

Statistics and Probability

BIG Ideas

- Identify various sampling techniques.
- Count outcomes using the Fundamental Counting Principle.
- Determine probabilities.

Key Vocabulary

combination (p. 657)

compound event (p. 663)

permutation (p. 655)

sample (p. 642)

Real-World Link

U.S. Senate The United States Senate forms committees to focus on different issues. These committees are made up of senators from various states and political parties. You can use probability to find how many ways these committees can be formed.

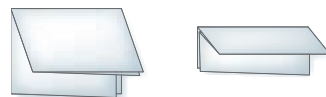
FOLDABLES™ Study Organizer

Statistics and Probability Make this Foldable to help you organize what you learn about statistics and probability. Begin with a sheet of $8\frac{1}{2}'' \times 11''$ paper.

- 1 Fold** in half lengthwise.



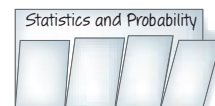
- 2 Fold** the top to the bottom twice.



- 3 Open.** Cut along the second fold to make four tabs.



- 4 Label** as shown.



GET READY for Chapter 12

Diagnose Readiness You have two options for checking Prerequisite Skills.

Option 2



Take the Online Readiness Quiz at algebra1.com.

Option 1

Take the Quick Check below. Refer to the Quick Review for help.

QUICK Check

Determine the probability of each event if you randomly select a cube from a bag containing 6 red cubes, 4 yellow cubes, 3 blue cubes, and 1 green cube.

(Prerequisite Skill)

1. $P(\text{red})$ 2. $P(\text{blue})$ 3. $P(\text{not red})$

4. **GAMES** Paul is going to roll a game cube with 3 sides painted red, two painted blue, and 1 painted green. What is the probability that a red side will land face up?

Find each product. (Prerequisite Skill)

5. $\frac{4}{5} \cdot \frac{3}{4}$

6. $\frac{5}{12} \cdot \frac{6}{11}$

7. $\frac{7}{20} \cdot \frac{4}{19}$

8. $\frac{4}{32} \cdot \frac{7}{32}$

9. $\frac{13}{52} \cdot \frac{4}{52}$

10. $\frac{56}{100} \cdot \frac{24}{100}$

Write each fraction as a percent. Round to the nearest tenth. (Prerequisite Skill)

11. $\frac{7}{8}$

12. $\frac{33}{80}$

13. $\frac{107}{125}$

14. $\frac{625}{1024}$

15. **CONCERTS** At a local concert, 585 of 2000 people were under the age of 18. What percentage of the audience were under 18? Round to the nearest tenth.

QUICK Review

EXAMPLE 1

Determine the probability of selecting a green cube if you randomly select a cube from a bag containing 6 red cubes, 4 yellow cubes, and 1 green cube.

There is 1 green cube and a total of 11 cubes in the bag.

$$\frac{1}{11} = \frac{\text{number of green cubes}}{\text{total number of cubes}}$$

The probability of selecting a green cube is $\frac{1}{11}$.

EXAMPLE 2

Find $\frac{5}{4} \cdot \frac{2}{3}$.

$$\frac{5}{4} \cdot \frac{2}{3} = \frac{5 \cdot 2}{4 \cdot 3}$$

Multiply both the numerators and the denominators.

$$= \frac{10}{12} \text{ or } \frac{5}{6}$$

Simplify.

EXAMPLE 3

Write the fraction $\frac{14}{17}$ as a decimal. Round to the nearest tenth.

$$\frac{14}{17} = 0.823 \quad \text{Simplify and round.}$$

$$0.823 \times 100 \quad \text{Multiply the decimal by 100.}$$

$$= 82.3 \quad \text{Simplify.}$$

$\frac{14}{17}$ written as a percent is 82.3%.

Main Ideas

- Identify various sampling techniques.
- Recognize a biased sample.

New Vocabulary

sample
 population
 random sample
 simple random sample
 stratified random sample
 systematic random sample
 biased sample
 convenience sample
 voluntary response sample

GET READY for the Lesson

Manufacturing music CDs involves burning copies from a master. However, not every burn is successful. Because it is costly to check every CD, manufacturers monitor production by randomly checking CDs for defects.

Sampling Techniques A **sample** is some portion of a larger group, called the **population**, selected to represent that group. Sample data are often used to estimate a characteristic within an entire population, such as voting preferences prior to elections. A **random sample** of a population is selected so that it is representative of the entire population. The sample is chosen without any preference. There are several ways to pick a random sample.

KEY CONCEPT		Random Samples
Type	Definition	Example
Simple Random Sample	A simple random sample is a sample that is as equally likely to be chosen as any other sample from the population.	The 26 students in a class are each assigned a different number from 1 to 26. Then three of the 26 numbers are picked at random.
Stratified Random Sample	In a stratified random sample, the population is first divided into similar, nonoverlapping groups. A sample is then selected from each group.	The students in a school are divided into freshmen, sophomores, juniors, and seniors. Then two students are randomly selected from each group of students.
Systematic Random Sample	In a systematic random sample, the items are selected according to a specified time or item interval.	Every 2 minutes, an item is pulled off the assembly line. or Every twentieth item is pulled off the assembly line.

EXAMPLE Classify a Random Sample

1 ECOLOGY Ten lakes in Minnesota are selected randomly. Then 2 liters of water are drawn from each of the ten lakes.

- a. Identify the sample and suggest a population from which it was selected.

The sample is ten 2-liter containers of lake water, one from each of 10 lakes. The population is lake water from all of the lakes in Minnesota.

b. Classify the sample as *simple*, *stratified*, or *systematic*.

This is a simple random sample. Each of the ten lakes was equally likely to have been chosen from the list.

CHECK Your Progress

BARBECUE Refer to the information at the left. The cooks lined up randomly within their category, and every tenth cook in each category was selected.

1A. Identify the sample and a population from which it was selected.

1B. Classify the sample as *simple*, *stratified*, or *systematic*.

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Biased Sample Random samples are unbiased. In a **biased sample**, one or more parts of a population are favored over others.

EXAMPLE Identify Sample as Biased or Unbiased

2 Identify each sample as *biased* or *unbiased*. Explain your reasoning.

a. **MANUFACTURING** Every 1000th bolt is pulled from the production line and measured for length.

The sample is chosen using a specified interval. This is an unbiased sample because it is a systematic random sample.

b. **MUSIC** Every tenth customer in line for a certain rock band's concert tickets is asked about his or her favorite rock band.

The sample is a biased sample because customers in line for concert tickets are more likely to name the band giving the concert as a favorite.

CHECK Your Progress

2. **POLITICS** A journalist visited a senior center and chose 10 individuals randomly to poll about various political topics.



Real-World Link

Each year, Meridian, Texas, hosts The National Championship Barbecue Cook-Off. In 2003, there were 190 cooks in the competition, and they competed in one of three categories: brisket, chicken, or pork spare ribs.

Source: bbq.htcomp.net

Two popular forms of samples that are often biased include convenience samples and voluntary response samples.

KEY CONCEPT		Biased Samples
Type	Definition	Example
Convenience Sample	A convenience sample includes members of a population who are easily accessed.	To check spoilage, a produce worker selects 10 apples from the top of the bin. The 10 apples are unlikely to represent all of the apples in the bin.
Voluntary Response Sample	A voluntary response sample involves only those who want to participate in the sampling.	A radio call-in show records that 75% of its 40 callers voiced negative opinions about a local football team. Those 40 callers are unlikely to represent the entire local population. Volunteer callers are more likely to have strong opinions and are typically more negative than the entire population.

EXAMPLE Identify and Classify a Biased Sample

3 BUSINESS The travel account records from 4 of the 20 departments in a corporation are to be reviewed. The accountant states that the first 4 departments to voluntarily submit their records will be reviewed.

a. Identify the sample and a population from which it was selected.

The sample is the travel account records from 4 departments in the corporation. The population is the travel account records from all 20 departments in the corporation.

b. Classify the sample as *convenience* or *voluntary response*.

Since the departments voluntarily submit their records, this is a voluntary response sample.

CHECK Your Progress

POLL A principal asks the students in her school to write down the name of a favorite teacher on an index card. She then tabulates the results from the first 20 and the last 20 cards received.

3A. Identify the sample and a population from which it was selected.

3B. Classify the sample as *convenience* or *voluntary response*.

EXAMPLE Identify the Sample

4 NEWS REPORTING Rafael needs to determine whether students in his school believe that an arts center should be added to the school. He polls 15 of his friends who sing in the chorale. Twelve of them think the school needs an arts center, so Rafael reports that 80% of the students surveyed support the project.

a. Identify the sample.

The sample is a group of students from the chorale.

b. Suggest a population from which the sample was selected.

The population for the survey is all of the students in the school.

c. State whether the sample is *unbiased* (random) or *biased*. If unbiased, classify it as *simple*, *stratified*, or *systematic*. If biased, classify it as *convenience* or *voluntary response*.

The sample was from the chorale. So the reported support is not likely to be representative of the student body. The sample is biased. Since Rafael polled only his friends, it is a convenience sample.

CHECK Your Progress

ELECTIONS To estimate the leading candidate, a candidate's committee randomly sends a survey to the people on their mailing list. The returns indicate that their candidate is leading by a margin of 58% to 42%.

4A. Identify the sample.

4B. Suggest a population from which the sample was selected.

4C. State whether the sample is *unbiased* (random) or *biased*. If unbiased, classify it as *simple*, *stratified*, or *systematic*. If biased, classify it as *convenience* or *voluntary response*.

Study Tip

Random Sample

A sample is *random* if every member of the population has an equal probability of being chosen for the sample.

Examples 1–4 (pp. 642–644) Identify each sample, suggest a population from which it was selected, and state whether it is *unbiased* (random) or *biased*. If unbiased, classify the sample as *simple*, *stratified*, or *systematic*. If biased, classify as *convenience* or *voluntary response*.

- 1. NEWSPAPERS** The local newspaper asks readers to write letters stating their preferred candidates for mayor.
- 2. SCHOOL** A teacher needs a sample of work from four students in her first-period math class to display at the school open house. She selects the work of the first four students who raise their hands.
- 3. BUSINESS** A hardware store wants to assess the strength of nails it sells. Store personnel select 25 boxes at random from among all of the boxes on the shelves. From each of the 25 boxes, they select one nail at random and subject it to a strength test.
- 4. SCHOOL** A class advisor hears complaints about an incorrect spelling of the school name on pencils sold at the school store. The advisor goes to the store and asks Namid to gather a sample of pencils and look for spelling errors. Namid grabs the closest box of pencils and counts out 12 pencils from the top of the box. She checks the pencils, returns them to the box, and reports the results to the advisor.

Exercises

HOMEWORK HELP	
For Exercises	See Examples
5–18	1–4

Identify each sample, suggest a population from which it was selected, and state whether it is *unbiased* (random) or *biased*. If unbiased, classify the sample as *simple*, *stratified*, or *systematic*. If biased, classify as *convenience* or *voluntary response*.

- 5. SCHOOL** Pieces of paper with the names of three sophomores are drawn from a hat containing identical pieces of paper with all sophomores' names.
- 6. FOOD** Twenty shoppers outside a fast-food restaurant are asked to name their preferred cola between two choices.
- 7. RECYCLING** An interviewer goes from house to house on weekdays between 9 A.M. and 4 P.M. to determine how many people recycle.
- 8. POPULATION** Ten people from each of the 86 counties in a state are chosen at random and asked their opinion on a state issue.
- 9. SCOOTERS** A scooter manufacturer is concerned about quality control. The manufacturer checks the first five scooters off the line in the morning and the last five off the line in the afternoon for defects.
- 10. SCHOOL** To determine who will speak for her class at the school board meeting, Ms. Finchie used the numbers appearing next to her students' names in her grade book. She writes each of the numbers on an identical piece of paper and shuffles the pieces of papers in a box. Without seeing the contents of the box, one student draws 3 pieces of paper from the box. The students with these numbers will speak for the class.



Real-World Link

In 2002, Washington led the nation in cherry production by growing 194 million pounds of cherries.

Source: USDA

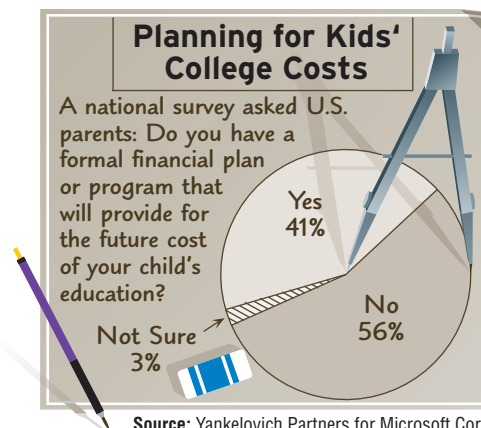
Identify each sample, suggest a population from which it was selected, and state whether it is *unbiased* (random) or *biased*. If unbiased, classify the sample as *simple*, *stratified*, or *systematic*. If biased, classify as *convenience* or *voluntary response*.

