{2m} \cdot \dfrac{18m^2}{9m}$

16. $\dfrac{5a + 10}{10x^2} \cdot \dfrac{4x^3}{a^2 + 11a + 18}$

17. $\dfrac{4n + 8}{n^2 - 25} \cdot \dfrac{n - 5}{5n + 10}$

18. $\dfrac{x + 1}{3x^2 - 5x - 2} \cdot \dfrac{x - 2}{x^2 - 1}$

19. $\dfrac{x^2 - x - 6}{x^2 - 9} \cdot \dfrac{x^2 + 7x + 12}{x^2 + 4x + 4}$

20. $\dfrac{a^2 + 7a + 10}{a + 1} \cdot \dfrac{3a + 3}{a + 2}$

21. **TOYS** If a remote control car is advertised to travel at a speed of 44 feet per second, how fast can the car travel in miles per hour? (Lesson 11-3)

22. **MULTIPLE CHOICE** Which expression *best* represents the length of the rectangle? (Lesson 11-4)

$A = x^2 - 9 \qquad \dfrac{x^2 - x - 12}{x - 4}$

A $x + 4$

B $x + 3$

C $x - 3$

D $x - 4$

Find each quotient. (Lessons 11-4 and 11-5)

23. $\dfrac{a}{a + 3} \div \dfrac{a + 11}{a + 3}$

24. $\dfrac{4z + 8}{z + 3} \div (z + 2)$

25. $\dfrac{b^2 - 9}{4b} \div (b - 3)$

26. $\dfrac{m^2 - 16}{5m} \div (m + 4)$

27. $\dfrac{(2x - 1)(x - 2)}{(x - 2)(x - 3)} \div \dfrac{(2x - 1)(x + 5)}{(x - 3)(x - 1)}$

28. $(9xy^2 - 15xy + 3) \div 3xy$

29. $(2x^2 - 7x - 16) \div (2x + 3)$

30. $\dfrac{y^2 - 19y + 9}{y - 4}$

31. **DECORATING** Anoki wants to put a decorative border 3 feet above the floor around his bedroom walls. If the border comes in 5-yard rolls, how many rolls of border should Anoki buy? (Lesson 11-5)
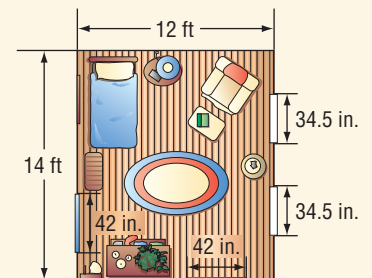
12 ft
14 ft
34.5 in.
34.5 in.
42 in.
42 in.

Rational Expressions with Like Denominators

GET READY for the Lesson

The graph at the right shows the results of a survey that asked families how often they eat takeout. To determine what fraction of those surveyed eat takeout more than once a week, you can use addition. Remember that percents can be written as fractions with denominators of 100.

How Many Times a Week Families Eat Takeout

8% Daily

30% 2–3 times a week

22% once a week

40% less often

Source: *Reader's Digest*
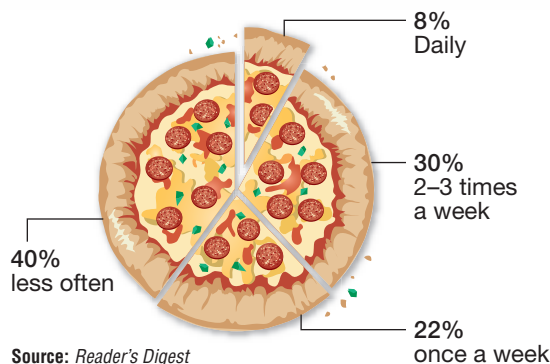
2–3 times a week	plus	daily	equals	more than once a week.
$\frac{30}{100}$	$+$	$\frac{8}{100}$	$=$	$\frac{38}{100}$

Thus, $\frac{38}{100}$ or 38% eat takeout more than once a week.

Add Rational Expressions Recall that to add fractions with like denominators, you add the numerators and then write the sum over the common denominator. You can add rational expressions with like denominators in the same way. Answers should always be expressed in simplest form.

EXAMPLE Numbers in Denominator

1 Find $\frac{3n}{12} + \frac{7n}{12}$.

$$\frac{3n}{12} + \frac{7n}{12} = \frac{3n + 7n}{12} \qquad \text{The common denominator is 12.}$$

$$= \frac{10n}{12} \qquad \text{Add the numerators.}$$

$$= \frac{\overset{5}{10n}}{\underset{6}{12}} \text{ or } \frac{5n}{6} \qquad \text{Divide by the common factor, 2, and simplify.}$$

CHECK Your Progress Find each sum.

1A. $\frac{8x}{6} + \frac{5x}{6}$ **1B.** $\frac{4x}{5dy} + \frac{7}{5dy}$

Sometimes the denominators of rational expressions are binomials. As long as each rational expression has exactly the same binomial as its denominator, the process of addition is the same.

Binomials in Denominator

② Find $\dfrac{2x}{x+1} + \dfrac{2}{x+1}$.

$$\dfrac{2x}{x+1} + \dfrac{2}{x+1} = \dfrac{2x+2}{x+1} \qquad \text{The common denominator is } x+1.$$

$$= \dfrac{2(x+1)}{x+1} \qquad \text{Factor the numerator.}$$

$$= \dfrac{2(\overset{1}{x+1})}{\underset{1}{x+1}} \qquad \text{Divide by the common factor, } x+1.$$

$$= \dfrac{2}{1} \text{ or } 2 \qquad \text{Simplify.}$$

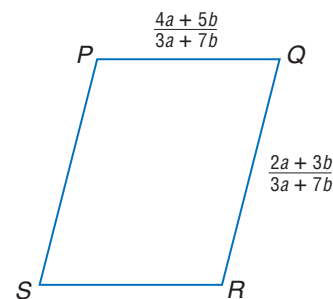**✓CHECK Your Progress** **Find each sum.**

2A. $\dfrac{3y}{3+y} + \dfrac{y^2}{3+y}$
2B. $\dfrac{15x}{33x+9} + \dfrac{3}{33x+9}$

EXAMPLE **Find a Perimeter**

③ **GEOMETRY** Find an expression for the perimeter of parallelogram *PQRS*.

Remember that opposite sides of a parallelogram have the same length.

$$P = 2\ell + 2w \qquad \text{Perimeter formula}$$

$$= 2\left(\dfrac{4a+5b}{3a+7b}\right) + 2\left(\dfrac{2a+3b}{3a+7b}\right) \qquad \ell = \dfrac{4a+5b}{3a+7b}, \ w = \dfrac{2a+3b}{3a+7b}$$

$$= \dfrac{2(4a+5b) + 2(2a+3b)}{3a+7b} \qquad \text{The common denominator is } 3a+7b.$$

$$= \dfrac{8a+10b+4a+6b}{3a+7b} \qquad \text{Distributive Property}$$

$$= \dfrac{12a+16b}{3a+7b} \qquad \text{Combine like terms.}$$

$$= \dfrac{4(3a+4b)}{3a+7b} \qquad \text{Factor.}$$

The perimeter can be represented by the expression $\dfrac{4(3a+4b)}{3a+7b}$.

✓CHECK Your Progress

Find an expression for the perimeter of each figure.

3A.

$\dfrac{2t+1}{t+2}$ $\dfrac{2t+1}{t+2}$

$\dfrac{2t}{t+2}$

3B.

$\dfrac{8h-14}{h+1}$

$\dfrac{5h+1}{h+1}$ $\dfrac{5h+1}{h+1}$

$\dfrac{7h}{h+1}$

The parallelogram figure:
P $\dfrac{4a+5b}{3a+7b}$ Q

$\dfrac{2a+3b}{3a+7b}$

S   R

Study Tip

Common Misconceptions
You may be tempted to divide out common factors like the 3*a* in the final step of Example 3. But remember that every term of the numerator and the denominator must be multiplied or divided by a number for the fraction to remain equivalent.

online **Personal Tutor at** algebra1.com

Subtract Rational Expressions To subtract rational expressions with like denominators, subtract the numerators and write the difference over the common denominator. Recall that to subtract an expression, you add its additive inverse. As with addition, answers should always be expressed in simplest form.

> **EXAMPLE** Subtract Rational Expressions
>
> **4** **a.** Find $\dfrac{3x + 4}{x - 2} - \dfrac{x - 1}{x - 2}$.
>
> $\dfrac{3x + 4}{x - 2} - \dfrac{x - 1}{x - 2} = \dfrac{(3x + 4) - (x - 1)}{x - 2}$ The common denominator is $x - 2$.
>
> $= \dfrac{(3x + 4) + [-(x - 1)]}{x - 2}$ The additive inverse of $(x - 1)$ is $-(x - 1)$.
>
> $= \dfrac{3x + 4 - x + 1}{x - 2}$ Distributive Property
>
> $= \dfrac{2x + 5}{x - 2}$ Simplify.
>
> **b.** Find $\dfrac{3m - 5}{m + 4} - \dfrac{4m + 2}{m + 4}$.
>
> $\dfrac{3m - 5}{m + 4} - \dfrac{4m + 2}{m + 4} = \dfrac{(3m - 5) - (4m + 2)}{m + 4}$ The common denominator is $m + 4$.
>
> $= \dfrac{(3m - 5) + [-(4m + 2)]}{m + 4}$ The additive inverse of $(4m + 2)$ is $-(4m + 2)$.
>
> $= \dfrac{3m - 5 - 4m - 2}{m + 4}$ Distributive Property
>
> $= \dfrac{-m - 7}{m + 4}$ Simplify.

CHECK Your Progress **Find each difference.**

4A. $\dfrac{2h + 4}{h + 1} - \dfrac{5 + h}{h + 1}$

4B. $\dfrac{17h + 4}{15h - 5} - \dfrac{2h - 6}{15h - 5}$

Study Tip

Common Misconception

Adding the additive inverse will help you avoid the following error in the numerator.

$(3x + 4) - (x - 1) =$
$3x + 4 - x - 1.$

Sometimes you must express a denominator as its additive inverse to have like denominators.

> **EXAMPLE** Inverse Denominators
>
> **5** Find $\dfrac{2m}{m - 9} + \dfrac{4m}{9 - m}$.
>
> $\dfrac{2m}{m - 9} + \dfrac{4m}{9 - m} = \dfrac{2m}{m - 9} + \dfrac{4m}{-(m - 9)}$ Rewrite $9 - m$ as $-(m - 9)$.
>
> $= \dfrac{2m}{m - 9} - \dfrac{4m}{m - 9}$ Rewrite so the denominators are the same.
>
> $= \dfrac{2m - 4m}{m - 9}$ The common denominator is $m - 9$.
>
> $= \dfrac{-2m}{m - 9}$ Subtract.

CHECK Your Progress **Find each sum.**

5A. $\dfrac{3n}{n - 4} + \dfrac{6n}{4 - n}$

5B. $\dfrac{t^2}{t - 3} + \dfrac{3}{3 - t}$

Math Online **Extra Examples at algebra1.com**

Find each sum.

Example 1
(p. 608)

1. $\dfrac{a+2}{4} + \dfrac{a-2}{4}$
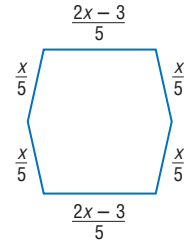
2. $\dfrac{12z}{7} + \dfrac{-5z}{7}$

Example 2
(p. 609)

3. $\dfrac{2-n}{n-1} + \dfrac{1}{n-1}$

4. $\dfrac{4t-1}{1-4t} + \dfrac{2t+3}{1-4t}$

Example 3
(p. 609)

5. Find an expression for the perimeter of the figure.

Find each difference.