11. **FARMING** An 8-ounce jar was filled with corn from a storage silo by dipping the jar into the pile of corn. The corn in the jar was then analyzed for moisture content.
12. **COURTS** The gender makeup of district court judges in the United States is to be estimated from a sample. All judges are grouped geographically by federal reserve districts. Within each of the 11 federal reserve districts, all judges' names are assigned a distinct random number. In each district, the numbers are then listed in order. A number between 1 and 20 inclusive is selected at random, and the judge with that number is selected. Then every 20th name after the first selected number is also included in the sample.
13. **TELEVISION** A television station asks its viewers to share their opinions about a proposed golf course to be built just outside the city limits. Viewers can call one of two 800 numbers. One number represents a "yes" vote, and the other number represents a "no" vote.
14. **GOVERNMENT** To discuss leadership issues shared by all United States Senators, the President asks four of his closest colleagues in the Senate to meet with him.
15. **FOOD** To sample the quality of the Bing cherries throughout the produce department, the produce manager picks up a handful of cherries from the edge of one case and checks to see if these cherries are spoiled.
16. **MANUFACTURING** During the manufacture of high-definition televisions, units are checked for defects. Within the first 10 minutes of a work shift, a television is randomly chosen from the line of completed sets. For the rest of the shift, every 15th television on the line is checked for defects.
17. **BUSINESS** To get reaction about a benefits package, a company uses a computer program to randomly pick one person from each of its departments.
18. **MOVIES** A magazine is trying to determine the most popular actor of the year. It asks its readers to mail the name of their favorite actor to their office.

COLLEGE For Exercises 19 and 20, use the following information.

The graph at the right reveals that 56% of survey respondents did not have a formal financial plan for a child's college tuition.

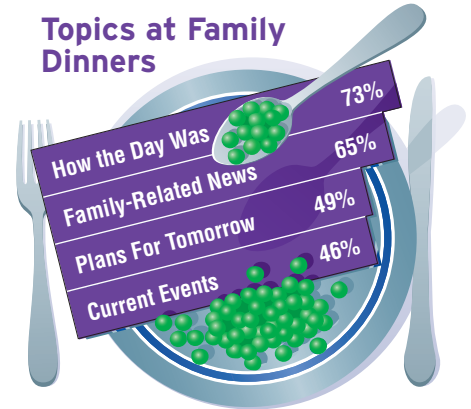
19. Write a statement to describe what you do know about the sample.
20. What additional information would you like to have about the sample to determine whether the sample is biased?



DESIGN A SURVEY For Exercises 21–23, describe an unbiased way to conduct each survey.

21. **SCHOOL** Suppose you want to sample the opinion of the students in your school about a new dress code.
22. **ELECTIONS** Suppose you are running for mayor of your city and want to know if you are likely to be elected.
23. **PICK A TOPIC** Write a question you would like to conduct a survey to answer. Then describe an unbiased way to conduct your survey.

24. **FAMILY** Study the graph at the right. Describe the information that is revealed in the graph. What information is there about the type or size of the sample?



Source: National Pork Producers Council

25. **FARMING** Suppose you are a farmer and want to know if your tomato crop is ready to harvest. Describe an unbiased way to determine whether the crop is ready to harvest.

26. **MANUFACTURING** Suppose you want to know whether the infant car seats manufactured by your company meet the government standards for safety. Describe an unbiased way to determine whether the seats meet the standards.

EXTRA PRACTICE
See pages 742, 755.
Math online
Self-Check Quiz at algebra1.com

H.O.T. Problems

27. **REASONING** Describe how the following three types of sampling techniques are similar and how they are different.

- simple random sample
- stratified random sample
- systematic random sample

28. **REASONING** Explain the difference between a convenience sample and a voluntary response sample.

29. **OPEN ENDED** Give a real-world example of a biased sample.

30. **CHALLENGE** The following is a proposal for surveying a stratified random sample of the student body.

Divide the student body according to those who are on the basketball team, those who are in the band, and those who are in the drama club. Then take a simple random sample from each of the three groups. Conduct the survey using this sample.

Study the proposal. Describe its strengths and weaknesses. Is the sample a stratified random sample? Explain.

31. **Writing in Math** Refer to the information on page 642 to explain why sampling is important in manufacturing. Describe two different ways, one biased and one unbiased, to pick which CDs to check.

32. To predict the candidate who will win the seat in city council, which method would give the newspaper the *most* accurate result?

- A Ask every fifth person that passes a reporter in the mall.
- B Use a list of registered voters and call every 20th person.
- C Publish a survey and ask readers to reply.
- D Ask reporters at the newspaper.

33. **REVIEW** Which equation *best* represents the relationship between x and y ?

x	y
-1	8
0	5
1	2
2	-1
3	-4
4	-7

- F $y = 8 - 3x$
- G $y = 5x - 3$
- H $y = -3x + 5$
- J $y = 3x + 5$

Spiral Review

Solve each equation. (Lesson 11-9)

34. $\frac{10}{3y} - \frac{5}{2y} = \frac{1}{4}$

35. $\frac{3}{r+4} - \frac{1}{r} = \frac{1}{r}$

36. $\frac{1}{4m} + \frac{2m}{m-3} = 2$

Simplify. (Lesson 11-8)

37. $\frac{2 + \frac{5}{x}}{\frac{x}{3} + \frac{5}{6}}$

38. $\frac{a + \frac{35}{a+12}}{a+7}$

39. $\frac{t^2 - 4}{\frac{t^2 + 5t + 6}{t-2}}$

40. **GEOMETRY** The sides of a triangle have measures of $4\sqrt{24}$ centimeters, $5\sqrt{6}$ centimeters, and $3\sqrt{54}$ centimeters. What is the perimeter of the triangle? Write in simplest form. (Lesson 10-2)

Solve each equation by using the Quadratic Formula. Approximate any irrational roots to the nearest tenth. (Lesson 9-4)

41. $x^2 - 6x - 40 = 0$

42. $6b^2 + 15 = -19b$

43. $2d^2 = 9d + 3$

Find each product. (Lesson 7-6)

44. $(y + 5)(y + 7)$

45. $(c - 3)(c - 7)$

46. $(x + 4)(x - 8)$

GET READY for the Next Lesson

PREREQUISITE SKILL Find each product.

47. $3 \cdot 2 \cdot 1$

48. $11 \cdot 10 \cdot 9$

49. $6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

50. $8 \cdot 7 \cdot 6 \cdot 5$

51. $19 \cdot 18 \cdot 17$

52. $30 \cdot 29 \cdot 28 \cdot 27$

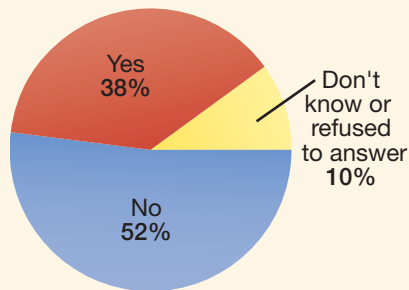
READING MATH

Survey Questions

Even though taking a random sample eliminates bias or favoritism in the choice of a sample, questions may be worded to influence people's thoughts in a desired direction. Two different surveys on Internet sales tax had different results.

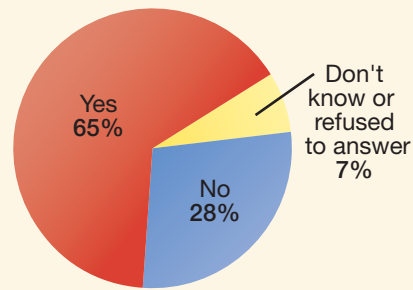
Question 1

Should there be sales tax on purchases made on the Internet?



Question 2

Do you think people should or should not be required to pay the same sales tax for purchases made over the Internet as those bought at a local store?



Notice the difference in Questions 1 and 2. Question 2 includes more information. Pointing out that customers pay sales tax for items bought at a local store may give the people answering the survey a reason to answer "yes." Asking the question in that way probably led people to answer the way they did.

Because they are random samples, the results of both of these surveys are accurate. However, the results could be used in a misleading way by someone with an interest in the issue. For example, an Internet retailer would prefer to state the results of Question 1. Be sure to think about survey questions carefully so the results can be interpreted correctly.

Reading to Learn

For Exercises 1–2, tell whether each question is likely to bias the results. Explain your reasoning.

- On a survey on environmental issues:
 - "Due to diminishing resources, should a law be made to require recycling?"
 - "Should the government require citizens to participate in recycling efforts?"
- On a survey on education:
 - "Should schools fund extracurricular sports programs?"
 - "The budget of the River Valley School District is short of funds. Should taxes be raised in order for the district to fund extracurricular sports programs?"
- Suppose you want to determine whether to serve hamburgers or pizza at the class party.
 - Write a survey question that would likely produce biased results.
 - Write a survey question that would likely produce unbiased results.

Main Ideas

- Count outcomes using a tree diagram.
- Count outcomes using the Fundamental Counting Principle.

New Vocabulary

tree diagram
 sample space
 event
 Fundamental Counting Principle
 factorial

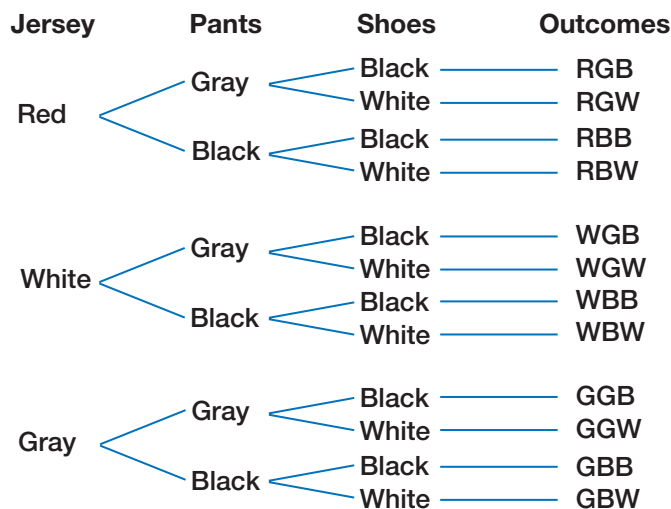
GET READY for the Lesson

The Atlantic Coast Conference (ACC) football championship is decided by the number of conference wins. If there is a tie, the team with more nonconference wins is champion. If Florida State plays 3 nonconference games, a tree diagram can be used to show the different records they could have for those games.

Tree Diagrams One method used for counting the number of possible outcomes is to draw a **tree diagram**. The last column of a tree diagram shows all of the possible outcomes. The list of all possible outcomes is called the **sample space**, while any collection of one or more outcomes in the sample space is called an **event**.

EXAMPLE Tree Diagram

- 1** A soccer team uses red jerseys for road games, white jerseys for home games, and gray jerseys for practice games. The team uses gray or black pants, and black or white shoes. Use a tree diagram to determine the number of possible uniforms.



The tree diagram shows that there are 12 possible uniforms.

CHECK Your Progress

- 1.** At the cafeteria, you have several options for a sandwich. You can choose either white (W) or wheat (E) bread. You can choose turkey (T), ham (H), or roast beef (R). You can choose mustard (M) or mayonnaise (A). Use a tree diagram to determine the number of possibilities for your sandwich.

The Fundamental Counting Principle The number of possible uniforms in Example 1 can also be found by multiplying the number of choices for each item. If the team can choose from 3 different colored jerseys, 2 different colored pants, and 2 different colored pairs of shoes, there are $3 \cdot 2 \cdot 2$, or 12, possible uniforms. This example illustrates the **Fundamental Counting Principle**.

Study Tip

Fundamental Counting Principle

This rule for counting outcomes can be extended to any number of events.

KEY CONCEPT

Fundamental Counting Principle

If an event M can occur in m ways and is followed by an event N that can occur in n ways, then the event M followed by event N can occur in $m \cdot n$ ways.

EXAMPLE Fundamental Counting Principle

- 2** The Uptown Deli offers a lunch special in which you can choose from 10 different sandwiches, 12 different side dishes, and 7 different beverages. How many different lunch specials can you order?

Multiply to find the number of lunch specials.

sandwich choices	.	side dish choices	.	beverage choices	=	number of specials
10		12		7		840

CHECK Your Progress

- 2.** When ordering a certain car, there are 7 colors for the exterior, 8 colors for the interior, and 4 choices of interior fabric. How many different possibilities are there for color and fabric when ordering this car?

EXAMPLE Counting Arrangements

- 3** Mackenzie is setting up a display of the ten most popular video games from the previous week. If she places the games side-by-side on a shelf, in how many different ways can she arrange them?

Multiply the number of choices for each position.

- Mackenzie has ten games from which to choose for the first position.
- After choosing a game for the first position, there are nine games left from which to choose for the second position.
- There are now eight choices for the third position.
- This process continues until all positions have been filled.

The number of arrangements is

$$n = 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \text{ or } 3,628,800.$$

There are 3,628,800 different ways to arrange the video games.

CHECK Your Progress

- 3.** Student Council has a president, vice-president, treasurer, secretary, and two representatives from each of the four grades. For the school assembly they were all required to sit in a row up on the stage. In how many different ways can they arrange themselves?

Study Tip

Technology

You can use a TI 83/84 Plus graphing calculator to find $10!$ by pushing 10 **MATH**, scroll to PRB, 4 **ENTER**.

The expression $n = 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$ used in Example 3 can be written as $10!$ using a **factorial**.

KEY CONCEPT

Fundamental Counting Principle

Words The expression $n!$, read n factorial, where n is greater than zero, is the product of all positive integers beginning with n and counting backward to 1.

Symbols $n! = n \cdot (n - 1) \cdot (n - 2) \cdot \dots \cdot 3 \cdot 2 \cdot 1$

Example $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$ or 120

By definition, $0! = 1$.

EXAMPLE Factorial

4 Find the value of each expression.

a. $6!$

$$\begin{aligned} 6! &= 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \\ &= 720 \end{aligned}$$

b. $10!$

$$\begin{aligned} 10! &= 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \\ &= 3,628,800 \end{aligned}$$

CHECK Your Progress

4A. $5!$

4B. $8!$



Real-World Link

In 2005, there were 658 roller coasters in the United States.

Type	Number
Wood	121
Steel	537
Inverted	45
Stand Up	9
Suspended	9
Bobsled	4

Source: Roller Coaster Database

EXAMPLE Use Factorials to Solve a Problem

5 **ROLLER COASTERS** Zach and Kurt are going to an amusement park. They cannot decide in which order to ride the 12 roller coasters in the park.

a. In how many different orders can they ride all of the roller coasters if they ride each once?

Use a factorial.

$$\begin{aligned} 12! &= 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 && \text{Definition of factorial} \\ &= 479,001,600 && \text{Simplify.} \end{aligned}$$

b. If they only have time to ride 8 of the roller coasters, how many ways can they do this?

Use the Fundamental Counting Principle to count the sample space.

$$\begin{aligned} s &= 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 && \text{Fundamental Counting Principle} \\ &= 19,958,400 && \text{Simplify.} \end{aligned}$$

CHECK Your Progress

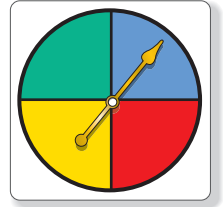
José needs to speak with six college representatives.

5A. In how many different orders can he speak to these people if he only speaks to each person once?

5B. He decides that he will not have time to talk to two of the people. In how many ways can he speak to the others?

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Examples 1–3 For Exercises 1–3, suppose the spinner at the right is spun three times.
(pp. 650–651)



Example 4
(p. 652)

Example 5
(p. 652)

1. Draw a tree diagram to show the sample space.
2. How many outcomes involve both green and blue?
3. How many outcomes are possible?
4. Find the value of $8!$.
5. **SCHOOL** In a science class, each student must choose a lab project from a list of 15, write a paper on one of 6 topics, and give a presentation about one of 8 subjects. How many ways can students choose to do their assignments?

Exercises

HOMEWORK HELP	
For Exercises	See Examples
6, 7, 16	1
8–10	2, 3
11–14	4
15, 17–20	5

Draw a tree diagram to show the sample space for each event. Determine the number of possible outcomes.

6. earning an A, B, or C in English, math, and science classes
7. buying a computer with a choice of a CD-ROM, a CD recorder, or a DVD drive, one of 2 monitors, and either a printer or a scanner

For Exercises 8–10, determine the possible number of outcomes.

8. Three dice, one red, one white, and one blue are rolled. How many outcomes are possible?
9. How many outfits are possible if you choose one each of 5 shirts, 3 pairs of pants, 3 pairs of shoes, and 4 jackets?
10. **TRAVEL** Suppose four different airlines fly from Seattle to Denver. Those same four airlines and two others fly from Denver to St. Louis. In how many ways can a traveler use these airlines to book a flight from Seattle to St. Louis?

Find the value of each expression.

11. $4!$ 12. $7!$ 13. $11!$ 14. $13!$

COMMUNICATIONS For Exercises 15 and 16, use the following information.

A new 3-digit area code is needed to accommodate new telephone numbers.

15. If the first digit must be odd, the second digit must be a 0 or a 1, and the third digit can be anything, how many area codes are possible?
16. Draw a tree diagram to show the different area codes using 4 or 5 for the first digit, 0 or 1 for the second digit, and 7, 8, or 9 for the third digit.

SOCCER For Exercises 17–19, use the following information.

The Columbus Crew is playing FC Dallas in a best three out of five championship soccer series.

17. What are the possible outcomes of the series?
18. How many outcomes require only four games be played to determine the champion?
19. How many ways can FC Dallas win the championship?

20. **GAMES** William has been dealt seven different cards in a game he is playing. How many different ways are there for him to play his cards if he is required to play one card at a time?

EXTRA PRACTICE
See page 742, 755.

Math online
Self-Check Quiz at algebra1.com

H.O.T. Problems

21. **OPEN ENDED** Give a real-world example of an event that has $7 \cdot 6$ or 42 outcomes.
22. **CHALLENGE** To get to and from school, Tucker can walk, ride his bike, or get a ride with a friend. Suppose that one week he walked 60% of the time, rode his bike 20% of the time, and rode with his friend 20% of the time, not necessarily in that order. How many outcomes represent this situation? Assume that he returns home the same way that he went to school.
23. **Writing in Math** Refer to the information on page 650 to explain how possible win/loss records can be determined in football. Demonstrate how to find the number of possible outcomes for a team's four home games.

STANDARDIZED TEST PRACTICE

24. A car manufacturer offers a sports car in 4 different models with 6 different option packages. Each model is available in 12 different colors. How many different possibilities are available for this car?
- A 48 B 54 C 76 D 288

25. **REVIEW** Ko collects soup can labels to raise money for his school. He receives \$4 for every 75 labels that he collects. If Ko wants to raise \$96, how many labels does he need?
- F 7200 G 1800 H 900 J 24

Spiral Review

PRINTING For Exercises 26–28, use the following information.

To determine the quality of calendars printed at a local shop, the last 10 calendars printed each day are examined. (Lesson 12-1)

26. Identify the sample.
27. Suggest a population from which it was selected.
28. State whether it is *unbiased* (random) or *biased*. If unbiased, classify the sample as *simple*, *stratified*, or *systematic*. If biased, classify as *convenience* or *voluntary response*.

Solve each equation. (Lesson 11-9)

29. $\frac{-4}{a+1} + \frac{3}{a} = 1$

30. $\frac{3}{x} + \frac{4x}{x-3} = 4$

31. $\frac{d+3}{d+5} + \frac{2}{d-9} = \frac{5}{2d+10}$

Find each sum. (Lesson 11-7)

32. $\frac{2x+1}{3x-1} + \frac{x+4}{x-2}$

33. $\frac{4n}{2n+6} + \frac{3}{n+3}$

GET READY for the Next Lesson

PREREQUISITE SKILL Colored marbles are placed in a bag and selected at random. There are 12 yellow, 15 red, 11 green, 16 blue, and 14 black marbles in the bag. Find each probability.

34. $P(\text{green})$ 35. $P(\text{black})$ 36. $P(\text{yellow or blue})$
37. $P(\text{green or red})$ 38. $P(\text{purple})$ 39. $P(\text{yellow or red or green})$

Main Ideas

- Determine probabilities using permutations.
- Determine probabilities using combinations.

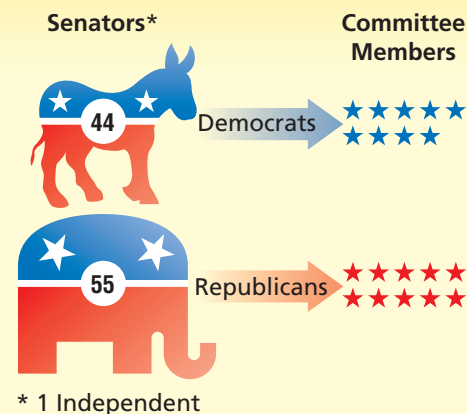
New Vocabulary

permutation
combination

GET READY for the Lesson

The United States Senate forms various committees by selecting senators from both political parties. The Senate Health, Education, Labor, and Pensions Committee of the 109th Congress was made up of 10 Republican senators and 9 Democratic senators. How many different ways could the committee have been selected? The members of the committee were selected in no particular order. This is an example of a combination.

Senate Health, Education, Labor, and Pensions Committee

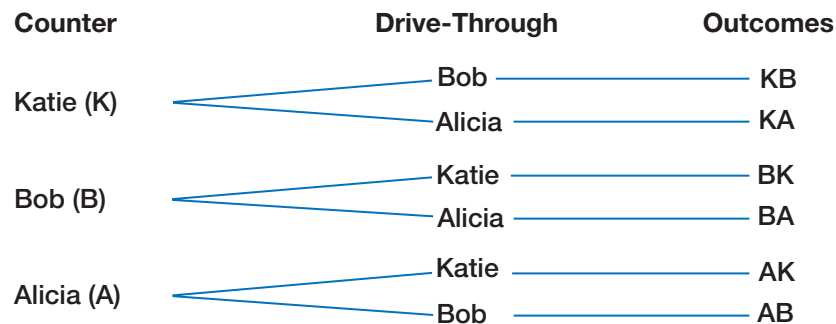


Permutations An arrangement or listing in which order or placement is important is called a **permutation**.

EXAMPLE Tree Diagram Permutation

- 1 EMPLOYMENT** The manager of a coffee shop needs to hire two employees, one to work at the counter and one to work at the drive-through window. Katie, Bob, and Alicia all applied for a job. How many possible ways can the manager place them?

Use a tree diagram to show the possible arrangements.



There are 6 different ways for the 3 applicants to hold the 2 positions.

CHECK Your Progress

- 1. MUSIC** At Rock City Music Store, customers can purchase CDs, cassettes, and downloads. They can choose from rock, jazz, hip-hop, and gospel. How many possible ways are there for a customer to buy music?

Study Tip

Common Misconception

When arranging two objects *A* and *B* using a permutation, the arrangement *AB* is different from the arrangement *BA*.

In Example 1, the positions are in a specific order, so each arrangement is unique. The symbol ${}_3P_2$ denotes the number of permutations when arranging 3 applicants in 2 positions. You can also use the Fundamental Counting Principle to determine the number of permutations.

$$\begin{aligned}
 {}_3P_2 &= \begin{array}{c} \text{ways to choose} \\ \text{first employee} \end{array} 3 \cdot \begin{array}{c} \text{ways to choose} \\ \text{second employee} \end{array} 2 \\
 &= 3 \cdot 2 \cdot \frac{1}{1} \quad \frac{1}{1} = 1 \\
 &= \frac{3 \cdot 2 \cdot 1}{2 \cdot 1} \quad \text{Multiply.} \\
 &= \frac{3!}{2!} \quad 3 \cdot 2 \cdot 1 = 3!, 2 \cdot 1 = 2!
 \end{aligned}$$

In general, ${}_nP_r$ is used to denote the number of permutations of n objects taken r at a time.

KEY CONCEPT

Permutation

Word The number of permutations of n objects taken r at a time is the quotient of $n!$ and $(n - r)!$.

Symbols ${}_nP_r = \frac{n!}{(n - r)!}$

EXAMPLE Permutation and Probability

2 A word processing program requires a user to enter a 7-digit registration code made up of the digits 1, 2, 4, 5, 6, 7, and 9. Each number has to be used, and no number can be used more than once.

a. How many different registration codes are possible?

Since the order of the numbers in the code is important, this situation is a permutation of 7 digits taken 7 at a time.

$${}_nP_r = \frac{n!}{(n - r)!} \quad \text{Definition of permutation}$$

$${}_7P_7 = \frac{7!}{(7 - 7)!} \quad n = 7, r = 7.$$

$${}_7P_7 = \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{1} \text{ or } 5040 \quad 5040 \text{ codes are possible.}$$

b. What is the probability that the first three digits of the code are even?

Use the Fundamental Counting Principle to determine the number of ways for the first three digits to be even.

- There are three even digits and four odd digits.
- The number of choices for the first three digits, if they are even, is $3 \cdot 2 \cdot 1$.
- The number of choices for the remaining odd digits is $4 \cdot 3 \cdot 2 \cdot 1$.
- The number of favorable outcomes is $3 \cdot 2 \cdot 1 \cdot 4 \cdot 3 \cdot 2 \cdot 1$ or 144.

Study Tip

Permutations

The number of permutations of n objects taken n at a time is $n!$. ${}_nP_n = n!$

$$P(\text{first 3 digits even}) = \frac{144}{5040} \begin{array}{l} \leftarrow \text{number of favorable outcomes} \\ \leftarrow \text{number of possible outcomes} \end{array}$$

$$= \frac{1}{35} \quad \text{Simplify.}$$

The probability that the first three digits are even is $\frac{1}{35}$ or about 3%.

CHECK Your Progress

BICYCLES A combination bike lock requires a three-digit code made up of the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. No number can be used more than once.

- 2A.** How many different combinations are possible?
2B. What is the probability that all of the digits are odd numbers?

Combinations An arrangement or listing in which order is not important is called a **combination**. For example, if you are choosing 2 salad ingredients from a list of 10, the order in which you choose the ingredients does not matter.

KEY CONCEPT

Combination

Words The number of combinations of n objects taken r at a time is the quotient of $n!$ and $(n - r)!r!$.

Symbols ${}_n C_r = \frac{n!}{(n - r)!r!}$

Study Tip

Common Misconception

Not all everyday uses of the word *combination* are descriptions of mathematical combinations. For example, the combination to a lock is described by a permutation.

EXAMPLE Combinations and Probability

3 SCHOOL A group of 7 seniors, 5 juniors, and 4 sophomores have volunteered to be peer tutors. Mr. DeLuca needs to choose 12 students out of the group.

- a.** How many ways can the 12 students be chosen?

The order in which the students are chosen does not matter, so we must find the number of combinations of 16 students taken 12 at a time.

$${}_n C_r = \frac{n!}{(n - r)!r!} \quad \text{Definition of combination}$$

$${}_{16} C_{12} = \frac{16!}{(16 - 12)!12!} \quad n = 16, r = 12$$

$$= \frac{16!}{4!12!} \quad 16 - 12 = 4$$

$$= \frac{16 \cdot 15 \cdot 14 \cdot 13 \cdot \overset{1}{12!}}{4! \cdot \underset{1}{12!}} \quad \text{Divided by the GCF, 12!}$$

$$= \frac{43,680}{24} \text{ or } 1820 \quad \text{Simplify.}$$

There are 1820 ways to choose 12 students out of 16.

(continued on the next page)



- b.** If the students are chosen randomly, what is the probability that 4 seniors, 4 juniors, and 4 sophomores will be selected?

To find the probability, there are three questions to consider.

- How many ways can 4 seniors be chosen from 7?
- How many ways can 4 juniors be chosen from 5?
- How many ways can 4 sophomores be chosen from 4?