Example 4
(p. 610)

6. $\dfrac{5a}{12} - \dfrac{7a}{12}$

7. $\dfrac{7}{n-3} - \dfrac{4}{n-3}$

Example 5
(p. 610)

8. $\dfrac{3m}{m-2} - \dfrac{6}{2-m}$

9. $\dfrac{x^2}{x-y} - \dfrac{y^2}{y-x}$

Exercises

Find each sum.

10. $\dfrac{m}{3} + \dfrac{2m}{3}$

11. $\dfrac{x+3}{5} + \dfrac{x+2}{5}$

12. $\dfrac{2y}{y+3} + \dfrac{6}{y+3}$

13. $\dfrac{3r}{r+5} + \dfrac{15}{r+5}$

14. $\dfrac{k-5}{k-1} + \dfrac{4}{k-1}$

15. $\dfrac{n-2}{n+3} + \dfrac{-1}{n+3}$

16. What is the sum of $\dfrac{12x-7}{3x-2}$ and $\dfrac{9x-5}{2-3x}$?

17. Find the sum of $\dfrac{11x-5}{2x+5}$ and $\dfrac{11x+12}{2x+5}$.

GEOMETRY For Exercises 18 and 19, use the following information.
Each figure has a perimeter of x units.

a.
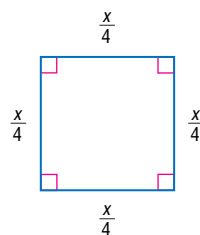
b.
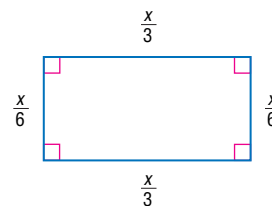
c.

18. Find the ratio of the area of each figure to its perimeter.

19. Which figure has the greatest ratio?

Find each difference.

20. $\dfrac{5x}{7} - \dfrac{3x}{7}$

21. $\dfrac{x+4}{5} - \dfrac{x+2}{5}$

22. $\dfrac{5}{3x-5} - \dfrac{3x}{3x-5}$

23. $\dfrac{8}{3t-4} - \dfrac{6t}{3t-4}$

24. $\dfrac{2x}{x-2} - \dfrac{2x}{2-x}$

25. $\dfrac{5y}{y-3} - \dfrac{5y}{3-y}$

26. POPULATION The population of Aurora, Illinois, is 31,536 greater than the population of Springfield, Illinois. Write an expression for the fraction of the Aurora population that is under 19 years old if the population of Springfield is n.

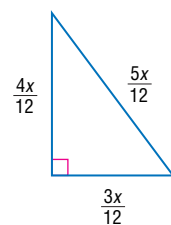| Aurora Population	
Age	**Number of People**
Under 5 years	15,095
5 to 9 years	13,256
10 to 14 years	10,873
15 to 19 years	10,042

Source: U.S. Census Bureau

27. CONSERVATION The freshman class chose to plant spruce and pine trees at a wildlife sanctuary for a service project. Some students can plant 140 trees on Saturday, and others can plant 20 trees after school on Monday and again on Tuesday. Write an expression for the fraction of the trees that could be planted on these days if n represents the number of spruce trees and there are twice as many pine trees.

28. SCHOOL Most schools create daily attendance reports to keep track of their students. The school office manager knows that, of the 960 students, 45 are absent due to illness, 10 are excused for appointments, and both the wrestling team and the choir are at competitions. Though she doesn't know exactly how many people attended the competitions, she does know that there are twice as many people in the choir as there are on the wrestling team. Write an expression that gives the percentage of students who are absent.

Find each sum or difference.

29. $\dfrac{4}{7m-2} + \dfrac{7m}{2-7m}$

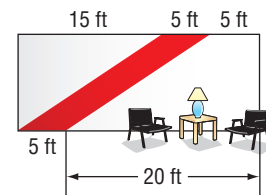**30.** $\dfrac{b-15}{2b+12} - \dfrac{-3b+8}{2b+12}$

31. $\dfrac{a+5}{6} - \dfrac{a+3}{6}$

32. $\dfrac{10a-12}{2a-6} - \dfrac{6a}{6-2a}$

33. $\dfrac{2}{x+7} - \dfrac{-5}{x+7}$

34. $\dfrac{15x}{5x+1} + \dfrac{-3}{-1-5x}$

35. GEOMETRIC DESIGN The Jerome Student Center has a square room that is 25 feet wide and 25 feet long. The walls are 10 feet high, and each wall is painted white with a red diagonal stripe as shown. What fraction of the walls are painted red?

HIKING For Exercises 36 and 37, use the following information.
A tour guide recommends that hikers carry a gallon of water on hikes to the bottom of the Grand Canyon. Water weighs 62.4 pounds per cubic foot, and one cubic foot of water contains 7.48 gallons.

36. Tanika plans to carry two 1-quart bottles and four 1-pint bottles for her hike. Write a rational expression for this amount of water written as a fraction of a cubic foot.

37. How much does this amount of water weigh?

38. OPEN ENDED Describe a real-life situation that could be expressed by adding two rational expressions that are fractions. Explain what the denominator and numerator represent in both expressions.

39. FIND THE ERROR Russell and Ginger are finding the difference of $\frac{7x + 2}{4x - 3}$ and $\frac{x - 8}{3 - 4x}$. Who is correct? Explain your reasoning.

Russell

$$\frac{7x+2}{4x-3} - \frac{x-8}{3-4x} = \frac{7x+2}{4x-3} + \frac{x-8}{4x-3}$$

$$= \frac{7x+x+2-8}{4x-3}$$

$$= \frac{8x-6}{4x-3}$$

$$= \frac{2(4x-3)}{4x-3}$$

$$= 2$$
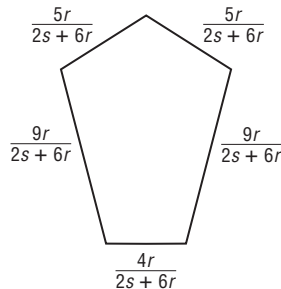
Ginger

$$\frac{7x+2}{4x-3} - \frac{x-8}{3-4x} = \frac{-2-7x}{3-4x} + \frac{x-8}{3-4x}$$

$$= \frac{-2-8-7x+x}{3-4x}$$

$$= \frac{-6-8x}{3-4x}$$

$$= \frac{-2(3-4x)}{3-4x}$$

$$= -2$$

40. CHALLENGE Which of the following rational numbers is *not* equivalent to the others?

a. $\frac{3}{2-x}$　　　　**b.** $\frac{-3}{x-2}$　　　　**c.** $-\frac{3}{2-x}$　　　　**d.** $-\frac{3}{x-2}$

41. *Writing in Math* Use the chart on page 608 to explain how rational expressions can be used to interpret graphics. Include an explanation of how the numbers in the graphic relate to rational expressions and a description of how to add rational expressions with denominators that are additive inverses.

STANDARDIZED TEST PRACTICE

42. Which is an expression for the perimeter of the pentagon?

A $\dfrac{16r}{s + 3r}$

B $\dfrac{16r}{2s + 6r}$

C $\dfrac{32}{s + 3r}$

D $\dfrac{32}{4s + 12r}$

$\frac{5r}{2s + 6r}$　$\frac{5r}{2s + 6r}$　$\frac{9r}{2s + 6r}$　$\frac{9r}{2s + 6r}$　$\frac{4r}{2s + 6r}$

43. REVIEW Shelby sells cosmetics door-to-door. She makes $5 an hour and 17% commission on the total dollar value on whatever she sells. To the nearest dollar, how much money will she make if she sells $300 dollars worth of product and works 30 hours?

F $201　　　　**H** $255

G $226　　　　**J** $283

Spiral Review

Find each quotient. (Lessons 11-4 and 11-5)

44. $\dfrac{x^3 - 7x + 6}{x - 2}$

45. $\dfrac{56x^3 + 32x^2 - 63x - 36}{7x + 4}$

46. $\dfrac{b^2 - 9}{4b} \div (b - 3)$

47. $\dfrac{x}{x + 2} \div \dfrac{x^2}{x^2 + 5x + 6}$

GET READY for the Next Lesson

PREREQUISITE SKILL Find the least common multiple for each set of numbers.

48. 4, 9, 12　　　　**49.** 45, 10, 6　　　　**50.** 16, 20, 25　　　　**51.** 36, 48, 60

Rational Expressions with Unlike Denominators

Main Ideas

- Add rational expressions with unlike denominators.
- Subtract rational expressions with unlike denominators.

New Vocabulary

least common multiple (LCM)

least common denominator (LCD)

GET READY for the Lesson

The President of the United States is elected every four years, and senators are elected every six years. A certain senator is elected in 2004, the same year as a presidential election, and is reelected in subsequent elections. In what year is the senator's reelection the same year as a presidential election?

Add Rational Expressions The least number of years that will pass until the next election for both a specific senator and the President is the least common multiple of 4 and 6. The **least common multiple (LCM)** is the least number that is a multiple of two or more numbers. You can also find the LCM of a set of polynomials.

EXAMPLE LCMs of Polynomials

1 **a.** Find the LCM of $15m^2b^3$ and $18mb^2$.

Find the prime factors of each coefficient and variable expression.

$15m^2b^3 = 3 \cdot 5 \cdot m \cdot m \cdot b \cdot b \cdot b$ $18mb^2 = 2 \cdot 3 \cdot 3 \cdot m \cdot b \cdot b$

Use each prime factor the greatest number of times it appears in any of the factorizations.

$15m^2b^3 = 3 \cdot 5 \cdot m \cdot m \cdot b \cdot b \cdot b$ $18mb^2 = 2 \cdot 3 \cdot 3 \cdot m \cdot b \cdot b$

$\text{LCM} = 2 \cdot 3 \cdot 3 \cdot 5 \cdot m \cdot m \cdot b \cdot b \cdot b$ or $90m^2b^3$

b. Find the LCM of $x^2 + 8x + 15$ and $x^2 + x - 6$.

Express each polynomial in factored form.

$x^2 + 8x + 15 = (x + 3)(x + 5)$ $x^2 + x - 6 = (x - 2)(x + 3)$

Use each factor the greatest number of times it appears.

$\text{LCM} = (x + 3)(x + 5)(x - 2)$

CHECK Your Progress

Find the LCM of each pair of polynomials.

1A. $28m^2n$ and $12m^2n^3p$ **1B.** $x^2 - 2x - 8$ and $x^2 - 5x - 14$

To add fractions with unlike denominators, you need to rename the fractions using the least common multiple (LCM) of the denominators, known as the **least common denominator (LCD).** You can add rational expressions with unlike denominators in the same way.

KEY CONCEPT   *Add Rational Expressions*

Use the following steps to add rational expressions with unlike denominators.

Step 1 Find the LCD.

Step 2 Change each rational expression into an equivalent expression with the LCD as the denominator.

Step 3 Add rational expressions with like denominators.

Step 4 Simplify if necessary.

EXAMPLE   **Polynomial Denominators**

2 **a.** Find $\dfrac{a+1}{a} + \dfrac{a-3}{3a}$.

Factor each denominator and find the LCD.

$$a = a$$
$$3a = 3 \cdot a \qquad \text{LCD} = 3a$$

Since the denominator of $\dfrac{a-3}{3a}$ is already $3a$, only $\dfrac{a+1}{a}$ needs to be renamed.

$\dfrac{a+1}{a} + \dfrac{a-3}{3a} = \dfrac{3(a+1)}{3(a)} + \dfrac{a-3}{3a}$ Multiply $\dfrac{a+1}{a}$ by $\dfrac{3}{3}$.

$\qquad\qquad = \dfrac{3a+3}{3a} + \dfrac{a-3}{3a}$ Distributive Property

$\qquad\qquad = \dfrac{3a+3+a-3}{3a}$ Add the numerators.

$\qquad\qquad = \dfrac{\overset{1}{4a}}{\underset{1}{3a}} \text{ or } \dfrac{4}{3}$ Divide by the common factor, a, and simplify.

b. Find $\dfrac{y-2}{y^2+4y+4} + \dfrac{y-2}{y+2}$.

$\dfrac{y-2}{y^2+4y+4} + \dfrac{y-2}{y+2} = \dfrac{y-2}{(y+2)^2} + \dfrac{y-2}{y+2}$ Factor the denominators.

$\qquad\qquad = \dfrac{y-2}{(y+2)^2} + \dfrac{y-2}{y+2} \cdot \dfrac{y+2}{y+2}$ The LCD is $(y+2)^2$.

$\qquad\qquad = \dfrac{y-2}{(y+2)^2} + \dfrac{y^2-4}{(y+2)^2}$ $(y-2)(y+2) = y^2-4$

$\qquad\qquad = \dfrac{y-2+y^2-4}{(y+2)^2}$ Add the numerators.

$\qquad\qquad = \dfrac{y^2+y-6}{(y+2)^2} \text{ or } \dfrac{(y-2)(y+3)}{(y+2)^2}$ Simplify.

Study Tip

Checking Answers

You can check to see that you have simplified rational expressions involving variables by substituting a few values. If the results are different for the original expression and the simplified expression, check for an error in arithmetic.

Find each sum.

2A. $\dfrac{4d^2}{d} + \dfrac{d+2}{d^2}$

2B. $\dfrac{b+3}{b} + \dfrac{b-5}{b+1}$

Subtract Rational Expressions As with addition, to subtract rational expressions with unlike denominators, you must first rename the expressions using a common denominator.

EXAMPLE **Polynomials in Denominators**

③ Find each difference.

a. $\dfrac{4}{3a-6} - \dfrac{a}{a+2}$

$\dfrac{4}{3a-6} - \dfrac{a}{a+2} = \dfrac{4}{3(a-2)} - \dfrac{a}{a+2}$ Factor.

$= \dfrac{4(a+2)}{3(a-2)(a+2)} - \dfrac{3a(a-2)}{3(a-2)(a+2)}$ The LCD is $3(a-2)(a+2)$.

$= \dfrac{4(a+2) - 3a(a-2)}{3(a-2)(a+2)}$ Subtract the numerators.

$= \dfrac{4a+8 - 3a^2 + 6a}{3(a-2)(a+2)}$ Multiply.

$= \dfrac{-3a^2 + 10a + 8}{3(a-2)(a+2)}$ Add like terms.

b. $\dfrac{h-2}{h^2+4h+4} - \dfrac{h-4}{h^2-4}$

$\dfrac{h-2}{(h+2)^2} - \dfrac{h-4}{(h+2)(h-2)} = \dfrac{(h-2)}{(h+2)^2} \cdot \dfrac{(h-2)}{(h-2)} - \dfrac{(h-4)}{(h+2)(h-2)} \cdot \dfrac{(h+2)}{(h+2)}$

$= \dfrac{(h-2)(h-2)}{(h+2)^2(h-2)} - \dfrac{(h-4)(h+2)}{(h+2)^2(h-2)}$ The LCD is $(h+2)^2(h-2)$.

$= \dfrac{h^2-4h+4}{(h+2)^2(h-2)} - \dfrac{h^2-2h-8}{(h+2)^2(h-2)}$ Multiply.

$= \dfrac{(h^2-4h+4) - (h^2-2h-8)}{(h+2)^2(h-2)}$ Subtract.

$= \dfrac{h^2-h^2-4h+2h+4+8}{(h+2)^2(h-2)}$ Combine like terms.

$= \dfrac{-2h+12}{(h-2)(h+2)^2}$ Simplify.

Find each sum.

3A. $\dfrac{5}{2\ell+2} + \dfrac{\ell}{\ell+5}$

3B. $\dfrac{k-3}{k^2+k-12} + \dfrac{k}{k^2-9}$

3C. $\dfrac{x}{x-3} - \dfrac{3}{x+2}$

3D. $\dfrac{m}{m^2-2m-8} - \dfrac{m+3}{m-4}$

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Example 1
(p. 614)

Find the LCM for each pair of expressions.

1. $5a^2, 7a$

2. $2x - 4, 3x - 6$

3. $n^2 + 3n - 4, (n - 1)^2$

4. MUSIC A music director wants to form a group of students to sing and dance at community events. Sometimes the music is 2-part, 3-part, or 4-part harmony, and she would like to have the same number of voices on each part. What is the least number of students that would allow for an even distribution on all these parts?

Example 2
(p. 615)

Find each sum.

5. $\dfrac{6}{5x} + \dfrac{7}{10x^2}$

6. $\dfrac{3z}{6w^2} + \dfrac{z}{4w}$

7. $\dfrac{2y}{y^2 - 25} + \dfrac{y + 5}{y - 5}$

8. $\dfrac{a + 2}{a^2 + 4a + 3} + \dfrac{6}{a + 3}$

Example 3
(p. 616)

Find each difference.

9. $\dfrac{a}{a - 4} - \dfrac{4}{a + 4}$

10. $\dfrac{b + 8}{b^2 - 16} - \dfrac{1}{b - 4}$

11. $\dfrac{2y}{y^2 + 7y + 12} - \dfrac{y + 2}{y + 4}$

12. $\dfrac{x}{x - 2} - \dfrac{3}{x^2 + 3x - 10}$

Exercises

Find the LCM for each pair of expressions.

13. a^2b, ab^3

14. $7xy, 21x^2y$

15. $x - 4, x + 2$

16. $2n - 5, n + 2$

17. $x^2 + 5x - 14, (x - 2)^2$

18. $p^2 - 5p - 6, p + 1$

HOMEWORK HELP	
For Exercises	**See Examples**
13–18	1
19–30	2
31–39	3

Find each sum.

19. $\dfrac{3}{x^2} + \dfrac{5}{x}$

20. $\dfrac{2}{a^3} + \dfrac{7}{a^2}$

21. $\dfrac{7}{6a^2} + \dfrac{5}{3a}$

22. $\dfrac{3}{7m} + \dfrac{4}{5m^2}$

23. $\dfrac{3}{x + 5} + \dfrac{4}{x - 4}$

24. $\dfrac{n}{n + 4} + \dfrac{3}{n - 3}$

25. $\dfrac{7a}{a + 5} + \dfrac{a}{a - 2}$

26. $\dfrac{6x}{x - 3} + \dfrac{x}{x + 1}$

27. $\dfrac{-3}{5 - a} + \dfrac{5}{a^2 - 25}$

28. $\dfrac{18}{y^2 - 9} + \dfrac{-7}{3 - y}$

29. $\dfrac{x}{x^2 + 2x + 1} + \dfrac{1}{x + 1}$

30. $\dfrac{2x + 1}{(x - 1)^2} + \dfrac{x - 2}{x^2 + 3x - 4}$

Find each difference.

31. $\dfrac{7}{3x} - \dfrac{3}{6x^2}$

32. $\dfrac{5a}{7x} - \dfrac{3a}{21x^2}$

33. $\dfrac{x^2 - 1}{x + 1} - \dfrac{x^2 + 1}{x - 1}$

34. $\dfrac{m - 1}{m + 1} - \dfrac{4}{2m + 5}$

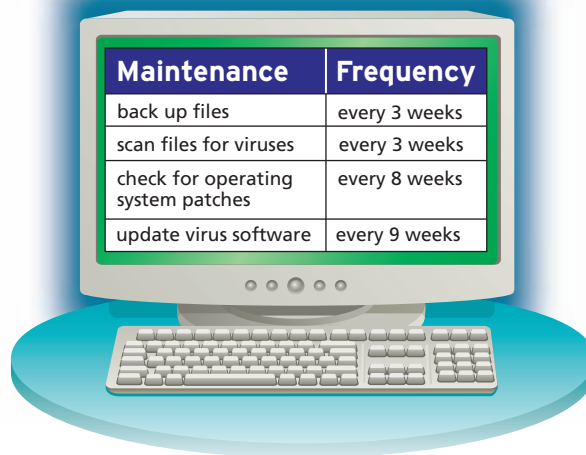**35.** $\dfrac{2x}{x^2 - 5x} - \dfrac{-3x}{x - 5}$

36. $\dfrac{-3}{a - 6} - \dfrac{-6}{a^2 - 6a}$

37. $\dfrac{n}{5 - n} - \dfrac{3}{n^2 - 25}$

38. $\dfrac{3a + 2}{6 - 3a} - \dfrac{a + 2}{a^2 - 4}$

39. $\dfrac{k}{2k + 1} - \dfrac{2}{k + 2}$

40. PET CARE Kendra takes care of pets while their owners are out of town. One week she has three dogs that all eat the same kind of dog food. The first dog eats a bag of food every 12 days, the second dog eats a bag every 15 days, and the third dog eats a bag every 16 days. How many bags of food should Kendra buy for one week?

41. CHARITY Maya, Makalla, and Monya can walk one mile in 12, 15, and 20 minutes respectively. They plan to participate in a walk-a-thon to raise money for a local charity. Sponsors have agreed to pay $2.50 for each mile that is walked. What is the total number of miles the girls will walk in one hour? How much money will they raise?

42. COMPUTERS Computer owners need to follow a regular maintenance schedule to keep their computers running effectively and efficiently. The table shows several items that should be performed on a regular basis. If Jamie got his computer 53 weeks ago and he has been appropriately maintaining it, how many weeks will it be until he has to perform all of the items on the same week?

Maintenance	Frequency
back up files	every 3 weeks
scan files for viruses	every 3 weeks
check for operating system patches	every 8 weeks
update virus software	every 9 weeks

Find each sum or difference.

43. $\dfrac{4}{15x^2} - \dfrac{5}{3x}$

44. $\dfrac{x^2}{4x^2 - 9} + \dfrac{x}{(2x + 3)^2}$

45. $\dfrac{11x}{3y^2} - \dfrac{7x}{6y}$

46. $\dfrac{k}{k + 5} - \dfrac{3}{k - 3}$

47. $\dfrac{a^2}{a^2 - b^2} + \dfrac{a}{(a - b)^2}$

48. $\dfrac{x^2 + 4x - 5}{x^2 - 2x - 3} - \dfrac{2}{x + 1}$

49. $\dfrac{3x}{x^2 + 3x + 2} - \dfrac{3x - 6}{x^2 + 4x + 4}$

50. $\dfrac{5x}{3x^2 + 19x - 14} - \dfrac{1}{9x^2 - 12x + 4}$

H.O.T. Problems

51. REASONING Describe how to find the LCD of two rational expressions with unlike denominators.

52. OPEN ENDED Write two rational expressions in which the LCD is twice the denominator of one of the expressions.

53. REASONING Explain how to rename rational expressions using their LCD.

54. CHALLENGE Janelle says that a shortcut for adding fractions with unlike denominators is to add the cross products for the numerator and write the denominator as the product of the denominators. For example, $\dfrac{2}{7} + \dfrac{5}{8} = \dfrac{2 \cdot 8 + 5 \cdot 7}{7 \cdot 8} = \dfrac{51}{56}$. Explain why the method will always work or provide a counterexample to show that it does not always work.

55. *Writing in Math* Use the information about elections on page 614 to explain how rational expressions can be used to describe elections. Include an explanation of how to determine the least common multiple of two or more rational expressions.
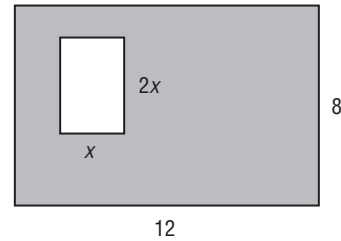
56. $\dfrac{x}{7x-3}+\dfrac{x+2}{15x+30}=$

A $\dfrac{22x-3}{210x-90}$

B $\dfrac{22x-3}{105x-45}$

C $\dfrac{41x-6}{210x-90}$

D $\dfrac{41x-6}{105x-45}$

57. REVIEW A rectangle with length x and width $2x$ is inside a rectangle with length 12 and width 8. Which expression represents the area of the shaded region in terms of x?