Using the Fundamental Counting Principle, the answer can be determined with the product of the three combinations.

$$\begin{array}{ccc} \text{ways to choose} & \text{ways to choose} & \text{ways to choose} \\ \text{4 seniors} & \text{4 juniors} & \text{4 sophomores} \\ \text{out of 7} & \text{out of 5} & \text{out of 4} \\ {}_7C_4 & \cdot {}_5C_4 & \cdot {}_4C_4 \end{array}$$

$$\begin{aligned} {}_7C_4 {}_5C_4 {}_4C_4 &= \frac{7!}{(7-4)!4!} \cdot \frac{5!}{(5-4)!4!} \cdot \frac{4!}{(4-4)!4!} && \text{Definition of combination} \\ &= \frac{7!}{3!4!} \cdot \frac{5!}{1!4!} \cdot \frac{4!}{0!4!} && \text{Simplify.} \\ &= \frac{7 \cdot 6 \cdot 5}{3!} \cdot \frac{5}{1} && \text{Divide by the GCF, 4!} \\ &= 175 && \text{Simplify.} \end{aligned}$$

Finally, there are 175 ways to choose this particular combination out of 1820 possible combinations.

$$\begin{aligned} P(4 \text{ seniors, 4 juniors, 4 sophomores}) &= \frac{175}{1820} && \leftarrow \begin{array}{l} \text{number of favorable outcomes} \\ \text{number of possible outcomes} \end{array} \\ &= \frac{5}{52} && \text{Simplify.} \end{aligned}$$

The probability that Mr. DeLuca will randomly select 4 seniors, 4 juniors, and 4 sophomores is $\frac{5}{52}$ or about 10%.

Study Tip

Combinations

The number of combinations of n objects taken n at a time is 1. ${}_nC_n = 1$

CHECK Your Progress

PARADE A group of 7 Army veterans, 5 Air Force veterans, 6 Navy veterans, and 4 Marine veterans have volunteered to march in the Memorial Day Parade.

- 3A.** In how many ways can 12 veterans be chosen to march?
- 3B.** If the 12 veterans are chosen randomly, what is the probability that 3 veterans from each branch of the military will be selected?

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CHECK Your Understanding

Example 1, 3 Determine whether each situation involves a *permutation* or *combination*.
(pp. 655, 657) Explain your reasoning.

1. choosing 6 books from a selection of 12 for summer reading
2. choosing digits for a personal identification number

Example 2 Evaluate each expression.

(pp. 656–657)

3. ${}_8P_5$

4. $({}_{10}P_5)({}_3P_2)$

For Exercises 5–7, use the following information.

The digits 0 through 9 are written on index cards. Three of the cards are randomly selected to form a three-digit code.

5. Does this situation represent a permutation or a combination? Explain.
6. How many different codes are possible?
7. What is the probability that all three digits will be odd?

Example 3
(pp. 657–658)

8. A diner offers a choice of two side items from the list with each entrée. How many ways can two items be selected?



9. **PROBABILITY** 15 marbles out of 20 must be randomly selected. There are 7 red marbles, 8 purple marbles, and 5 green marbles from which to choose. What is the probability that 5 of each color is selected?

Evaluate each expression.

10. ${}_7C_5$

11. $({}_6C_2)({}_4C_3)$

Exercises

HOMEWORK	HELP
For Exercises	See Examples
12–19, 32, 36, 38, 43	1, 3
20–31, 33–35, 37, 39–40, 44–45	2, 3

Determine whether each situation involves a *permutation* or *combination*. Explain your reasoning.

12. team captains for the soccer team
13. three mannequins in a display window
14. a hand of 10 cards from a selection of 52
15. the batting order of the New York Yankees
16. first-place and runner-up winners for the table tennis tournament
17. a selection of 5 DVDs from a group of eight
18. selection of 2 candy bars from six equally-sized bars
19. the selection of 2 trombones, 3 clarinets, and 2 trumpets for a jazz combo

Evaluate each expression.

20. ${}_{12}P_3$

21. ${}_4P_1$

22. ${}_6C_6$

23. ${}_7C_3$

24. ${}_{15}C_3$

25. ${}_{20}C_8$

26. ${}_{15}P_3$

27. ${}_{16}P_5$

28. $({}_7P_7)({}_7P_1)$

29. $({}_{20}P_2)({}_{16}P_4)$

30. $({}_3C_2)({}_7C_4)$

31. $({}_8C_5)({}_5P_5)$

SCHOOL For Exercises 32–35, use the following information.

Mrs. Moyer’s class has to choose 4 out of 12 people for an activity committee.

32. Does the selection involve a permutation or a combination? Explain.
33. How many different groups of students could be selected?
34. Suppose the students are selected for the positions of chairperson, activities planner, activity leader, and treasurer. How many different groups of students could be selected?
35. What is the probability that any one of the students is chosen to be the chairperson?



SOFTBALL For Exercises 36 and 37, use the following information.

The manager of a softball team needs to prepare a batting lineup using her nine starting players.

- 36. Is this situation a permutation or a combination?
- 37. How many different lineups can she make?

GAMES For Exercises 38–40, use the following information.

For a certain game, each player rolls five dice at the same time.

- 38. Do the outcomes of rolling the five dice represent a permutation or a combination? Explain.
- 39. How many outcomes are possible?
- 40. What is the probability that all five dice show the same number on a single roll?

Real-World Link

The game of softball was developed in 1888 as an indoor sport for practicing baseball during the winter months.

Source: encyclopedia.com




BUSINESS For Exercises 41 and 42, use the following information.

There are six positions available in the research department of a software company. Of the applicants, 15 are men and 10 are women.

- 41. In how many ways could 4 men and 2 women be chosen if each were equally qualified?
- 42. What is the probability that five women would be selected if the positions were randomly filled?

DINING For Exercises 43–45, use the following information.

For lunch in the school cafeteria, you can select one item from each category to get the daily combo.

Entree	Side Dish	Beverage
Burger Deli Sandwich Taco Pizza	Soup Salad French Fries	Lemonade Iced Tea Soft Drink
		

- 43. Find the number of possible meal combinations.
- 44. If a side dish is chosen at random, what is the probability that a student will choose soup?
- 45. What is the probability that a student will randomly choose a sandwich and soup?

SWIMMING For Exercises 46–48, use the following information.

A swimming coach plans to pick four swimmers out of a group of 6 to form the 400-meter freestyle relay team.

- 46. How many different teams can he form?
- 47. The swimmers have been chosen for the relay team. The coach must now decide in which order the four swimmers should swim. He timed the swimmers in each possible order and chose the best time. How many relays did the four swimmers have to swim so that the coach could collect all the data necessary?
- 48. If Tomás is chosen to be on the team, what is the probability that he will swim in the third leg?

Cross-Curricular Project

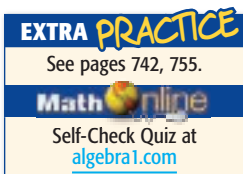
Math online You can use permutations and combinations to analyze data on U.S. amusement parks. Visit algebra1.com

49. **BASKETBALL** The coach had to select 5 out of 12 players on his basketball team to start the game. How many different groups of players could be selected to start the game?

SPORTS For Exercises 50 and 51, use the following information.

Central High School is competing against West High School at a track meet. Each team entered four girls to run the 1600-meter event. The top three finishers are awarded medals.

50. If there are only the runners from Central and West in this race, how many different ways can the runners place first, second, and third?
51. If all eight runners have an equal chance of placing, what is the probability that the first and second place finishers are from West and the third place finisher is from Central?



H.O.T. Problems

52. **OPEN ENDED** Describe the difference between a permutation and a combination. Then give an example of each.
53. **Which One Doesn't Belong?** Determine which situation does not belong. Explain your reasoning.

The five starters on a basketball team.

Choosing 10 colored marbles out of a bag.

Choosing 4 horses from 6 to run in the race.

Determining class rank in a senior class of 100 students.

54. **FIND THE ERROR** Eric and Alisa are taking a trip to Washington, D.C., to visit the Lincoln Memorial, the Jefferson Memorial, the Washington Monument, the White House, the Capitol Building, the Supreme Court, and the Pentagon. Both are finding the number of ways they can choose to visit 5 of these 7 sites. Who is correct? Explain your reasoning.

$$\begin{aligned} \text{Eric} \\ {}_7C_5 &= \frac{7!}{2!} \\ &= 2520 \end{aligned}$$

$$\begin{aligned} \text{Alisa} \\ {}_7C_5 &= \frac{7!}{2!5!} \\ &= 21 \end{aligned}$$

CHALLENGE For Exercises 55 and 56, use the following information.

Larisa is trying to solve a word puzzle. She needs to arrange the letters H, P, S, T, A, E, O into a two-word arrangement.

55. How many different arrangements of the letters can she make?
56. If each arrangement has an equal chance of occurring, what is the probability that she will form the words "tap shoe" on her first try?
57. *Writing in Math* Refer to the information on page 655 to explain how combinations can be used to form Senate committees. Discuss why the formation of a Senate committee is a combination and not a permutation. Explain how this would change if the selection of the committee was based on seniority.

58. Julie remembered that the 4 digits of her locker combination were 4, 9, 15, and 22, but not their order. What is the maximum number of attempts Julie could have to make before her locker opens?

- A 4
- B 16
- C 24
- D 256

59. **REVIEW** Jimmy has \$23 in a jar at home, and he is saving to buy a \$175 video game system. If he can save \$15 a week, which equation could be used to determine w , the number of weeks it will take Jimmy to buy the video game system?

- F $23 = 175 + 15w$
- G $23 = 15(w + 175)$
- H $175 = 15w + 23$
- J $175 = 23w + 15$

Spiral Review

60. The Sanchez family acts as a host family for a foreign exchange student during each school year. It is equally likely that they will host a girl or a boy. In how many different ways can they host boys and girls over the next four years? (Lesson 12-2)

61. **MANUFACTURING** Every 15 minutes, a CD player is taken off the assembly line and tested. State whether this sample is *unbiased* (random) or *biased*. If unbiased, classify the sample as *simple*, *stratified*, or *systematic*. If biased, classify as *convenience* or *voluntary response*. (Lesson 12-1)

Simplify each expression. (Lesson 11-2)

62. $\frac{x + 3}{x^2 + 6x + 9}$

63. $\frac{x^2 - 49}{x^2 - 2x - 35}$

64. $\frac{n^2 - n - 20}{n^2 + 9n + 20}$

Find the distance between each pair of points with the given coordinates. Express answers in simplest radical form and as decimal approximations rounded to the nearest hundredth if necessary. (Lesson 10-5)

65. (12, 20), (16, 34)

66. (-18, 7), (2, 15)

67. (-2, 5), $(-\frac{1}{2}, 3)$

Solve each equation by using the Quadratic Formula. Approximate irrational roots to the nearest hundredth. (Lesson 9-4)

68. $m^2 + 4m + 2 = 0$

69. $2s^2 + s - 15 = 0$

70. $2n^2 - n = 4$

GET READY for the Next Lesson

PREREQUISITE SKILL Find each sum or difference. (Pages 694-695)

71. $\frac{8}{52} + \frac{4}{52}$

72. $\frac{7}{32} + \frac{5}{8}$

73. $\frac{5}{15} + \frac{6}{15} - \frac{2}{15}$

74. $\frac{15}{24} + \frac{11}{24} - \frac{3}{4}$

75. $\frac{2}{3} + \frac{15}{36} - \frac{1}{4}$

76. $\frac{16}{25} + \frac{3}{10} - \frac{1}{4}$

Main Ideas

- Find the probability of two independent events or dependent events.
- Find the probability of two mutually exclusive or inclusive events.

New Vocabulary

simple event
 compound event
 independent events
 dependent events
 complements
 mutually exclusive
 inclusive

GET READY for the Lesson

The weather forecast for Saturday calls for rain in Chicago and Los Angeles. By using the probabilities for both cities, we can find other probabilities. What is the probability that it will rain in both cities? only in Chicago? Chicago or Los Angeles?

**Saturday Forecast:
Rain Likely**

Chicago
40%



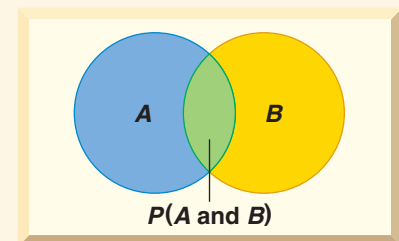
Los Angeles
80%

Independent and Dependent Events A single event, like rain in Los Angeles, is called a **simple event**. Suppose you wanted to determine the probability that it will rain in both Chicago and Los Angeles. This is an example of a **compound event**, which is made up of two or more simple events. The weather in Chicago does not affect the weather in Los Angeles. These two events are called **independent events** because the outcome of one event does not affect the outcome of the other.

KEY CONCEPT**Probability of Independent Events**

Words If two events, A and B , are independent, then the probability of both events occurring is the product of the probability of A and the probability of B .

Symbols $P(A \text{ and } B) = P(A) \cdot P(B)$

Model**EXAMPLE Independent Events**

- 1** Refer to the application above. Find the probability that it will rain in Chicago and Los Angeles.

$$\begin{aligned}
 &P(\text{Chicago and Los Angeles}) \\
 &= P(\text{Chicago}) \cdot P(\text{Los Angeles}) \quad \text{Probability of independent events} \\
 &= 0.4 \quad \cdot \quad 0.8 \quad \quad \quad 40\% = 0.4 \text{ and } 80\% = 0.8 \\
 &= 0.32 \quad \quad \quad \text{Multiply.}
 \end{aligned}$$

The probability that it will rain in Chicago and Los Angeles is 32%.

CHECK Your Progress

- 1.** Find the probability of rain in Chicago and no rain in Los Angeles.

When the outcome of one event affects the outcome of another event, the events are **dependent events**. For example, drawing a marble from a bag, not returning it, then drawing a second marble are dependent events because the drawing of the second marble is dependent on the drawing of the first marble.

Study Tip

More Than Two Dependent Events

Notice that the formula for the probability of dependent events can be applied to more than two events.

KEY CONCEPT

Probability of Dependent Events

Words If two events, A and B , are dependent, then the probability of both events occurring is the product of the probability of A and the probability of B after A occurs.

Symbols $P(A \text{ and } B) = P(A) \cdot P(B \text{ following } A)$

EXAMPLE Dependent Events

2 A bag contains 8 red marbles, 12 blue marbles, 9 yellow marbles, and 11 green marbles. Three marbles are randomly drawn from the bag one at a time and not replaced. Find each probability if the marbles are drawn in the order indicated.

a. $P(\text{red, blue, green})$

The selection of the first marble affects the selection of the next marble since there is one less marble from which to choose. So, the events are dependent.

$$\text{First marble: } P(\text{red}) = \frac{8}{40} \text{ or } \frac{1}{5} \quad \leftarrow \begin{array}{l} \text{number of red marbles} \\ \text{total number of marbles} \end{array}$$

$$\text{Second marble: } P(\text{blue}) = \frac{12}{39} \text{ or } \frac{4}{13} \quad \leftarrow \begin{array}{l} \text{number of blue marbles} \\ \text{number of marbles remaining} \end{array}$$

$$\text{Third marble: } P(\text{green}) = \frac{11}{38} \quad \leftarrow \begin{array}{l} \text{number of green marbles} \\ \text{number of marbles remaining} \end{array}$$

$$\begin{aligned} P(\text{red, blue, green}) &= P(\text{red}) \cdot P(\text{blue}) \cdot P(\text{green}) \\ &= \frac{1}{5} \cdot \frac{4}{13} \cdot \frac{11}{38} && \text{Substitution} \\ &= \frac{44}{2470} \text{ or } \frac{22}{1235} && \text{Multiply.} \end{aligned}$$

b. $P(\text{yellow, yellow, not green})$

Notice that after selecting a yellow marble, not only is there one fewer marble from which to choose, there is also one fewer yellow marble. Also, since the marble that is not green is selected after the first two marbles, there are $29 - 2$ or 27 marbles that are not green.

$$\begin{aligned} P(\text{yellow, yellow, not green}) &= P(\text{yellow}) \cdot P(\text{yellow}) \cdot P(\text{not green}) \\ &= \frac{9}{40} \cdot \frac{8}{39} \cdot \frac{27}{38} \\ &= \frac{1944}{59,280} \text{ or } \frac{81}{2470} \end{aligned}$$

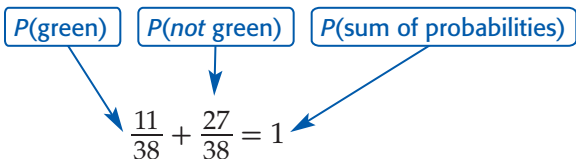
CHECK Your Progress

2A. $P(\text{red, green, green})$

2B. $P(\text{red, blue, not yellow})$

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In part b of Example 2, the events for drawing a marble that is green and for drawing a marble that is *not* green are called **complements**. Consider the probabilities for drawing the third marble.



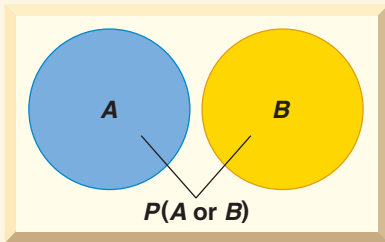
This is always true for any two complementary events.

Vocabulary Link

Complement
Everyday Use Two people are said to complement each other if their strengths and weaknesses balance each other.

Math Use A complement is one of two parts that make up a whole.

Mutually Exclusive and Inclusive Events Events that cannot occur at the same time are called **mutually exclusive**. Suppose you want to find the probability of rolling a 2 *or* a 4 on a die. Since a die cannot show both a 2 and a 4 at the same time, the events are mutually exclusive.

KEY CONCEPT		Mutually Exclusive Events
<p>Words If two events, A and B, are mutually exclusive, then the probability that either A or B occurs is the sum of their probabilities.</p> <p>Symbols $P(A \text{ or } B) = P(A) + P(B)$</p>	<p>Model</p> 	

EXAMPLE **Mutually Exclusive Events**

3 During a magic trick, a magician randomly draws one card from a standard deck of cards. What is the probability that the card drawn is a heart or a diamond?

Since a card cannot be both a heart and a diamond, the events are mutually exclusive.

$$P(\text{heart}) = \frac{13}{52} \text{ or } \frac{1}{4} \quad \begin{array}{l} \leftarrow \frac{\text{number of hearts}}{\text{total number of cards}} \end{array}$$

$$P(\text{diamond}) = \frac{13}{52} \text{ or } \frac{1}{4} \quad \begin{array}{l} \leftarrow \frac{\text{number of diamonds}}{\text{total number of cards}} \end{array}$$

$$P(\text{heart or diamond}) = P(\text{heart}) + P(\text{diamond}) \quad \text{Definition of mutually exclusive events}$$

$$= \frac{1}{4} + \frac{1}{4} \quad \text{Substitution}$$

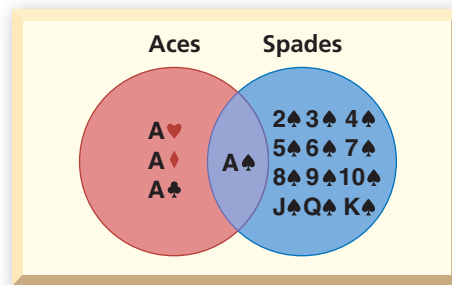
$$= \frac{2}{4} \text{ or } \frac{1}{2} \quad \text{Add.}$$

The probability of drawing a heart or a diamond is $\frac{1}{2}$.

CHECK Your Progress

3. What is the probability that the card drawn is an ace or a face card?

Suppose you want to find the probability of randomly selecting an ace or a spade from a standard deck of cards. Since it is possible to draw a card that is both an ace and a spade, these events are not mutually exclusive. They are called **inclusive** events. The following formula allows you to find the probability of inclusive events.



Study Tip

Inclusive Events

If the formula for mutually exclusive events was used to find the probability for inclusive events, events in the intersection would be counted twice.

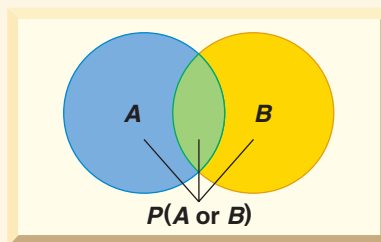
KEY CONCEPT

Mutually Exclusive Events

Words If two events, A and B , are inclusive, then the probability that either A or B occurs is the sum of their probabilities decreased by the probability of both occurring.

Symbols $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

Model



STANDARDIZED TEST EXAMPLE

- 4 GAMES** In the game of bingo, balls or tiles are numbered 1 through 75. These numbers correspond to columns on a bingo card, as shown in the table. A number is selected at random. What is the probability that it is a multiple of 4 or is in the O column?

B	I	N	G	O
1-15	16-30	31-45	46-60	61-75

- A $\frac{1}{5}$ B $\frac{2}{5}$ C $\frac{1}{2}$ D $\frac{4}{5}$

Since the numbers 64, 68, and 72 are multiples of 4 and they can be in the O column, these events are inclusive.

$P(\text{multiple of 4 or O column})$

$$\begin{aligned}
 & P(\text{multiple of 4}) + P(\text{O column}) - P(\text{multiple of 4 and O column}) \\
 = & \frac{18}{75} + \frac{15}{75} - \frac{3}{75} \quad \text{Substitution} \\
 = & \frac{18 + 15 - 3}{75} \quad \text{LCD is 75.} \\
 = & \frac{30}{75} \text{ or } \frac{2}{5} \quad \text{Simplify.}
 \end{aligned}$$

The correct choice is B.

CHECK Your Progress

4. Refer to the table above. What is the probability that a number selected is even or is in the N column?

- F $\frac{1}{5}$ G $\frac{2}{5}$ H $\frac{1}{2}$ J $\frac{4}{5}$

CHECK Your Understanding

Example 1 BUSINESS For Exercises 1–3, use the following information.

(p. 663)

Mr. Salyer is a buyer for an electronics store. He received a shipment of 5 hair dryers in which one is defective. He randomly chose 3 of the hair dryers to test.

1. Determine whether choosing the hair dryers are independent or dependent events.
2. What is the probability that he selected the defective dryer?
3. Suppose the defective dryer is one of the three that Mr. Salyer tested. What is the probability that the last one tested was the defective one?

Examples 1, 2
(pp. 663–664)

A bin contains colored chips as shown in the table. Find each probability.

Color	Number
Blue	8
Red	5
Green	6
Yellow	2

4. drawing a red chip, replacing it, then drawing a green chip
5. choosing green, then blue, then red, replacing each chip after it is drawn
6. selecting two yellow chips without replacement
7. choosing green, then blue, then red without replacing each chip

Examples 3, 4
(pp. 665–666)

A student is selected at random from a group of 12 male and 12 female students. There are 3 male students and 3 female students from each of the 9th, 10th, 11th, and 12th grades. Find each probability.

8. $P(9\text{th or }12\text{th grader})$
9. $P(\text{male or female})$
10. $P(10\text{th grader or female})$
11. $P(\text{male or not }11\text{th grader})$

Example 4
(p. 666)

12. STANDARDIZED TEST PRACTICE At the basketball game, 50% of the fans cheered for the home team. In the same crowd, 20% of the fans were waving banners. What is the probability that a fan cheered for the home team and waved a banner?

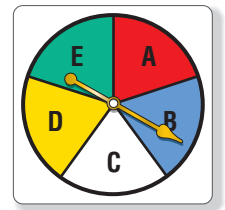
- A $\frac{1}{20}$
B $\frac{1}{10}$
C $\frac{1}{5}$
D $\frac{2}{5}$

Exercises

HOMEWORK HELP	
For Exercises	See Examples
13–19, 31–32	1
20–27	2
28–30	3
33–34	4

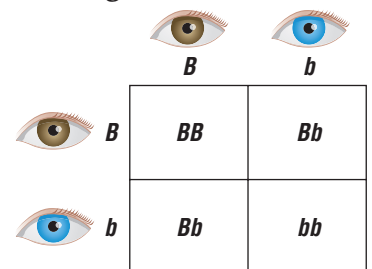
A die is rolled and a spinner like the one at the right is spun. Find each probability.

13. $P(3 \text{ and } D)$
14. $P(\text{an odd number and a vowel})$
15. $P(\text{a prime number and } A)$
16. $P(2 \text{ and } A, B, \text{ or } C)$



BIOLOGY For Exercises 17–19, use the diagram and following information.

Each person carries two types of genes for eye color. The gene for brown eyes (B) is dominant over the gene for blue eyes (b). That is, if a person has one gene for brown eyes and the other for blue, that person will have brown eyes. The Punnett square at the right shows the genes for two parents.



17. What is the probability that any child will have blue eyes?
18. What is the probability that the couple's two children both have brown eyes?
19. Find the probability that the first or the second child has blue eyes.

SAFETY For Exercises 20–23, use the following information.

A carbon monoxide detector system uses two sensors, A and B. If carbon monoxide is present, there is a 96% chance that sensor A will detect it, a 92% chance that sensor B will detect it, and a 90% chance that both sensors will detect it.

20. Draw a Venn diagram that illustrates this situation.
21. If carbon monoxide is present, what is the probability that it will be detected?
22. What is the probability that carbon monoxide would go undetected?
23. Do sensors A and B operate independently of each other? Explain.

A bag contains 2 red, 6 blue, 7 yellow, and 3 orange marbles. Once a marble is selected, it is not replaced. Find each probability.

24. $P(2 \text{ orange})$
25. $P(\text{blue, then red})$
26. $P(2 \text{ yellows in a row then orange})$
27. $P(\text{blue, then yellow, then red})$

ECONOMICS For Exercises 28–30, use the table below that compares the total number of hourly workers who earned the minimum wage of \$5.15 with those making less than minimum wage.

Number of Hourly Workers (thousands), 2005			
Age (years)	Total	At \$5.15	Below \$5.15
16–24	16,174	272	750
25+	57,765	249	733

Source: U.S. Bureau of Labor Statistics

28. If an hourly worker was chosen at random, what is the probability that he or she earned minimum wage? less than minimum wage?
29. What is the probability that a randomly-chosen hourly worker earned less than or equal to minimum wage?
30. If you randomly chose an hourly worker from each age group, which would you expect to have earned no more than minimum wage? Explain.

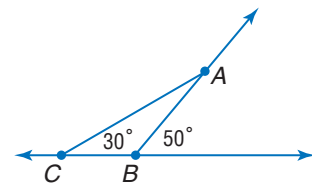
Raffle tickets numbered 1 through 30 are placed in a box. Tickets for a second raffle numbered 21 to 48 are placed in another box. One ticket is randomly drawn from each box. Find each probability.

31. Both tickets are even.
32. Both tickets are greater than 20 and less than 30.
33. The first ticket is greater than 10, and the second ticket is less than 40 or odd.
34. The first ticket is greater than 12 or prime, and the second ticket is a multiple of 6 or a multiple of 4.

GEOMETRY For Exercises 35–37, use the figure and the following information.

Two of the six angles in the figure are chosen at random.

35. What is the probability of choosing an angle inside $\angle ABC$ or an obtuse angle?
36. What is the probability of selecting a straight angle or a right angle?
37. Find the probability of picking a 20° angle or a 130° angle.