F $96-x$ **H** $96-x^2$

G $96-2x$ **J** $96-2x^2$

Spiral Review

Find each sum. (Lesson 11-6)

58. $\dfrac{3m}{2m+1}+\dfrac{3}{2m+1}$

59. $\dfrac{4x}{2x+3}+\dfrac{5}{2x+3}$

60. $\dfrac{2y}{y-3}+\dfrac{5}{3-y}$

Find each quotient. (Lesson 11-5)

61. $\dfrac{b^2+8b-20}{b-2}$

62. $\dfrac{t^3-19t+9}{t-4}$

63. $\dfrac{4m^2+8m-19}{2m+7}$

Determine the best method to solve each system of equations. Then solve the system. (Lesson 5-5)

64. $2x+3y=9$
$-x+5y=28$

65. $y=\dfrac{1}{4}x$
$-x+3y=-6$

66. CURRENCY The table shows the exchange rate for the American dollar to the British pound over the course of 30 days. Create a graph to display this data. Determine whether the graph shows a positive or negative correlation and draw a line of fit. (Lesson 5-7)

Day	5	10	15	20	25	30
British Pound to 1 American Dollar	0.5532	0.5621	0.5715	0.5832	0.5681	0.5721

GET READY for the Next Lesson

PREREQUISITE SKILL Find each quotient. (Lesson 11-4)

67. $\dfrac{x}{2}\div\dfrac{3x}{5}$

68. $\dfrac{a^2}{5b}\div\dfrac{4a}{10b^2}$

69. $\dfrac{x+7}{x}\div\dfrac{x+7}{x+3}$

70. $\dfrac{3n}{2n+5}\div\dfrac{12n^2}{2n+5}$

71. $\dfrac{3x}{x+2}\div(x-1)$

72. $\dfrac{x^2+7x+12}{x+6}\div(x+3)$

Mixed Expressions and Complex Fractions

Main Ideas

- Simplify mixed expressions.
- Simplify complex fractions.

New Vocabulary

- mixed expression
- complex fraction

GET READY for the Lesson

Katelyn bought $2\frac{1}{2}$ pounds of chocolate chip cookie dough. If the average cookie requires $1\frac{1}{2}$ ounces of dough, the number of cookies that Katelyn can bake can be found by simplifying the expression $\dfrac{2\frac{1}{2} \text{ pounds}}{1\frac{1}{2} \text{ ounces}}$.

Simplify Mixed Expressions Recall that a number like $2\frac{1}{2}$ is a mixed number because it is the sum of an integer, 2, and a fraction, $\frac{1}{2}$. An expression like $3 + \dfrac{x+2}{x-3}$ is called a **mixed expression** because it contains the sum of a monomial, 3, and a rational expression, $\dfrac{x+2}{x+3}$. Changing mixed expressions to rational expressions is similar to changing mixed numbers to improper fractions.

EXAMPLE Mixed Expression to Rational Expression

1 Simplify $3 + \dfrac{6}{x+3}$.

$$3 + \frac{6}{x+3} = \frac{3(x+3)}{x+3} + \frac{6}{x+3}$$ The LCD is $x + 3$.

$$= \frac{3(x+3)+6}{x+3}$$ Add the numerators.

$$= \frac{3x+9+6}{x+3} \text{ or } \frac{3x+15}{x+3}$$ Distributive Property

CHECK Your Progress Simplify each expression.

1A. $\dfrac{6y}{4y+8} + 5y$

1B. $15 - \dfrac{17x+5}{5x+10}$

Simplify Complex Fractions If a fraction has one or more fractions in the numerator or denominator, it is called a **complex fraction**. You simplify an algebraic complex fraction in the same way that you simplify a numerical complex fraction.

numerical complex fraction

$$\frac{\frac{8}{3}}{\frac{7}{5}} = \frac{8}{3} \div \frac{7}{5}$$

$$= \frac{8}{3} \cdot \frac{5}{7}$$

$$= \frac{40}{21}$$

algebraic complex fraction

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \div \frac{c}{d}$$

$$= \frac{a}{b} \cdot \frac{d}{c}$$

$$= \frac{ad}{bc}$$

KEY CONCEPT   *Simplifying a Complex Fraction*

Any complex fraction $\dfrac{\frac{a}{b}}{\frac{c}{d}}$, where $b \neq 0$, $c \neq 0$, and $d \neq 0$, can be expressed as $\dfrac{ad}{bc}$.

Real-World EXAMPLE   **Complex Fraction Involving Numbers**

2   **BAKING** **Refer to the application at the beginning of the lesson. How many cookies can Katelyn make with $2\frac{1}{2}$ pounds of dough?**

To find the total number of cookies, divide the amount of cookie dough by the amount of dough needed for each cookie.

$$\frac{2\frac{1}{2} \text{ pounds}}{1\frac{1}{2} \text{ ounces}} = \frac{2\frac{1}{2} \text{ pounds}}{1\frac{1}{2} \text{ ounces}} \cdot \frac{16 \text{ ounces}}{1 \text{ pound}}$$
     Convert pounds to ounces.
     Divide by common units.

$$= 16 \cdot \frac{2\frac{1}{2}}{1\frac{1}{2}}$$
     Simplify.

$$= \frac{\frac{16}{1} \cdot \frac{5}{2}}{\frac{3}{2}}$$
     Express each term as an improper fraction.

$$= \frac{\frac{80}{2}}{\frac{3}{2}}$$
     Multiply in the numerator.

$$= \frac{80 \cdot 2}{2 \cdot 3}$$
     $\dfrac{\frac{a}{b}}{\frac{c}{d}} = \dfrac{ad}{bc}$

$$= \frac{160}{6} \text{ or } 26\frac{2}{3}$$
     Simplify.

CHECK Your Progress

2. The Centralville High School Cooking Club has $12\frac{1}{2}$ pounds of flour with which to make tortillas. If there are $3\frac{3}{4}$ cups of flour in a pound and it takes about $\frac{1}{3}$ cup of flour per tortilla, how many tortillas can they make?

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Study Tip

Order of Operations

Recall that when applying the order of operations, simplify the numerator and denominator of a complex fraction before proceeding with division.

EXAMPLE **Complex Fraction Involving Monomials**

③ Simplify $\dfrac{\frac{x^2y^2}{a}}{\frac{x^2y}{a^3}}$.

$$\frac{\frac{x^2y^2}{a}}{\frac{x^2y}{a^3}} = \frac{x^2y^2}{a} \div \frac{x^2y}{a^3}$$ Rewrite as a division sentence.

$$= \frac{x^2y^2}{a} \cdot \frac{a^3}{x^2y}$$ Rewrite as multiplication by the reciprocal.

$$= \frac{\overset{1}{\cancel{x^2}}\overset{y}{\cancel{y^2}}}{\underset{1}{\cancel{a}}} \cdot \frac{\overset{a^2}{\cancel{a^3}}}{\underset{1}{\cancel{x^2}}\underset{1}{\cancel{y}}}$$ Divide by common factors x^2, y, and a.

$$= a^2y$$ Simplify.

CHECK Your Progress Simplify each expression.

3A. $\dfrac{\frac{g^3h}{b}}{\frac{gh^3i}{b^2}}$

3B. $\dfrac{\frac{-24m^3n^5}{p^2q}}{\frac{16pm^2}{n^4q}}$

EXAMPLE **Complex Fraction Involving Polynomials**

④ Simplify $\dfrac{a - \frac{15}{a-2}}{a+3}$.

The numerator contains a mixed expression. Rewrite it as a rational expression first.

$$\frac{a - \frac{15}{a-2}}{a+3} = \frac{\frac{a(a-2)}{a-2} - \frac{15}{a-2}}{a+3}$$ The LCD of the fractions in the numerator is $a - 2$.

$$= \frac{\frac{a^2 - 2a - 15}{a-2}}{a+3}$$ Simplify the numerator.

$$= \frac{\frac{(a+3)(a-5)}{a-2}}{a+3}$$ Factor.

$$= \frac{(a+3)(a-5)}{a-2} \div (a+3)$$ Rewrite as a division sentence.

$$= \frac{(a+3)(a-5)}{a-2} \cdot \frac{1}{a+3}$$ Multiply by the reciprocal of $a + 3$.

$$= \frac{(\overset{1}{\cancel{a+3}})(a-5)}{a-2} \cdot \frac{1}{\underset{1}{\cancel{a+3}}}$$ Divide by the GCF, $a + 3$.

$$= \frac{a-5}{a-2}$$ Simplify.

CHECK Your Progress Simplify each expression.

4A. $\dfrac{\frac{b}{b+3} + 2}{b^2 - 2b - 8}$

4B. $\dfrac{1 + \frac{2c^2 - 6c - 10}{c+7}}{2c+1}$

Math Online Extra Examples at **algebra1.com**

Example 1
(p. 620)

Write each mixed expression as a rational expression.

1. $3 + \frac{4}{x}$

2. $7 + \frac{5}{6y}$

3. $\frac{a-1}{3a} + 2a$

Example 2
(p. 622)

4. ENTERTAINMENT The student talent committee is arranging the performances for their holiday pageant. The first-act performances and their lengths are shown in the table. What is the average length of each performance?

Holiday Pageant Line-Up

Performance	Length (min)
A	7
B	$4\frac{1}{2}$
C	$6\frac{1}{2}$
D	$8\frac{1}{4}$
E	$10\frac{1}{5}$

Examples 3, 4
(p. 620)

Simplify each expression.

5. $\dfrac{3\frac{1}{2}}{4\frac{3}{4}}$

6. $\dfrac{\frac{x^3}{y^2}}{\frac{y^3}{x}}$

7. $\dfrac{\frac{x-y}{a+b}}{\frac{x^2-y^2}{a^2-b^2}}$

Exercises

Write each mixed expression as a rational expression.

8. $8 + \frac{3}{n}$

9. $4 + \frac{5}{a}$

10. $2x + \frac{x}{y}$

11. $6z + \frac{2z}{w}$

12. $2m - \frac{4+m}{m}$

13. $3a - \frac{a+1}{2a}$

14. $b^2 + \frac{a-b}{a+b}$

15. $r^2 + \frac{r-4}{r+3}$

16. $5n^2 + \frac{n+3}{n^2-9}$

17. $3s^2 - \frac{s+1}{s^2-1}$

18. $(x-5) + \frac{x+2}{x-3}$

19. $(p+4) + \frac{p+1}{p-4}$

20. PARTIES The student council is planning a party for the school volunteers. There are five 66-ounce unopened bottles of soda left from a recent dance. When poured over ice, $5\frac{1}{2}$ ounces of soda fills a cup. How many servings of soda can they get from the bottles they have?

21. SCIENCE When air is pumped into a bicycle tire, the pressure P required varies inversely as the volume of the air V and is given by the equation $P = \frac{k}{V}$. If the pressure is 30 lb/in^2 when the volume is $1\frac{2}{3}$ cubic feet, find the pressure when the volume is $\frac{3}{4}$ cubic feet.

Simplify each expression.

22. $\dfrac{5\frac{3}{4}}{7\frac{2}{3}}$

23. $\dfrac{8\frac{2}{7}}{4\frac{4}{5}}$

24. $\dfrac{\frac{a}{b^3}}{\frac{a^2}{b}}$

Simplify each expression.