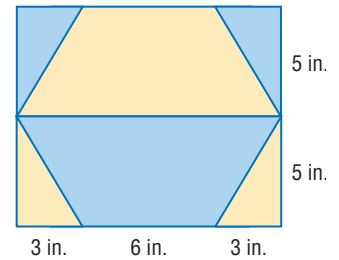
Real-World Link

The first federal minimum wage was set in 1938 at \$0.25 per hour. That was the equivalent of \$3.22 in 2005.

Source: U.S. Department of Labor

- 38. RESEARCH** Use the Internet or other reference to investigate various blood types. Use this information to determine the probability of a child having blood type O if the father has blood type A(Ai) and the mother has blood type B(Bi).

A dart is thrown at a dartboard like the one at the right. If the dart can land anywhere on the board, find the probability that it lands in each of the following.



EXTRA PRACTICE
See page 743, 755.
Math online
Self-Check Quiz at algebra1.com

- 39.** a triangle or a red region
40. a trapezoid or a blue region
41. a blue triangle or a red triangle
42. a square or a hexagon

H.O.T. Problems

- 43. OPEN ENDED** Explain how dependent events are different from independent events. Give specific examples in your explanation.
- 44. FIND THE ERROR** On the school debate team, 6 of the 14 girls are seniors, and 9 of the 20 boys are seniors. Chloe and Amber are both seniors on the team. Each girl calculated the probability that either a girl or a senior would randomly be selected to argue a position at a state debate. Who is correct? Explain your reasoning.

Chloe

$$\begin{aligned}
 P(\text{girl or senior}) &= \frac{14}{34} + \frac{15}{34} - \frac{6}{34} \\
 &= \frac{23}{34}
 \end{aligned}$$

Amber

$$\begin{aligned}
 P(\text{girl or senior}) &= \frac{6}{34} + \frac{15}{34} - \frac{14}{34} \\
 &= \frac{7}{34}
 \end{aligned}$$

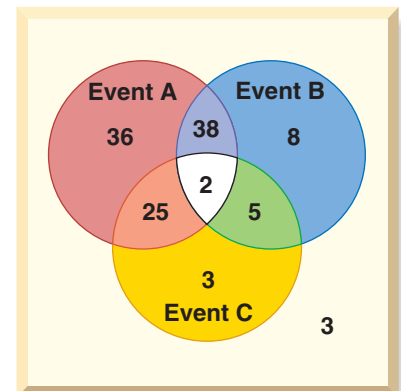
- 45. REASONING** Find a counterexample for the following statement.
If two events are independent, then the probability of both events occurring is less than 1.

CHALLENGE For Exercises 46–49, use the following information.

A sample of high school students were asked if they

- A) drive a car to school,
B) are involved in after-school activities, or
C) have a part-time job.

The results are shown in the Venn diagram.



- 46.** How many students were surveyed?
47. How many students said that they drive a car to school?
48. If a student is chosen at random, what is the probability that he or she does all three?
49. What is the probability that a randomly chosen student drives a car to school or is involved in after-school activities or has a part-time job?
- 50. Writing in Math** Refer to the information on page 663 to explain how probabilities are used by meteorologists. Illustrate how compound probabilities can be used to predict the weather.

STANDARDIZED TEST PRACTICE

51. A bag contains 8 red marbles, 5 blue marbles, 4 green marbles, and 7 yellow marbles. Five marbles are randomly drawn from the bag one at a time and not replaced. What is the probability that the first three marbles drawn are red?

- A $\frac{1}{27}$ C $\frac{7}{253}$
 B $\frac{28}{1771}$ D $\frac{7}{288}$

52. **REVIEW** The sum of a number x and -12 is 64. Which equation shows this relationship?

- F $x - 12 = 64$
 G $x + 12 = 64$
 H $12x = 64$
 J $64x = -12$

Spiral Review

CIVICS For Exercises 53 and 54, use the following information.

Stratford City Council wants to form a 3-person parks committee. Five people have applied to be on the committee. (Lesson 12-2)

53. How many committees are possible?
 54. What is the probability of any one person being selected if each has an equal chance?

55. **BUSINESS** A real estate developer built a strip mall with seven different-sized stores. Ten businesses have shown interest in renting space in the mall. The developer must decide which business would be best suited for each store. How many different arrangements are possible? (Lesson 12-1)

Find each quotient. Assume that no denominator has a value of 0. (Lesson 11-4)

56. $\frac{s}{s+7} \div \frac{s-5}{s+7}$

57. $\frac{2m^2 + 7m - 15}{m+2} \div \frac{2m-3}{m^2 + 5m + 6}$

Simplify. (Lesson 10-1)

58. $\sqrt{45}$

59. $\sqrt{128}$

60. $\sqrt{40b^4}$

61. $\sqrt{120a^3b}$

62. $3\sqrt{7} \cdot 6\sqrt{2}$

63. $\sqrt{3}(\sqrt{3} + \sqrt{6})$

GET READY for the Next Lesson

PREREQUISITE SKILL Express each fraction as a decimal. Round to the nearest thousandth. (pp. 700-701)

64. $\frac{9}{24}$

65. $\frac{2}{15}$

66. $\frac{63}{128}$

67. $\frac{5}{52}$

68. $\frac{8}{36}$

69. $\frac{11}{38}$

70. $\frac{81}{2470}$

71. $\frac{18}{1235}$

Identify each sample, suggest a population from which it was selected, and state whether it is *unbiased* (random) or *biased*. If unbiased, classify the sample as *simple*, *stratified*, or *systematic*. If biased, classify as *convenience* or *voluntary response*. (Lesson 12-1)

- Every other household in a neighborhood is surveyed to determine how to improve the neighborhood park.
- Every other household in a neighborhood is surveyed to determine the favorite candidate for the state's governor.

Find the number of outcomes for each event. (Lesson 12-2)

- A die is rolled, and two coins are tossed.
- A certain model of mountain bike comes in 5 sizes, 4 colors, with regular or off-road tires, and with a choice of 1 of 5 accessories.
- MULTIPLE CHOICE** There are seven teams in a league, but only four teams qualify for the post-season tournament. How many ways can the four spaces on the tournament bracket be filled by the teams in the league? (Lesson 12-2)

- A 210
B 420
C 840
D 5040

Find each value. (Lesson 12-3)

- ${}_{13}C_8$
- ${}_9P_6$
- $({}_5C_2)({}_7C_4)$
- $({}_{10}P_5)({}_{13}P_8)$

- SCHOOL** The students in Ms. Kish's homeroom had to choose 4 out of the 7 people who were nominated to serve on the Student Council. How many different groups of students could be selected?
- FLOWERS** A vase holds 5 carnations, 6 roses, and 3 lilies. Eliza picks four at random to give to her grandmother. What is the probability of selecting two roses and two lilies? (Lesson 12-3)

- MULTIPLE CHOICE** In a standard 52-card deck, what is the probability of randomly drawing an ace or a club? (Lesson 12-4)

F $\frac{1}{13}$

G $\frac{4}{13}$

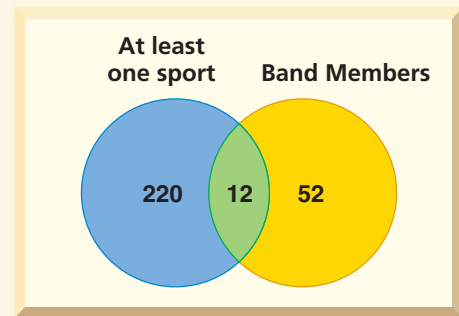
H $\frac{1}{4}$

J $\frac{17}{52}$

A ten-sided die, numbered 1 through 10, is rolled. Find each probability. (Lesson 12-4)

- $P(\text{odd or greater than 4})$
- $P(\text{less than 3 or greater than 7})$

- ACTIVITIES** There are 650 students in a high school. The Venn diagram shows the number of students involved in the band and at least one sport. What is the probability a student is randomly selected who participates in one of these extracurricular activities? (Lesson 12-4)



- MULTIPLE CHOICE** A teacher took a survey of his class of 28 students about their favorite foods. Thirteen students chose pizza, 7 chose ice cream, 5 chose steak, and 3 chose chicken. If the teacher randomly selects one person's favorite food for a party, what is the probability that he chooses ice cream or steak? (Lesson 12-4)

A $\frac{2}{7}$

C $\frac{4}{7}$

B $\frac{3}{7}$

D $\frac{5}{7}$

Main Ideas

- Use random variables to compute probability.
- Use probability distributions to solve real-world problems.

New Vocabulary

random variable
discrete random variable
probability distribution

GET READY for the Lesson

The owner of a pet store asked customers how many pets they owned. The results of this survey are shown in the table.

Number of Pets	Number of Customers
0	3
1	37
2	33
3	18
4	9



Random Variables and Probability A **random variable** is a variable with a value that is the numerical outcome of a random event. A **discrete random variable** has a finite number of possible outcomes. In the situation above, we can let the random variable X represent the number of pets owned. Thus, X can equal 0, 1, 2, 3, or 4.

EXAMPLE Random Variable

1 Refer to the application above.

- a.** Find the probability that a randomly chosen customer has 2 pets.

There is only one outcome in which there are 2 pets owned, and there are 100 survey results.

$$P(X = 2) = \frac{\text{2 pets owned}}{\text{customers surveyed}} \\ = \frac{33}{100}$$

The probability is $\frac{33}{100}$ or 33%.

- b.** Find the probability that a randomly chosen customer has at least 3 pets.

There are $18 + 9$ or 27 customers who own at least 3 pets.

$$P(X \geq 3) = \frac{27}{100}$$

The probability is $\frac{27}{100}$ or 27%.

Reading Math

Notation The notation $P(X = 2)$ means the same as $P(2 \text{ pets})$, the probability of a customer having 2 pets.

CHECK Your Progress

GRADES On an algebra test, there are 7 students with As, 9 students with Bs, 11 students with Cs, 3 students with Ds, and 2 students with Fs.

- 1A.** Find the probability that a randomly chosen student has a C.
1B. Find the probability that a randomly chosen student has at least a B.

Probability Distributions The probability of every possible value of the random variable X is called a **probability distribution**.

KEY CONCEPT

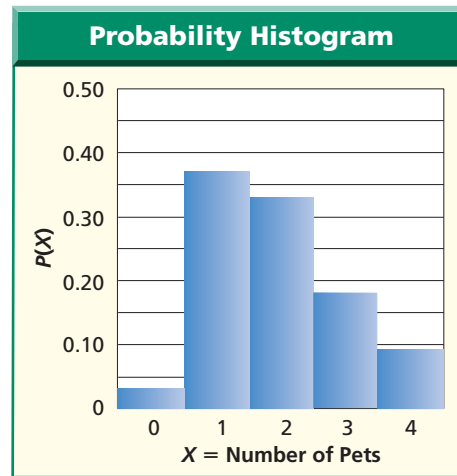
Properties of Probability Distributions

1. The probability of each value of X is greater than or equal to 0 and less than or equal to 1.
2. The probabilities of all of the values of X add up to 1.

The probability distribution for a random variable can be given in a table or in a **probability histogram**. The probability distribution and a probability histogram for the application at the beginning of the lesson are shown below.



Probability Distribution Table	
$X =$ Number of Pets	$P(X)$
0	0.03
1	0.37
2	0.33
3	0.18
4	0.09



EXAMPLE Probability Distribution

2 CARS The table shows the probability distribution of the number of vehicles per household for the Columbus, Ohio, area.

Vehicles per Household Columbus, OH	
$X =$ Number of Vehicles	Probability
0	0.10
1	0.42
2	0.36
3+	0.12

Source: U.S. Census Bureau

a. Show that the distribution is valid.

Check to see that each property holds.

1. For each value of X , the probability is greater than or equal to 0 and less than or equal to 1.
2. $0.10 + 0.42 + 0.36 + 0.12 = 1$, so the probabilities add up to 1.

b. What is the probability that a household has fewer than 2 vehicles?

Recall that the probability of a compound event is the sum of the probabilities of each individual event.

The probability of a household having fewer than 2 vehicles is the sum of the probability of 0 vehicles and the probability of 1 vehicle.

$$P(X < 2) = P(X = 0) + P(X = 1) \quad \text{Sum of individual probabilities}$$

$$= 0.10 + 0.42 \text{ or } 0.52 \quad P(X = 0) = 0.10, P(X = 1) = 0.42$$

(continued on the next page)

Real-World Link

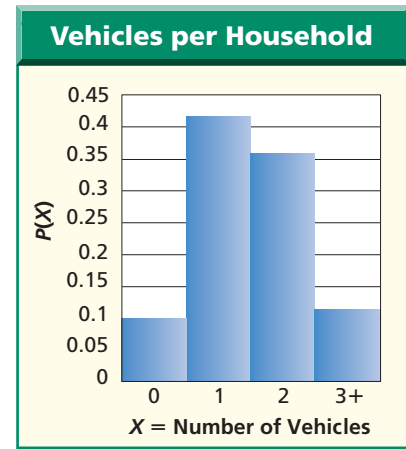
In 1900, there were 8000 registered cars in the United States. By 2002, there were over 136 million. This is an increase of more than 1,699,900.

Source: *The World Almanac*



c. Make a probability histogram of the data.

Draw and label the vertical and horizontal axes. Remember to use equal intervals on each axis. Include a title.



CHECK Your Progress

The table shows the probability distribution of adults who have played golf by age range.

- 2A. Show that the distribution is valid.
- 2B. What is the probability that an adult golfer is 35 years or older?
- 2C. Make a probability histogram of the data.

X = Age	Probability
18–24	0.13
25–34	0.18
35–44	0.21
45–54	0.19
55–64	0.12
65+	0.17

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CHECK Your Understanding

Example 1 (p. 672) For Exercises 1–3, use the table that shows the possible sums when rolling two dice and the number of ways each sum can be found.

Sum of Two Dice	2	3	4	5	6	7	8	9	10	11	12
Ways to Achieve Sum	1	2	3	4	5	6	5	4	3	2	1

- 1. Draw a table to show the sample space of all possible outcomes.
- 2. Find the probabilities for $X = 4$, $X = 5$, and $X = 6$.
- 3. What is the probability that the sum of two dice is greater than 6 on three separate rolls?

Example 2 (pp. 673–674) **GRADES** For Exercises 4–6, use the table that shows a class’s grade distribution, where A = 4.0, B = 3.0, C = 2.0, D = 1.0, and F = 0.

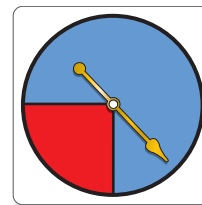
X = Grade	0	1.0	2.0	3.0	4.0
Probability	0.05	0.10	0.40	0.40	0.05

- 4. Show that the probability distribution is valid.
- 5. What is the probability that a student passes the course?
- 6. What is the probability that a student chosen at random from the class receives a grade of B or better?

Exercises

HOMEWORK HELP	
For Exercises	See Examples
7–8, 11, 15	1
9–10, 12–14, 16–17	2

For Exercises 7–10, the spinner shown is spun three times.



7. Write the sample space with all possible outcomes.
8. Find the probability distribution X , where X represents the number of times the spinner lands on blue for $X = 0$, $X = 1$, $X = 2$, and $X = 3$.
9. Make a probability histogram.
10. Do all possible outcomes have an equal chance of occurring? Explain.

SALES For Exercises 11–14, use the following information.

A music store manager takes an inventory of the top 10 CDs sold each week. After several weeks, the manager has enough information to estimate sales and make a probability distribution table.

Number of Top 10 CDs Sold Each Week	0–100	101–200	201–300	301–400	401–500
Probability	0.10	0.15	0.40	0.25	0.10

11. Define a random variable and list its values.
12. Show that this is a valid probability distribution.
13. In a given week, what is the probability that fewer than 400 CDs sell?
14. In a given week, what is the probability that more than 200 CDs sell?

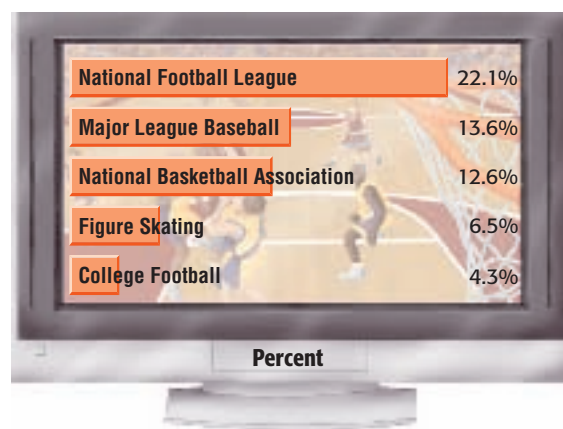
EDUCATION For Exercises 15–17, use the table, which shows the education level of persons aged 25 and older in the United States.

$X =$ Level of Education	Probability
Some High School	0.154
High School Graduate	0.320
Some College	0.172
Associate's Degree	0.082
Bachelor's Degree	0.179
Advanced Degree	0.093

15. If a person was randomly selected, what is the probability that he or she completed at most some college?
16. Make a probability histogram of the data.
17. Explain how you can find the probability that a randomly selected person has earned at least a bachelor's degree.

SPORTS For Exercises 18 and 19, use the graph that shows the sports most watched by women on TV.

Top 5 Sports Watched by Women



Source: ESPN Sports Poll

18. Determine whether this is a valid probability distribution. Justify your answer.
19. Based on the graph, in a group of 35 women how many would you expect to say they watch figure skating?

EXTRA PRACTICE

See pages 743, 755.

Math online

Self-Check Quiz at algebra1.com

H.O.T. Problems.....

20. **OPEN ENDED** Describe real-life data that could be displayed in a probability histogram.

- 21. CHALLENGE** Suppose a married couple keeps having children until they have a girl. Let the random variable X represent the number of children in their family. Assume that the probability of having a boy or a girl is each $\frac{1}{2}$.
- Calculate the probability distribution for $X = 1, 2, 3,$ and 4 .
 - Find the probability that the couple will have more than 4 children.
- 22. Writing in Math** Refer to the information on page 672 to explain how a pet store owner could use a probability distribution. How could the owner create a probability distribution and use it to establish a frequent buyer program?

STANDARDIZED TEST PRACTICE

- 23.** The table shows the probability distribution for the number of heads when four coins are tossed. What is the probability that no more than two heads show on a random toss?

- A 0.3125
B 0.375
C 0.6875
D 0.875

$X =$ Number of Heads	Probability $P(X)$
0	0.0625
1	0.25
2	0.375
3	0.25
4	0.0625

- 24. REVIEW** Mr. Perez works 40 hours a week at The Used Car Emporium. He earns \$7 an hour and 10% commission on every car he sells. If his hourly wage is increased to \$7.50 and his commission to 13%, how much money would he earn in a week if he sold \$20,000 worth of cars?
- F \$1700 H \$2300
G \$2200 J \$2900

Spiral Review

A card is drawn from a standard deck of 52 cards. Find each probability. (Lesson 12-4)

25. $P(\text{ace or } 10)$ 26. $P(3 \text{ or diamond})$ 27. $P(\text{odd number or spade})$

Evaluate. (Lesson 12-3)

28. ${}_{10}C_7$ 29. ${}_{12}C_5$ 30. $({}_6P_3)({}_5P_3)$

SAVINGS For Exercises 31–32, use the following information.

Selena is investing her \$900 tax refund in a certificate of deposit that matures in 4 years. The interest rate is 4.25% compounded quarterly. (Lesson 9-6)

31. Determine the balance in the account after 4 years.
32. Her friend Monique invests the same amount of money at the same interest rate, but her bank compounds interest monthly. Determine how much she will have after 4 years.
33. Which type of compounding appears more profitable? Explain.

GET READY for the Next Lesson

PREREQUISITE SKILL Write each fraction as a percent rounded to the nearest whole number. (pages 702–703)

34. $\frac{16}{80}$ 35. $\frac{20}{52}$ 36. $\frac{30}{114}$ 37. $\frac{57}{120}$ 38. $\frac{72}{340}$ 39. $\frac{54}{162}$

Main Ideas

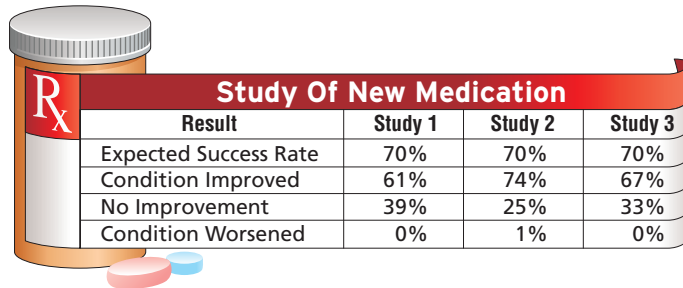
- Use theoretical and experimental probability to represent and solve problems involving uncertainty.
- Perform probability simulations to model real-world situations involving uncertainty.

New Vocabulary

theoretical probability
 experimental probability
 relative frequency
 empirical study
 simulation

GET READY for the Lesson

Researchers at a pharmaceutical company expect a new drug to work successfully in 70% of patients. To test the drug's effectiveness, the company performs three clinical studies of 100 volunteers who use the drug for six months. The results of the studies are shown in the table.



Result	Study 1	Study 2	Study 3
Expected Success Rate	70%	70%	70%
Condition Improved	61%	74%	67%
No Improvement	39%	25%	33%
Condition Worsened	0%	1%	0%

Theoretical and Experimental Probability The probability we have used to describe events in previous lessons is theoretical probability. **Theoretical probabilities** are determined mathematically and describe what should happen. In the situation above, the expected success rate of 70% is a theoretical probability.

A second type of probability is **experimental probability**, which is determined using data from tests or experiments. Experimental probability is the ratio of the number of times an outcome occurred to the total number of events or trials. This ratio is also known as the **relative frequency**.

$$\text{experimental probability} = \frac{\text{frequency of an outcome}}{\text{total number of trials}}$$

EXAMPLE Experimental Probability

- 1 MEDICAL RESEARCH** Refer to the application at the beginning of the lesson. What is the experimental probability that the drug was successful for a patient in Study 1?

$$\text{experimental probability} = \frac{61}{100} \quad \leftarrow \begin{array}{l} \text{frequency of successes} \\ \text{total number of patients} \end{array}$$

The experimental probability of Study 1 is $\frac{61}{100}$ or 61%.

CHECK Your Progress

1. Trevor says he is able to make at least 63% of free throws he takes. To prove this, he decides to take 50 free throws, of which he made 33. Did his experimental probability support his assertion?

It is often useful to perform an experiment repeatedly, collect and combine the data, and analyze the results. This is known as an **empirical study**.

EXAMPLE Empirical Study



Real-World Career...

Medical Scientist

Many medical scientists conduct research to advance knowledge of living organisms, including viruses and bacteria.



For more information, go to algebra1.com.

- 2 Refer to the application at the beginning of the lesson. What is the experimental probability of success for all three studies?

The number of successful outcomes of the three studies was $61 + 74 + 67$ or 202 out of the 300 total patients.

$$\text{experimental probability} = \frac{202}{300} \text{ or } \frac{101}{150}$$

The experimental probability of the three studies was $\frac{101}{150}$ or about 67%.

CHECK Your Progress

2. Refer to Check Your Progress 1. Trevor decided to shoot 50 free throws two more times. He makes 29 of the first 50 free throws and 34 of the second 50. What is the experimental probability of all three tests?

Performing Simulations A **simulation** allows you to find an experimental probability by using objects to act out an event that would be difficult or impractical to perform.

Algebra Lab

Simulations

COLLECT THE DATA

- Roll a die 20 times. Record the value on the die after each roll.
- Determine the experimental probability distribution for X , the value on the die.
- Combine your results with the rest of the class to find the experimental probability distribution for X given the new number of trials. ($20 \cdot$ the number of students in your class)



ANALYZE THE DATA

1. Find the theoretical probability of rolling a 2.
2. Find the theoretical probability of rolling a 1 or a 6.
3. Find the theoretical probability of rolling a value less than 4.
4. Compare the experimental and theoretical probabilities. Which pair of probabilities was closer to each other: your individual probabilities or your class's probabilities?
5. Suppose each person rolls the die 50 times. Explain how this would affect the experimental probabilities for the class.
6. What can you conclude about the relationship between the number of experiments in a simulation and the experimental probability?

Reading Math

Law of Large Numbers

The *Law of Large Numbers* states that as the number of trials increases, the experimental probability gets closer to the theoretical probability.

You can conduct simulations of the outcomes for many problems by using one or more objects such as dice, coins, marbles, or spinners. The objects you choose should have the same number of outcomes as the number of possible outcomes of the problem, and all outcomes should be equally likely.

Study Tip

Alternate Simulation

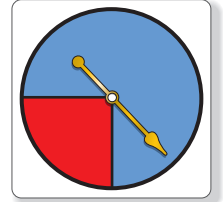
You could also use a 6-sided die to simulate this, where 1, 2, and 3 represent field goals made and 4 is field goals missed. If you roll a 5 or a 6, ignore it and roll again.

EXAMPLE Simulation

3 In one season, Malcolm made 75% of the field goals he attempted.

a. What could be used to simulate his kicking a field goal? Explain.

You could use a spinner like the one at the right, where 75% of the spinner represents making a field goal.



b. Describe a way to simulate his next 8 attempts.

Spin the spinner once to simulate a kick. Record the result, then repeat this 7 more times.

CHECK Your Progress

In a trivia game, Becky answered an average of two out of three questions correctly.

3A. What could be used to simulate her correctly answering a question? Explain.

3B. Describe a way to simulate the next 12 questions.

EXAMPLE Theoretical and Experimental Probability

4 DOGS Ali raises purebred dogs. One of her dogs had a litter of four puppies. What is the most likely mix of male and female puppies?

Assume that $P(\text{male}) = P(\text{female}) = \frac{1}{2}$.

a. What objects can be used to simulate the possible outcomes of the puppies?

Each puppy can be male or female, so there are $2 \cdot 2 \cdot 2 \cdot 2$ or 16 possible outcomes for the litter. Use a simulation that also has 2 outcomes for each of 4 events. One possible simulation would be to toss four coins, one for each puppy, with heads representing female and tails representing male.

b. Find the theoretical probability that there are two female and two male puppies.

There are 16 possible outcomes, and the number of combinations that have two female and two male puppies is ${}_4C_2$ or 6. So the theoretical probability is $\frac{6}{16}$ or $\frac{3}{8}$.

c. The results of a simulation Ali performed are shown in the table at the right. What is the experimental probability that there are three male puppies?

Ali performed 50 trials and 12 of those resulted in three males. So, the experimental probability is $\frac{12}{50}$ or 24%.

Outcomes	Frequency
4 female, 0 male	3
3 female, 1 male	13
2 female, 2 male	18
1 female, 3 male	12
0 female, 4 male	4

(continued on the next page)



- d. How does the experimental probability compare to the theoretical probability of a litter with three males?