25. $\dfrac{\dfrac{n^3}{m^2}}{\dfrac{n^2}{m^2}}$

26. $\dfrac{\dfrac{x+4}{y-2}}{\dfrac{x^2}{y^2}}$

27. $\dfrac{\dfrac{s^3}{t^2}}{\dfrac{s+t}{s-t}}$

28. $\dfrac{\dfrac{y^2-1}{y^2+3y-4}}{y+1}$

29. $\dfrac{\dfrac{a^2-2q-3}{a^2-1}}{a-3}$

30. $\dfrac{\dfrac{n^2+2n}{n^2+9n+18}}{\dfrac{n^2-5n}{n^2+n-30}}$

31. $\dfrac{\dfrac{x^2+4x-21}{x^2-9x+18}}{\dfrac{x^2+3x-28}{x^2-10x+24}}$

32. $\dfrac{x-\dfrac{15}{x-2}}{x-\dfrac{20}{x-1}}$

33. $\dfrac{n+\dfrac{35}{n+12}}{n-\dfrac{63}{n-2}}$

SIRENS For Exercises 34 and 35, use the following information.
As an ambulance approaches, the siren sounds different than if it were sitting still. If the ambulance is moving toward you at v miles per hour and blowing its siren at a frequency of f, then you hear the siren as if it were blowing at a frequency of h. This can be defined by the equation $h = \dfrac{f}{1-\dfrac{v}{s}}$, where s is the speed of sound, approximately 760 miles per hour.

34. Simplify the complex fraction in the formula.

35. Suppose a siren blows at 45 cycles per minute and is moving toward you at 65 miles per hour. Find the frequency of the siren as you hear it.

36. POPULATION According to a recent census, Union City, New Jersey, was the most densely populated city in the U.S., and Anchorage City, Alaska, was one of the least. The population of Union City was 67,088, and the population of Anchorage City was 260,283. The land area of Union City is about 1.3 square miles, and the land area of Anchorage City is about 1,697.3 square miles. How many more people were there per square mile in Union City than in Anchorage City?

37. What is the product of $\dfrac{2b^2}{5c}$ and the quotient of $\dfrac{4b^3}{2c}$ and $\dfrac{7b^3}{8c^2}$?

H.O.T. Problems

38. OPEN ENDED Think of a real-world complex fraction and explain how you would simplify it.

39. CHALLENGE Which expressions are equivalent to 0?

a. $\dfrac{a}{1-\dfrac{3}{a}}+\dfrac{a}{\dfrac{3}{a}-1}$

b. $\dfrac{a-\dfrac{1}{3}}{b}-\dfrac{a+\dfrac{1}{3}}{b}$

c. $\dfrac{\dfrac{1}{2}+2a}{b-1}+\dfrac{2a+\dfrac{1}{2}}{1-b}$

40. FIND THE ERROR Bolton and Lian found the LCD of $\dfrac{4}{2x+1}-\dfrac{5}{x+1}+\dfrac{2}{x-1}$. Who is correct? Explain your reasoning.

Bolton

$\dfrac{4}{2x+1}-\dfrac{5}{x+1}+\dfrac{2}{x-1}$

LCD: $(2x+1)(x+1)(x-1)$

Lian

$\dfrac{4}{2x+1}-\dfrac{5}{x+1}+\dfrac{2}{x-1}$

LCD: $2(x+1)(x-1)$

41. *Writing in Math* Refer to the information on page 620. Explain how complex fractions are used in baking. Include an explanation of the process used to simplify a complex fraction.
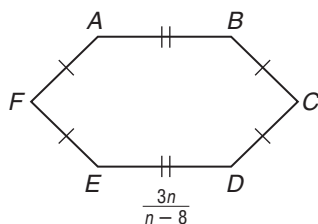
42. The perimeter of hexagon *ABCDEF* is 12. Which expression can be used to represent the measure of $\overline{BC}$?

A $\dfrac{6n - 96}{n - 8}$

B $\dfrac{9n - 96}{n - 8}$

C $\dfrac{3n - 48}{2n - 16}$

D $\dfrac{9n - 96}{4n - 32}$

43. REVIEW Ms. Roberts is draining her cylindrical above-ground pool. The pool has a radius of 12 feet and a height of 4 feet. If water is pumped out at a constant rate of 5.5 gallons per minute, about how long will it take to drain the pool? ($1\ \text{ft}^3 = 7.5$ gal)

F 43.9 min   H 35.2 h

G 3.8 h   J 41.1 h

Spiral Review

Find each sum. (Lesson 11-7)

44. $\dfrac{12x}{4y^2} + \dfrac{8}{6y}$

45. $\dfrac{a}{a - b} + \dfrac{b}{2b + 3a}$

46. $\dfrac{a + 3}{3a^2 - 10a - 8} + \dfrac{2a}{a^2 - 8a + 16}$

Find each difference. (Lesson 11-6)

47. $\dfrac{7}{x^2} - \dfrac{3}{x^2}$

48. $\dfrac{x}{(x - 3)^2} - \dfrac{3}{(x - 3)^2}$

49. $\dfrac{2}{t^2 - t - 2} - \dfrac{t}{t^2 - t - 2}$

FAMILIES For Exercises 50–52, refer to the graph. (Lesson 7-1)

50. Write each number in the graph using scientific notation.

51. How many times as great is the amount spent on food as the amount spent on clothing? Express your answer in scientific notation.

52. What percent of the total amount is spent on housing?
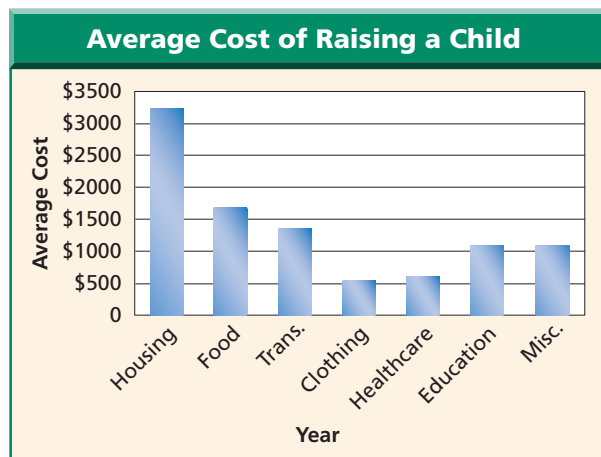
Source: University of Minnesota

PREREQUISITE SKILL Solve each equation. (Lessons 2-2, 2-3, and 2-4)

53. $-12 = \dfrac{x}{4}$

54. $1.8 = g - 0.6$

55. $\dfrac{3}{4}n - 3 = 9$

56. $7x^2 = 28$

57. $3.2 = \dfrac{-8 + n}{-7}$

58. $\dfrac{-3n - (-4)}{-6} = -9$

Rational Equations and Functions

GET READY for the Lesson

The Washington, D.C., Metrorail is one of the safest subway systems in the world, serving a population of more than 3.5 million. It is vital that a rail system of this size maintain a consistent schedule. Rational equations can be used to determine the exact positions of trains at any given time.

Washington Metropolitan Area Transit Authority	
Train	Distance
● Red Line	19.4 mi
● Orange Line	24.14 mi
● Blue Line	19.95 mi
● Green Line	20.59 mi
● Yellow Line	9.46 mi

Solve Rational Equations **Rational equations** are equations that contain rational expressions. You can use cross products to solve rational equations, but *only* when both sides of the equation are single fractions.

EXAMPLE **Use Cross Products**

① Solve $\dfrac{12}{x+5} = \dfrac{4}{(x+2)}$.

$$\dfrac{12}{x+5} = \dfrac{4}{(x+2)} \qquad \text{Original equation}$$

$$12(x+2) = 4(x+5) \qquad \text{Cross multiply.}$$

$$12x + 24 = 4x + 20 \qquad \text{Distributive Property}$$

$$8x = -4 \qquad \text{Add } -4x \text{ and } -24 \text{ to each side.}$$

$$x = -\dfrac{4}{8} \text{ or } -\dfrac{1}{2} \qquad \text{Divide each side by 8.}$$

Study Tip

Proportions
If the rational equation is a proportion, you can cross multiply to solve.

CHECK Your Progress

Solve each equation.

1A. $\dfrac{7}{y-3} = \dfrac{3}{y+1}$

1B. $\dfrac{13}{f+10} = \dfrac{2}{7f}$

Another method you can use to solve rational equations is to multiply each side of the equation by the LCD of all of the fractions on both sides of the equation. This will eliminate all of the fractions. This method works for any rational equation.

EXAMPLE **Use the LCD**

② Solve $\dfrac{n-2}{n} - \dfrac{n-3}{n-6} = \dfrac{1}{n}$.

$$\dfrac{n-2}{n} - \dfrac{n-3}{n-6} = \dfrac{1}{n} \qquad \text{Original equation}$$

$$n(n-6)\left(\frac{n-2}{n}-\frac{n-3}{n-6}\right)=n(n-6)\left(\frac{1}{n}\right)$$ The LCD is $n(n-6)$.

$$\left(\frac{\overset{1}{\cancel{n}}(n-6)}{1}\cdot\frac{n-2}{\cancel{n}}\right)-\left(\frac{n(\overset{1}{\cancel{n-6}})}{1}\cdot\frac{n-3}{\cancel{n-6}}\right)=\frac{\overset{1}{\cancel{n}}(n-6)}{1}\cdot\frac{1}{\cancel{n}}$$ Distributive Property

$$(n-6)(n-2)-n(n-3)=n-6$$ Simplify.

$$(n^2-8n+12)-(n^2-3n)=n-6$$ Multiply.

$$n^2-8n+12-n^2+3n=n-6$$ Subtract.

$$-5n+12=n-6$$ Simplify.

$$-6n=-18$$ Subtract.

$$n=3$$ Divide.

✓CHECK Your Progress **Solve each equation.**

2A. $\dfrac{2b-5}{b-2}-2=\dfrac{3}{b+2}$ **2B.** $1+\dfrac{1}{c+2}=\dfrac{28}{c^2+2c}$

Recall that to find the roots of a quadratic function, find the values of x when $y=0$. The roots of a rational function are found similarly.

EXAMPLE **Rational Functions**

3 Find the roots of $f(x)=\dfrac{x^2-x-12}{x-2}$.

$$f(x)=\frac{x^2-x-12}{x-2}$$ Original function

$$0=\frac{x^2-x-12}{x-2}$$ $f(x)=0$

$$0=\frac{(x-4)(x+3)}{x-2}$$ Factor.

When $x=4$ and -3, the numerator becomes zero, so $f(x)=0$. Therefore, the roots of the function are 4 and -3.

✓CHECK Your Progress **Find the roots of each function.**

3A. $f(x)=\dfrac{x^2+3x-18}{x-3}$ **3B.** $f(x)=\dfrac{x^2+6x+8}{x^2+x-2}$

Rational equations can be used to solve <mark>**work problems**</mark>.

EXAMPLE **Work Problem**

4 **LAWN CARE** It takes Abbey two hours to mow and trim Mr. Morely's lawn. When Jamal worked with her, the job took only 1 hour and 20 minutes. How long would it have taken Jamal to do the job himself?

Explore Since it takes Abbey two hours to do the yard, she can finish $\frac{1}{2}$ the job in one hour. Thus, her rate of work is $\frac{1}{2}$ of the job per hour. The amount of work Jamal can do in one hour can be represented by $\frac{1}{t}$. To determine how long it takes Jamal to do the job alone, use the formula Abbey's portion of the job + Jamal's portion of the job = 1 completed yard.

Plan　The time that both of them worked was $1\frac{1}{3}$ hours. Each rate multiplied by this time results in the amount of work done by each person.

Solve

Abbey's portion	plus	Jamal's portion	equals	1 job.
$\frac{1}{2}\left(\frac{4}{3}\right)$	$+$	$\frac{1}{t}\left(\frac{4}{3}\right)$	$=$	1

$$\frac{4}{6} + \frac{4}{3t} = 1 \qquad \text{Multiply.}$$

$$6t\left(\frac{4}{6} + \frac{4}{3t}\right) = 6t \cdot 1 \qquad \text{The LCD is } 6t.$$

$$\overset{1}{6t}\left(\frac{4}{\underset{1}{6}}\right) + \overset{2}{6t}\left(\frac{4}{\underset{1}{3t}}\right) = 6t \qquad \text{Distributive Property}$$

$$4t + 8 = 6t \qquad \text{Simplify.}$$

$$8 = 2t \qquad \text{Add } -4t \text{ to each side.}$$

$$4 = t \qquad \text{Divide each side by 2.}$$

Check　The time that it would take Jamal to do the yard by himself is 4 hours. This seems reasonable because the combined efforts of the two took longer than half of Abbey's usual time.

✔CHECK Your Progress

4. Lupe can paint a 60 square foot wall in 40 minutes. Working with her friend Steve, the two of them can paint the wall in 25 minutes. How long would it take Steve to do the job himself?

Online Personal Tutor at algebra1.com

Rational equations can also be used to solve **rate problems**.
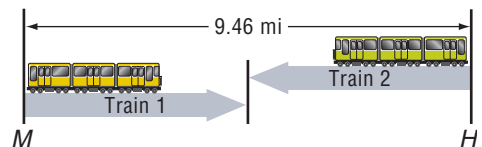
EXAMPLE　**Rate Problem**

5 TRANSPORTATION　Refer to the application at the beginning of the lesson. The Yellow Line runs between Huntington and Mt. Vernon Square. Suppose one train leaves Mt. Vernon Square at noon and arrives at Huntington 24 minutes later, and a second train leaves Huntington at noon and arrives at Mt. Vernon Square 28 minutes later. At what time do the two trains pass each other?

Determine the rates of both trains. The total distance is 9.46 miles.

Train 1　$\dfrac{9.46 \text{ mi}}{24 \text{ min}}$　　　Train 2　$\dfrac{9.46 \text{ mi}}{28 \text{ min}}$

Next, since both trains left at the same time, the time both have traveled when they pass will be the same. And since they started at opposite ends of the route, the sum of their distances is equal to the total route, 9.46 miles.

	r	t	d
Train 1	$\dfrac{9.46 \text{ mi}}{24 \text{ min}}$	t min	$\dfrac{9.46t}{24}$ mi
Train 2	$\dfrac{9.46 \text{ mi}}{28 \text{ min}}$	t min	$\dfrac{9.46t}{28}$ mi

$$\frac{9.46t}{24} + \frac{9.46t}{28} = 9.46 \qquad \text{The sum of the distances is 9.46.}$$

$$168\left(\frac{9.46t}{24} + \frac{9.46t}{28}\right) = 168 \cdot 9.46 \qquad \text{The LCD is 168.}$$

$$\frac{\overset{7}{\cancel{168}}}{1} \cdot \frac{9.46t}{\cancel{24}_1} + \frac{\overset{6}{\cancel{168}}}{1} \cdot \frac{9.46t}{\cancel{28}_1} = 1589.28 \qquad \text{Distributive Property}$$

$$66.22t + 56.76t = 1589.28 \qquad \text{Simplify.}$$

$$122.98t = 1589.28 \qquad \text{Add.}$$

$$t = 12.92 \qquad \text{Divide each side by 122.98.}$$

The trains passed at about 12.92 or about 13 minutes after leaving their stations. This would be 12:13 P.M.

✓ CHECK Your Progress

5. Debbie leaves the house walking at a rate of 3 miles per hour. After 10 minutes, her mother realizes that Debbie has forgotten her homework and leaves the house riding a bicycle at a rate of 10 miles per hour. How many minutes after Debbie initially left the house will her mother catch up to her?

Extraneous Solutions Multiplying each side of an equation by the LCD of two rational expressions can yield results that are not solutions to the original equation. Recall that such solutions are called **extraneous solutions**. Rational equations can have both valid solutions and extraneous solutions.

EXAMPLE Extraneous Solutions

6 Solve $\dfrac{3x}{x-1} + \dfrac{6x-9}{x-1} = 6$.

$$\frac{3x}{x-1} + \frac{6x-9}{x-1} = 6 \qquad \text{Original equation}$$

$$(x-1)\left(\frac{3x}{x-1} + \frac{6x-9}{x-1}\right) = (x-1)6 \qquad \text{The LCD is } x-1.$$

$$(x \overset{1}{\cancel{-1}})\left(\frac{3x}{x \cancel{-1}}\right) + (x \overset{1}{\cancel{-1}})\left(\frac{6x-9}{x \cancel{-1}}\right) = (x-1)6 \qquad \text{Distributive Property}$$

$$3x + 6x - 9 = 6x - 6 \qquad \text{Simplify.}$$

$$9x - 9 = 6x - 6 \qquad \text{Add like terms.}$$

$$3x = 3 \qquad \text{Add 9 to each side.}$$

$$x = 1 \qquad \text{Divide each side by 3.}$$

Notice that 1 is an excluded value for x. If we substitute 1 for x in the original equation, we get undefined expressions. Since $x = 1$ is an extraneous solution, this equation has no solution.

✓ CHECK Your Progress Solve each equation.

6A. $\dfrac{3x}{x-4} - \dfrac{4x+4}{x-4} = -1$

6B. $\dfrac{n^2 - 3n}{n^2 - 4} - \dfrac{10}{n^2 - 4} = 2$

Examples 1, 2, 6
(pp. 626–627, 629)

Solve each equation. State any extraneous solutions.

1. $\dfrac{2}{x} = \dfrac{3}{x+1}$

2. $\dfrac{3x}{5} + \dfrac{3}{2} = \dfrac{7x}{10}$

3. $\dfrac{x+2}{x-2} - \dfrac{2}{x+2} = -\dfrac{7}{3}$

4. $\dfrac{n^2 - n - 6}{n^2 - n} - \dfrac{n-5}{n-1} = \dfrac{n-3}{n^2 - n}$

Example 3
(p. 627)

Find the roots of each function.

5. $f(x) = \dfrac{x^2 - 8x + 15}{x^2 + 5x - 6}$

6. $f(x) = \dfrac{x^2 - x - 6}{x^2 + 8x + 12}$

Example 4
(pp. 627–628)

7. BASEBALL Omar has 32 hits in 128 times at bat. He wants his batting average to be .300. His current average is $\dfrac{32}{128}$ or .250. How many at bats does Omar need to reach his goal if he gets a hit in each of his next b at bats?

Example 5
(pp. 628–629)

8. LANDSCAPING Kumar is filling a 3.5-gallon bucket to water plants at a faucet that flows at a rate of 1.75 gallons a minute. If he were to add a hose that flows at a rate of 1.45 gallons per minute, how many minutes would it take him to fill the bucket? Round to the nearest tenth of a minute.

Exercises

Solve each equation. State any extraneous solutions.

HOMEWORK HELP	
For Exercises	**See Examples**
9–12	1
13–18	2
19–22	6
23–26	3
27, 28	4
29–32	5

9. $\dfrac{4}{a} = \dfrac{3}{a-2}$

10. $\dfrac{3}{x} = \dfrac{1}{x-2}$

11. $\dfrac{x-3}{x} = \dfrac{x-3}{x-6}$

12. $\dfrac{x}{x+1} = \dfrac{x-6}{x-1}$

13. $\dfrac{2n}{3} + \dfrac{1}{2} = \dfrac{2n-3}{6}$

14. $\dfrac{5}{4} + \dfrac{3y}{2} = \dfrac{7y}{6}$

15. $\dfrac{a-1}{a+1} - \dfrac{2a}{a-1} = -1$

16. $\dfrac{m-1}{m+1} - \dfrac{2m}{m-1} = -1$

17. $\dfrac{4x}{2x+3} - \dfrac{2x}{2x-3} = 1$

18. $\dfrac{a}{3a+6} - \dfrac{a}{5a+10} = \dfrac{2}{5}$

19. $\dfrac{2n}{n-1} + \dfrac{n-5}{n^2-1} = 1$

20. $\dfrac{5}{5-p} - \dfrac{p^2}{p-5} = -8$

21. $\dfrac{x^2 - x - 6}{x+2} + \dfrac{x^3 + x^2}{x} = 3$

22. $\dfrac{x - \frac{6}{5}}{x} - \dfrac{x - 10\frac{1}{5}}{x-5} = \dfrac{x+21}{x^2 - 5x}$

Find the roots of each function.

23. $f(x) = \dfrac{x^2 - x - 12}{x^2 + 2x - 35}$

24. $f(x) = \dfrac{x^2 + 3x - 4}{x^2 + 9x + 20}$

25. $f(x) = \dfrac{x^3 + x^2 - 6x}{x-1}$

26. $f(x) = \dfrac{x^3 - 4x^2 - 12x}{x+2}$

27. CAR WASH Ian and Nadya can each wash a car and clean its interior in about 2 hours, but Raul needs 3 hours to do the work. If the three work together, how long will it take to clean seven cars?

28. JOBS Ron works as a dishwasher and can wash 500 plates in two hours and 15 minutes. Occasionally, the busser, Chris, helps. Together they can finish 500 plates in 77 minutes. About how long would it take Chris to finish all of the plates by himself?

SWIMMING POOLS For Exercises 29 and 30, use the following information.
The pool in Kara's backyard is cleaned and ready to be filled for the summer. It measures 15 feet long and 10 feet wide with an average depth of 4 feet.

29. If Kara's hose runs at a rate of 5 gallons per minute, how long will it take to fill the pool?

30. Kara's neighbor's hose runs at a rate of 9 gallons per minute. How long will it take to fill the pool using both hoses?

BOATING For Exercises 31 and 32, use the following information.
Jim and Mateo live across a lake from each other at a distance of about 3 miles. Jim can row his boat to Mateo's house in 1 hour and 20 minutes. Mateo can drive his power boat the same distance in a half hour.

31. If they leave their houses at the same time and head for each other, how long will it be before they meet?

32. How far from the nearest shore will they be when they meet?

33. QUIZZES Each week, Mandy's algebra teacher gives a 10-point quiz. After 5 weeks, Mandy has earned a total of 36 points for an average of 7.2 points per quiz. She would like to raise her average to 9 points. On how many quizzes must she score 10 points in order to reach her goal?

34. PAINTING Morgan can paint a standard-sized house in about 5 days. For his latest assignment, Morgan is going to hire two assistants. At what rate must these assistants work for Morgan to meet a deadline of two days?

GRAPHING CALCULATOR For Exercises 35–37, use a graphing calculator.
For each rational function, a) describe the shape of the graph, b) use factoring to simplify the function, and c) determine the roots of the function.

35. $f(x) = \dfrac{x^2 - x - 30}{x - 6}$ **36.** $f(x) = \dfrac{x^3 + x^2 - 2x}{x + 2}$ **37.** $f(x) = \dfrac{x^3 + 6x^2 + 12x}{x}$

AIRPLANES For Exercises 38 and 39, use the following information.
Headwinds push against a plane and reduce its total speed, while tailwinds push on a plane and increase its total speed. Let w = the speed of the wind, r = the speed set by the pilot, and s = the total speed.

38. Write an equation for the total speed with a headwind and an equation for the total speed with a tailwind.

39. Use the rate formula to write an equation for the distance traveled by a plane with a headwind and another equation for the distance traveled by a plane with a tailwind. Then solve each equation for time instead of distance.

H.O.T. Problems

40. OPEN ENDED Write an expression that models a real-world situation where work is being done.

41. REASONING Find a counterexample for the following statement.
The solution of a rational equation can never be zero.

42. CHALLENGE Solve $\dfrac{\frac{x+3}{x-2} \cdot \frac{x^2+x-2}{x+5}}{x-1} + 2 = 0$.

43. *Writing in Math* Refer to the information on page 626. How are rational equations important in the operation of a subway system? Include in your answer an explanation of how rational equations can be used to approximate the time that trains will pass each other if they leave distant stations and head toward each other.

44. A group of band students went to a restaurant after the football game. They agreed to split the bill equally. When the bill arrived, three people discovered that they had forgotten their wallets. The others in the group agreed to make up the difference by paying an extra $2.70 each. If the total bill was $117, how many band students went to dinner?

   A 13   **C** 10

   B 11   **D** 9

45. REVIEW Lorenzo's math test scores are shown below.

   85%, 92%, 95%

   If he earns a 92% on the next test, then

   F the median would decrease.

   G the mean would decrease.

   H the median would increase.

   J the mean would increase.

Spiral Review

Simplify each expression. (Lesson 11-8)

46. $\dfrac{\frac{x^2 + 8x + 15}{x^2 + x - 6}}{\frac{x^2 + 2x - 15}{x^2 - 2x - 3}}$

47. $\dfrac{\frac{a^2 - 6a + 5}{a^2 + 13a + 42}}{\frac{a^2 - 4a + 3}{a^2 + 3a - 18}}$

48. $\dfrac{x + 2 + \frac{2}{x + 5}}{x + 6 + \frac{6}{x + 1}}$

Find each difference. (Lesson 11-7)

49. $\dfrac{3}{2m - 3} - \dfrac{m}{6 - 4m}$

50. $\dfrac{y}{y^2 - 2y + 1} - \dfrac{1}{y - 1}$

**51.** $\dfrac{a + 2}{a^2 - 9} - \dfrac{2a}{6a^2 - 17a - 3}$

52. CHEMISTRY One solution is 50% glycol, and another is 30% glycol. How much of each solution should be mixed to make a 100 gallon solution that is 45% glycol? (Lesson 2-9)

Cross-Curricular Project

Math and Science

Building the Best Roller Coaster It is time to complete your project. Use the information and data you have gathered about the building and financing of a roller coaster to prepare a portfolio or Web page. Be sure to include graphs and/or tables in the presentation.
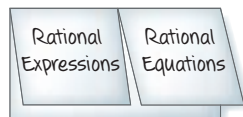
Math Online Cross-Curricular Project at **algebra1.com**
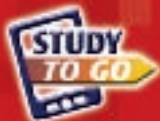
FOLDABLES
Study Organizer

GET READY to Study

Be sure the following Key Concepts are noted in your Foldable.

Rational Expressions | Rational Equations

Key Concepts

Inverse Variation (Lesson 11-1)
- You can use $\frac{x_1}{x_2} = \frac{y_1}{y_2}$ to solve problems involving inverse variation.

Rational Expressions (Lessons 11-2 to 11-4)
- Excluded values are values of a variable that result in a denominator of zero.
- Multiplying rational expressions is similar to multiplying rational numbers.
- Divide rational expressions by multiplying by the reciprocal of the divisor.

Dividing Polynomials (Lesson 11-5)
- To divide a polynomial by a monomial, divide each term of the polynomial by the monomial.

Rational Expressions (Lessons 11-6 and 11-7)
- Add (or subtract) rational expressions with like denominators by adding (or subtracting) the numerators and writing the sum (or difference) over the denominator.
- Rewrite rational expressions with unlike denominators using the least common denominator (LCD). Then add or subtract.

Complex Fractions (Lesson 11-8)
- Simplify complex fractions by writing them as division problems.

Solving Rational Equations (Lesson 11-9)
- Use cross product rule to solve rational equations with a single fraction on each side of the equals sign.

Key Vocabulary

complex fraction (p. 621)
excluded values (p. 583)
extraneous solution (p. 629)
inverse variation (p. 577)
least common denominator (p. 615)
least common multiple (p. 614)
mixed expression (p. 620)
product rule (p. 578)
rate problem (p. 628)
rational equation (p. 626)
rational expression (p. 583)

Vocabulary Check

State whether each sentence is *true* or *false*. If *false*, replace the underlined word or number to make a true sentence.

1. A <u>mixed</u> expression is a fraction whose numerator and denominator are polynomials.

2. The complex fraction $\frac{\frac{4}{5}}{\frac{2}{3}}$ can be simplified as $\frac{6}{5}$.

3. The equation $\frac{x}{x-1} + \frac{2x-3}{x-1}$ has an extraneous <u>1</u>.

4. The mixed expressions $6 - \frac{a-2}{a+3}$ can be rewritten as $\underline{\frac{5a+16}{a+3}}$.

5. The least common multiple for $(x^2 - 144)$ and $(x + 12)$ is <u>$(x + 12)$</u>.

6. The equation $x_1 y_1 = x_2 y_2$ is called the <u>product rule</u> for inverse variations.

7. The excluded values for $\frac{4x}{x^2 - x - 12}$ are <u>-3 and 4</u>.

8. When the product of two values remains constant, the relationship forms an <u>inverse variation</u>.

Lesson-by-Lesson Review

11-1 Inverse Variation (pp. 577–582)

Write an inverse variation equation that relates x and y. Assume that y varies inversely as x. Then solve.

9. If $y = 28$ when $x = 42$, find y when $x = 56$.

10. If $y = 35$ when $x = 175$, find x when $y = 75$.

11. **PHYSICS** If a 135-pound person sits 5 feet from the center of a seesaw and a 108-pound person is on the other end, how far from the center should the 108-pound person sit to balance?

Example 1 If y varies inversely as x and $y = 24$ when $x = 30$, find x when $y = 10$.

Let $x_1 = 30$, $y_1 = 24$, and $y_2 = 10$. Solve for x_2. Use a proportion.

$\dfrac{x_1}{x_2} = \dfrac{y_2}{y_1}$ Proportion for inverse variations

$\dfrac{30}{x_2} = \dfrac{10}{24}$ $x_1 = 30, y_1 = 24$, and $y_2 = 10$

$720 = 10x$ Cross multiply.

$72 = x_2$ Divide each side by 10.

Thus, $x = 72$ when $y = 10$.

11-2 Operations with Radical Expressions (pp. 583–588)

Simplify each expression.

12. $\dfrac{3x^2 y}{12xy^3 z}$

13. $\dfrac{n^2 - 3n}{n - 3}$

14. $\dfrac{a^2 - 25}{a^2 + 3a - 10}$

15. $\dfrac{x^2 + 10x + 21}{x^3 + x^2 - 42x}$

16. $\dfrac{b^2 - 5b + 6}{b^2 - 13b + 36}$

17. $\dfrac{3x^3}{3x^3 + 6x^2}$

Example 2 Simplify $\dfrac{x + 4}{x^2 + 12x + 32}$.

$\dfrac{x + 4}{x^2 + 12x + 32} = \dfrac{\overset{1}{\cancel{x + 4}}}{\underset{1}{(\cancel{x + 4})(x + 8)}}$ Factor.

$= \dfrac{1}{x + 8}$ Simplify.

11-3 Multiplying Rational Expressions (pp. 590–594)

Find each product.

18. $\dfrac{7b^2}{9} \cdot \dfrac{6a^2}{b}$

19. $\dfrac{5x^2 y}{8ab} \cdot \dfrac{12a^2 b}{25x}$

20. $(3x + 30) \cdot \dfrac{10}{x^2 - 100}$

21. $\dfrac{3a - 6}{a^2 - 9} \cdot \dfrac{a + 3}{a^2 - 2a}$

22. $\dfrac{b^2 + 19b + 84}{b - 3} \cdot \dfrac{b^2 - 9}{b^2 + 15b + 36}$

Example 3 Find $\dfrac{6m^2 n^4}{12} \cdot \dfrac{3m^3 n^2}{mn}$.

$\dfrac{6m^2 n^4}{12} \cdot \dfrac{3m^3 n^2}{mn}$

$= \dfrac{2mn^3}{4} \cdot \dfrac{m^2 n}{1}$ Divide by GCF 3mn.

$= \dfrac{2m^3 n^4}{4}$ or $\dfrac{m^3 n^4}{2}$ Multiply.

11-4 **Dividing Rational Expressions** (pp. 595–599)

Find each quotient.

23. $\dfrac{p^3}{2q} \div \dfrac{p^3}{4q}$

24. $\dfrac{y^2}{y+4} \div \dfrac{3y}{y^2-16}$

25. $\dfrac{3x-12}{y+4} \div (y^2 - 6y + 8)$

26. $\dfrac{2m^2 + 7m - 15}{m+5} \div \dfrac{9m^2 - 4}{3m+2}$

27. PIZZA On average, Americans eat 18 acres of pizza a day. If an average slice of pizza is about 5 square inches, how many pieces of pizza is this? (*Hint*: 43,560 square feet per acre)

Example 4 Find $\dfrac{y^2-16}{y^2-64} \div \dfrac{y+4}{y-8}$.

$\dfrac{y^2-16}{y^2-64} \div \dfrac{y+4}{y-8}$

$= \dfrac{y^2-16}{y^2-64} \cdot \dfrac{y-8}{y+4}$ Multiply by the reciprocal of $\dfrac{y+4}{y-8}$.

$= \dfrac{(y-4)(y+4)}{(y-8)(y+8)} \cdot \dfrac{y-8}{y+4}$ Factor.

$= \dfrac{(y-4)\cancel{(y+4)}}{\cancel{(y-8)}(y+8)} \cdot \dfrac{\cancel{y-8}}{\cancel{y+4}}$ Simplify.

$= \dfrac{y-4}{y+8}$

11-5 **Dividing Polynomials** (pp. 601–606)

Find each quotient.

28. $(4a^2b^2c^2 - 8a^3b^2c + 6abc^2) \div 2ab^2$

29. $(x^3 + 7x^2 + 10x - 6) \div (x + 3)$

30. $(48b^2 + 8b + 7) \div (12b - 1)$

31. $(4t^2 + 17t - 1) \div (4t + 1)$

32. GEOMETRY The volume of a prism with a triangular base is $x^3 + 6.5x^2 + 8.5x - 6$. If the height of the prism is $2x - 1$, what is the area of the triangular base?

Example 5 Find $(x^3 - 2x^2 - 22x + 21) \div (x - 3)$.

$$
\begin{array}{r}
x^2 + x - 19 \\
x - 3 \overline{)\, x^3 - 2x^2 - 22x + 21}
\end{array}
$$

    $\underline{x^3 - 3x^2}$ Multiply x^2 and $x - 3$.

      $x^2 - 22x$ Subtract.

      $\underline{x^2 - \;\;3x}$ Multiply x and $x - 3$.

       $-19x + 21$ Subtract.

       $\underline{-19x + 57}$ Multiply -19 and $x - 3$.

         -36 Subtract.

The quotient is $x^3 + x - 19 - \dfrac{36}{x-3}$.

11-6 **Rational Expressions with Like Denominators** (pp. 608–613)

Find each sum or difference.

33. $\dfrac{m+4}{5} + \dfrac{m-1}{5}$

34. $\dfrac{-5}{2n-5} + \dfrac{2n}{2n-5}$

35. $\dfrac{a^2}{a-b} + \dfrac{-b^2}{a-b}$

36. $\dfrac{7a}{b^2} - \dfrac{5a}{b^2}$

37. $\dfrac{2x}{x-3} - \dfrac{6}{x-3}$

38. $\dfrac{m^2}{m-n} - \dfrac{2mn-n^2}{m-n}$

Example 6 Find $\dfrac{n^2 + 10n}{n+5} + \dfrac{25}{n+5}$.

$\dfrac{n^2 + 10n}{n+5} + \dfrac{25}{n+5}$

$= \dfrac{n^2 + 10n + 25}{n+5}$ Add the numerators.

$= \dfrac{(n+5)(n+5)}{(n+5)}$ Factor.

$= n + 5$ Simplify.

11-7 **Rational Expressions with Unlike Denominators** (pp. 614–619)

Find each sum or difference.

39. $\dfrac{2c}{3d^2} + \dfrac{3}{2cd}$ **40.** $\dfrac{r^2 + 21r}{r^2 - 9} + \dfrac{3r}{r + 3}$

41. $\dfrac{7}{3a} - \dfrac{3}{6a^2}$ **42.** $\dfrac{2x}{2x + 8} - \dfrac{4}{5x + 20}$

Example 7 Find $\dfrac{3}{y + 1} - \dfrac{y}{y + 3}$.

$$\dfrac{3}{y + 1} - \dfrac{y}{y + 3}$$

$$= \dfrac{y + 3}{y + 3} \cdot \dfrac{3}{y + 1} - \dfrac{y}{y + 3} \cdot \dfrac{y + 1}{y + 1}$$

$$= \dfrac{3y + 9}{(y + 3)(y + 1)} - \dfrac{y^2 + y}{(y + 3)(y + 1)}$$

$$= \dfrac{-y^2 + 2y + 9}{(y + 3)(y + 1)}$$

11-8 **Mixed Expressions and Complex Fractions** (pp. 620–625)

Write each mixed expression as a rational expression.

43. $4 + \dfrac{x}{x - 2}$ **44.** $2 - \dfrac{x + 2}{x^2 - 4}$

Simplify each expression.

45. $\dfrac{x + \dfrac{35}{x + 2}}{x + \dfrac{42}{x + 13}}$ **46.** $\dfrac{y + 9 - \dfrac{6}{y + 4}}{y + 4 + \dfrac{2}{y + 1}}$

Example 8 Simplify $\dfrac{\dfrac{a^2 b^4}{c}}{\dfrac{a^3 b}{c^2}}$.

$$\dfrac{\dfrac{a^2 b^4}{c}}{\dfrac{a^3 b}{c^2}} = \dfrac{a^2 b^4}{c} \div \dfrac{a^3 b}{c^2} \quad \text{Rewrite as a division sentence.}$$

$$= \dfrac{a^2 b^4}{c} \cdot \dfrac{c^2}{a^3 b} \quad \text{Multiply by the reciprocal.}$$

$$= \dfrac{b^3 c}{a} \quad \text{Simplify.}$$

11-9 **Rational Equations and Functions** (pp. 626–632)

Solve each equation. State any extraneous solutions.

47. $\dfrac{4x}{3} + \dfrac{7}{2} = \dfrac{7x}{12} - 14$

48. $\dfrac{11}{2x} - \dfrac{2}{3x} = \dfrac{1}{6}$

49. $\dfrac{3}{x^2 + 3x} + \dfrac{x + 2}{x + 3} = \dfrac{1}{x}$

50. $\dfrac{1}{n + 4} - \dfrac{1}{n - 1} = \dfrac{2}{n^2 + 3n - 4}$

51. JOBS Normally, it takes Jeffery 1 hour 45 minutes to mow and trim an average lawn. When Lupe worked with him, an average lawn only took an hour. How long would it take Lupe to mow and trim an average yard on her own?

Example 9 Solve $\dfrac{5n}{6} + \dfrac{1}{n - 2} = \dfrac{n + 1}{3(n - 2)}$.

$$\dfrac{5n}{6} + \dfrac{1}{n - 2} = \dfrac{n + 1}{3(n - 2)}$$

$$6(n - 2)\left(\dfrac{5n}{6} + \dfrac{1}{n - 2}\right) = 6(n - 2)\dfrac{n + 1}{3(n - 2)}$$

$$6(n - 2)(5n) + \dfrac{6(n - 2)}{n - 2} = \dfrac{6(n - 2)(n + 1)}{3(n - 2)}$$

$$(n - 2)(5n) + 6 = 2(n + 1) \quad \text{Simplify.}$$

$$5n^2 - 12n + 4 = 0 \quad \text{Simplify.}$$

$$(5n - 2)(n - 2) = 0 \quad \text{Factor.}$$

$$n = \dfrac{2}{5} \text{ or } n = 2$$

When you check the value 2, you get a zero in the denominator. So, 2 is an extraneous solution.

WINTER For Exercises 1 and 2 use the following information.

An ice sculptor has a cube of ice. The length of each side of the cube is x inches. To begin a sculpture, he removes $\frac{3}{4}$ of an inch from the top. Then he removes $\frac{1}{2}$ inch from the width and length of the block.

1. Write an expression that represents the current volume of the block.
2. The sculptor scraps his idea and decides to divide the block into $x - 2$ blocks. What is the volume of the smaller blocks?

Solve each variation. Assume that y varies inversely as x.

3. If $y = 21$ when $x = 40$, find y when $x = 84$.
4. If $y = 22$ when $x = 4$, find x when $y = 16$.

5. **MULTIPLE CHOICE** Willie can type a 200 word essay in 6 hours. Myra can type the same essay in $4\frac{1}{2}$ hours. If they work together, how long will it take them to type the essay?

   A $2\frac{3}{5}$ hr   C $1\frac{4}{7}$ hr

   B $2\frac{4}{7}$ hr   D $1\frac{3}{5}$ hr

Simplify each rational expression. State the excluded values of the variables.

6. $\dfrac{5 - 2m}{6m - 15}$ 7. $\dfrac{3 + x}{2x^2 + 5x - 3}$

8. $\dfrac{4c^2 + 12c + 9}{2c^2 - 11c - 21}$ 9. $\dfrac{x + 4 + \frac{5}{x - 2}}{x + 6 + \frac{15}{x - 2}}$

10. $\dfrac{1 - \frac{9}{t}}{1 - \frac{81}{t^2}}$ 11. $\dfrac{\frac{5}{6} + \frac{u}{t}}{\frac{2u}{t} - 3}$

Perform the indicated operations.

12. $\dfrac{2x}{x - 7} - \dfrac{14}{x - 7}$

13. $\dfrac{n + 3}{2n - 8} \cdot \dfrac{6n - 24}{2n + 1} \div (z - 3)$

14. $(10m^2 + 9m - 36) \div (2m - 3)$

15. $\dfrac{x^2 + 4x - 32}{x + 5} \cdot \dfrac{x - 3}{x^2 - 7x + 12}$

16. $\dfrac{4x^2 + 11x + 6}{x^2 - x - 6} \div \dfrac{x^2 + 8x + 16}{x^2 + x - 12}$

17. $(10z^4 + 5z^3 - z^2) \div 5z^3$

18. $\dfrac{y}{7y + 14} + \dfrac{6}{-3y + 6}$

19. $\dfrac{x + 5}{x + 2} + 6$

Solve each equation.

20. $\dfrac{2}{3t} + \dfrac{1}{2} = \dfrac{3}{4t}$ 21. $\dfrac{2c}{c - 4} - 2 = \dfrac{4}{c + 5}$

22. **FINANCE** Barrington High School is raising money to build a house for Habitat for Humanity by doing lawn work for friends and neighbors. Scott can rake a lawn and bag the leaves in 5 hours, while Kalyn can do it in 3 hours. If Scott and Kalyn work together, how long will it take them to rake a lawn and bag the leaves?

23. **MULTIPLE CHOICE** Which expression can be used to represent the area of the triangle?
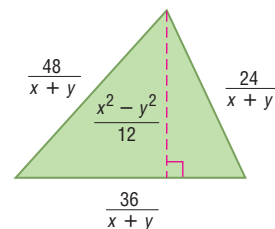
   F $2(x - y)$ H $4(x - y)$

   G $\dfrac{3}{2}(x - y)$ J $\dfrac{108}{x + y}$

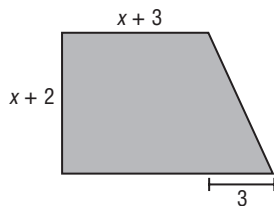**Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.**

1. Which expression is equivalent to $\dfrac{(3y^5)(6y^4)}{9y^2}$?

   A $2y^7$

   B y^{11}

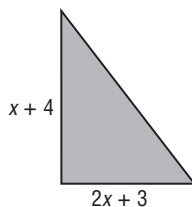   **C** $2y^{18}$

   D y^{18}

2. What is the area of the polygon below?

   F $x^2 + 5x + 6$

   G $x^2 + \dfrac{13}{2}x + 9$

   H $x^2 + 8x + 12$

   J $x^2 + \dfrac{13}{2}x + 12$

3. GRIDDABLE If $4x + y = 12$ and $x = -2$, then what value is y equal to?
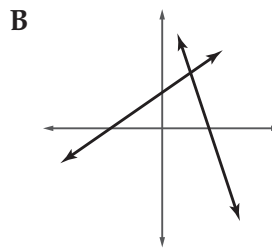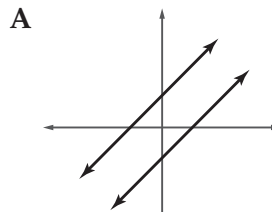
4. What is the area of the polygon below?

   A $2x^2 + 11x + 12$

   B $x^2 + \dfrac{11}{2}x + 6$

   C $3x + 7$

   D $x^2 + 6$

5. Which expression is equivalent to $\dfrac{16a^2b^3}{32ab^7}$?

   F $2ab^4$ **H** $\dfrac{a}{2b^4}$

   G $\dfrac{b^4}{2a}$ **J** $\dfrac{a^3}{2b^{10}}$

6. Which graph represents a system of equations with infinitely many solutions?

   A

   B

   C

   D
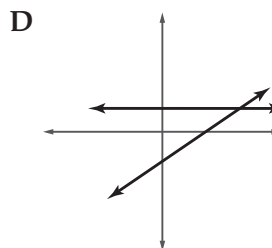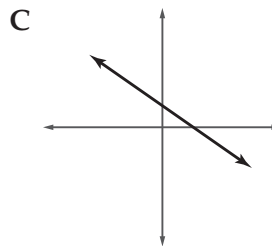
TEST-TAKING TIP

Question 6 If you don't know how to solve a problem, eliminate the answer choices you know are incorrect and then guess from the remaining choices. Even eliminating only one answer choice greatly increases your chance of guessing the correct answer.

Standardized Test Practice at **algebra1.com**

7. Lauren sold T-shirts for 10 days in a row as a fundraiser. In those 10 days, she sold 120 T-shirts for an average of 12 T-shirts per day. For how many days must she sell 20 T-shirts to bring her average to 18 T-shirts per day?
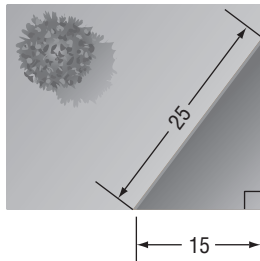
F 10   H 30
G 20   J 40

8. The graph of the equation $y = x^2 - x - 6$ is shown below.
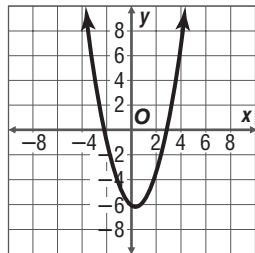
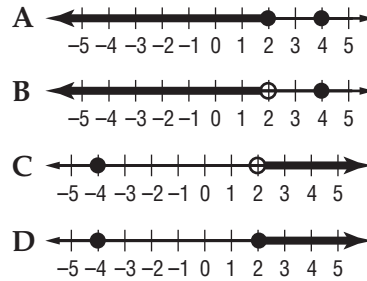For what value or values of x is $y = 0$?
A $x = -2$ only
B $x = -3$ only
C $x = -2$ and $x = 3$
D $x = 2$ and $x = -3$

9. Sally wants to sod a corner of her yard as pictured below. How much sod will she need to cover the triangular area?

F 150 ft^2 H 187.5 ft^2
G 300 ft^2 J 375 ft^2

10. Which graph below represents the solution set for $h = -4$ and $h > 2$?

11. GRIDDABLE Peter drives to his grandmother's house every Sunday. He lives 120 miles from his grandmother and he drives this distance in 2.4 hours. At this rate, how long, in hours, would it take him to drive 300 miles?

Pre-AP

Record your answers on a sheet of paper. Show your work.

12. A 12-foot ladder is placed against the side of a building so that the bottom of the ladder is 6 feet from the base of the building.

  a. Suppose the bottom of the ladder is moved closer to the base of the building. Does the height that the ladder reaches increase or decrease?

  b. What conclusion can you make about the height the ladder reaches and the distance between the bottom of the ladder and the base of the building?

  c. Does this relationship form an inverse variation? Explain your reasoning.

NEED EXTRA HELP?												
If You Missed Question...	1	2	3	4	5	6	7	8	9	10	11	12
Go to Lesson...	7-2	11-3	3-3	11-3	11-2	4-5	11-9	9-2	10-4	6-4	3-6	11-1