Theoretical probability:

$$P(3 \text{ males}) = \frac{{}^4C_3}{16} \quad \frac{\text{combinations with 3 male puppies}}{\text{possible outcomes}}$$

$$= \frac{4}{16} \text{ or } 25\% \quad \text{Simplify.}$$

The experimental probability, 24%, is very close to the theoretical probability.

CHECK Your Progress

QUALITY CONTROL Brandon inspects automobile frames as they come through on the assembly line. On average, he finds a weld defect in one out of ten of the frames each day. He sends these back to correct the defect.

- 4A. What objects can be used to model the possible outcomes of the automobiles per hour?
- 4B. What is the theoretical probability that there is one automobile found with defects in a certain hour?
- 4C. The results of a simulation Brandon performed are shown in the table at the right. What is the experimental probability that there will be one defect found in a certain hour?
- 4D. How does the experimental probability compare to the theoretical probability of one defect found in a certain hour?

Defects	Frequency
0	14
1	3
2	2
3	1

 **Online** Personal Tutor at algebra1.com

CHECK Your Understanding

Example 1 (p. 677) **GAMES** Games at the fair require the majority of people who play to lose in order for game owners to make a profit. Therefore, new games need to be tested to make sure they have sufficient difficulty. The results of three test groups are listed in the table. The owners would like a maximum of 33% of players to win the game. There were 50 participants in each test group.

1. What is the experimental probability that the participant was a winner in the second group?

Result	Group 1	Group 2	Group 3
Winners	13	15	19
Losers	37	35	31

Example 2 (p. 678)

2. What is the experimental probability of winning for all three groups?
3. A baseball player has a batting average of .300. That is, he gets a hit 30% of the time he is at bat. What could be used to simulate the player taking a turn at bat?

For Exercises 4–6, roll a die 25 times and record your results.

Example 3 (p. 679)

4. Based on your results, what is the probability of rolling a 3?
5. Based on your results, what is the probability of rolling a 5 or an odd number?
6. Compare your results to the theoretical probabilities.

Example 4 ASTRONOMY For Exercises 7–10, use the following information.

(p. 679)

Enrique is writing a report about meteorites and wants to determine the probability that a meteor reaching Earth’s surface hits land. He knows that 70% of Earth’s surface is covered by water. He places 7 blue marbles and 3 brown marbles in a bag to represent hitting water ($\frac{7}{10}$) and hitting land ($\frac{3}{10}$). He draws a marble from the bag, records the color, and then replaces the marble. Enrique drew 56 blue and 19 brown marbles.

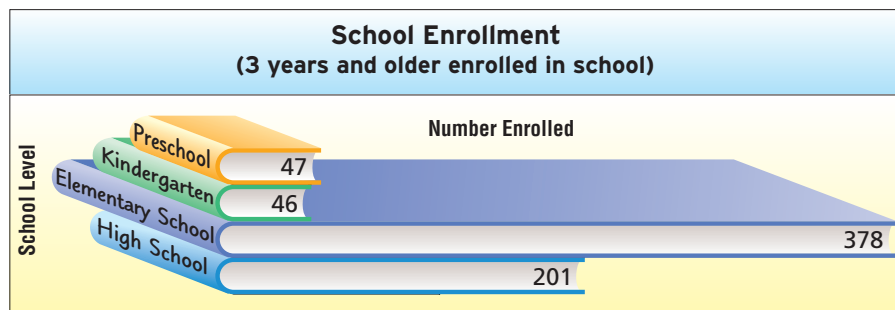
7. Did Enrique choose an appropriate simulation? Explain.
8. What is the theoretical probability that a meteorite reaching Earth’s surface hits land?
9. Based on his results, what is the probability that a meteorite hits land?
10. Using the experimental probability, how many of the next 500 meteorites that strike Earth would you expect to hit land?

Exercises

HOMEWORK HELP	
For Exercises	See Examples
11–13	1, 2
14–17	3
18–29	4

GOVERNMENT For Exercises 11–13, use the following information.

The Lewiston School Board sent surveys to randomly selected households to determine needs for the school district. The results of the survey are shown.



11. Find the experimental probability distribution for the number of people enrolled at each level.
12. Based on the survey, what is the probability that a student chosen at random is in elementary school or high school?
13. Suppose the school district is expecting school enrollment to increase by 1800 over the next 5 years due to new homes in the area. Of the new enrollment, how many will most likely be in kindergarten?
14. What could you use to simulate guessing on 15 true-false questions?
15. There are 12 cans of cola, 8 cans of diet cola, and 4 cans of root beer in a cooler. What could be used for a simulation to determine the probability of randomly picking any one type of soft drink?

For Exercises 16 and 17, use the following information.

A mall randomly gives each shopper one of 12 different gifts during a sale.

16. What could be used to perform a simulation of this situation? Explain.
17. How could you use this simulation to model the next 100 gifts handed out?

For Exercises 18 and 19, toss 3 coins, one at a time, 25 times and record your results. Find each probability based on your results.

18. What is the probability that any two coins will show heads?
19. What is the probability that the first and third coins show tails?

For Exercises 20–22, roll two dice 50 times and record the sums.

20. Based on your results, what is the probability that the sum is 8?
21. Based on your results, what is the probability that the sum is 7, or the sum is greater than 5?
22. If you roll the dice 25 more times, which sum would you expect to see about 10% of the time?

RESTAURANTS For Exercises 23–25, use the following information.

A family restaurant gives away a free toy with each child's meal. There are eight different toys that are randomly given. There is an equally likely chance of getting each toy each time.

23. What objects could be used to perform a simulation of this situation?
24. Conduct a simulation until you have one of each toy. Record your results.
25. Based on your results, how many meals must be purchased so that you get all 8 toys?

ANIMALS For Exercises 26–29, use the following information.

Refer to Example 4 on page 679. Suppose Ali's dog has a litter of 5 puppies.

26. List the possible outcomes of the genders of the puppies.
27. Perform a simulation and list your results in a table.
28. Based on your results, what is the probability that there are 3 females and two males in the litter?
29. What is the experimental probability that the litter has at least three males?

ENTERTAINMENT For Exercises 30–32, use the following information.

A CD changer contains 5 CDs with 14 songs each. When "Random" is selected, each CD is equally likely to be chosen as each song.

30. Use a graphing calculator to perform a simulation of randomly playing 20 songs from the 5 CDs. Record your answer.
KEYSTROKES: MATH ◀ 5 1 , 70 , 20) ENTER
31. Do the experimental probabilities for your simulation support the statement that each CD is equally likely to be chosen? Explain.
32. Based on your results, what is the probability that the first three songs played are on the third disc?

33. OPEN ENDED Describe a real-life situation that could be represented by a simulation. What objects would you use for this experiment?

34. CHALLENGE The captain of a football team believes that the coin the referee uses for the opening coin toss gives an advantage to one team. The referee has players toss the coin 50 times each and record their results. Based on the results, do you think the coin is fair? Explain your reasoning.

Player	1	2	3	4	5	6
Heads	38	31	29	27	26	30
Tails	12	19	21	23	24	20

35. Writing in Math Refer to the information on page 677 to explain how simulations can be used in health care. Include an explanation of experimental probability and why more trials are better than fewer trials when considering experimental probability.



Real-World Link

Labrador retrievers are the most popular breed of dog in the United States.

Source: American Kennel Club

EXTRA PRACTICE
See page 743, 755.
Math online
Self-Check Quiz at algebra1.com

H.O.T. Problems

STANDARDIZED TEST PRACTICE

36. Ramón tossed two coins and rolled a die. What is the probability that he tossed two tails and rolled a 3?

- A $\frac{1}{4}$ C $\frac{5}{12}$
 B $\frac{1}{6}$ D $\frac{1}{24}$

37. REVIEW Miranda bought a DVD boxed set for $\frac{1}{3}$ off the original price and another 10% off the sale price. If the original cost of the DVD set was \$44.99, what price did Miranda pay?

- F \$29.99 H \$27.79
 G \$28.34 J \$26.99

Spiral Review

For Exercises 38–40, use the probability distribution for the random variable X , the number of computers per household. (Lesson 12-5)

- 38.** Show that the probability distribution is valid.
39. If a household is chosen at random, what is the probability that it has at least 2 computers?
40. Determine the probability of randomly selecting a household with no more than one computer.

Computers per Household	
$X =$ Number of Computers	$P(X)$
0	0.579
1	0.276
2	0.107
3+	0.038

Source: U.S. Dept. of Commerce

For Exercises 41–43, use the following information.

A jar contains 18 nickels, 25 dimes, and 12 quarters. Three coins are randomly selected one at a time. Find each probability. (Lesson 12-4)

- 41.** picking three dimes, replacing each after it is drawn
42. a nickel, then a quarter, then a dime without replacing the coins
43. 2 dimes and a quarter, without replacing the coins, if order does not matter

Determine whether the following side measures would form a right triangle. (Lesson 10-4)

- 44.** 5, 7, 9 **45.** $3\sqrt{34}$, 9, 15 **46.** 36, 86.4, 93.6

Cross-Curricular Project

Algebra and Physical Science

Building the Best Roller Coaster It is time to complete your project. Use the information and data you have gathered about the building and financing of a roller coaster to prepare a portfolio or Web page. Be sure to include graphs, tables, and/or calculations in the presentation.



Cross-Curricular Project at algebra1.com

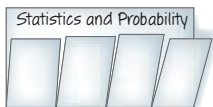


FOLDABLES™

Study Organizer

GET READY to Study

Be sure the following
Key Concepts are noted
in your Foldable.



Key Concepts

Sampling and Bias (Lesson 12-1)

- Simple random sample, stratified random sample, and systematic random sample are types of unbiased, or random, samples.
- Convenience sample and voluntary response sample are types of biased samples.

Counting Outcomes, Permutations, and Combinations (Lessons 12-2 and 12-3)

- If an event M can occur m ways and is followed by an event N that can occur n ways, the event M followed by event N can occur $m \cdot n$ ways.
- In a permutation, the order of objects is important. ${}_n P_r = \frac{n!}{(n-r)!}$
- In a combination, the order of objects is not important. ${}_n C_r = \frac{n!}{(n-r)!r!}$

Probability of Compound Events (Lesson 12-4)

- For independent events, use $P(A \text{ and } B) = P(A) \cdot P(B)$.
- For dependent events, use $P(A \text{ and } B) = P(A) \cdot P(B \text{ following } A)$.
- For mutually exclusive events, use $P(A \text{ or } B) = P(A) + P(B)$.
- For inclusive events, use $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$.

Probability Distributions and Simulations (Lessons 12-5 and 12-6)

- For each value of X , $0 \leq P(X) \leq 1$. The sum of the probabilities of each value of X is 1.
- Theoretical probability describes expected outcomes, while experimental probability describes tested outcomes.
- Simulations are used to perform experiments that would be difficult or impossible to perform in real life.

Key Vocabulary

biased sample (p. 643)	inclusive (p. 666)
combination (p. 657)	independent events (p. 663)
complements (p. 665)	population (p. 642)
compound event (p. 663)	random sample (p. 642)
convenience sample (p. 643)	sample (p. 642)
dependent events (p. 664)	simple random sample (p. 642)
discrete random variable (p. 672)	stratified random sample (p. 642)
empirical study (p. 678)	systematic random sample (p. 642)
event (p. 650)	voluntary response sample (p. 643)
experimental probability (p. 677)	
factorial (p. 652)	

Vocabulary Check

Choose the word or term that best completes each sentence.

1. The arrangement in which order is important is called a (combination, permutation).
2. The notation $10!$ refers to a (prime factor, factorial).
3. Rolling one die and then another die are (dependent, independent) events.
4. The sum of probabilities of complements equals (0, 1).
5. Randomly drawing a coin from a bag and then drawing another coin are dependent events if the coins (are, are not) replaced.
6. Events that cannot occur at the same time are (mutually exclusive, inclusive).
7. The sum of the probabilities in a probability distribution equals (0, 1).
8. (Theoretical, Experimental) probabilities are precise and predictable.

Lesson-by-Lesson Review

12-1 Sampling and Bias (pp. 642–648)

Identify the sample, suggest a population from which it was selected, and state whether the sample is *unbiased* (random) or *biased*. If unbiased, classify the sample as *simple*, *stratified*, or *systematic*. If biased, classify as *convenience* or *voluntary response*.

- 9. SCIENCE** A laboratory technician needs a sample of results of chemical reactions. She selects test tubes from the first 8 experiments performed on Tuesday.
- 10. CANDY BARS** To ensure that all of the chocolate bars are the appropriate weight, every 50th bar on the conveyor belt in the candy factory is removed and weighed.

Example 1 GOVERNMENT To determine whether voters support a new trade agreement, 5 people from the list of registered voters in each state and in the District of Columbia are selected at random. Identify the sample, suggest a population from which it was selected, and state whether the sample is *unbiased* (random) or *biased*. If unbiased, classify the sample as *simple*, *stratified*, or *systematic*. If biased, classify as *convenience* or *voluntary response*.

Since $5 \times 51 = 255$, the sample is 255 registered voters in the United States.

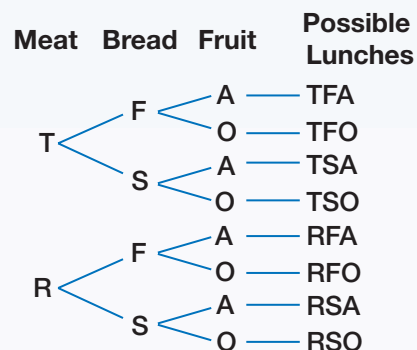
The sample is unbiased. It is an example of a stratified random sample.

12-2 Counting Outcomes (pp. 650–654)

Determine the number of outcomes for each event.

- 11. MOVIES** Samantha wants to watch 3 videos one rainy afternoon. She has a choice of 3 comedies, 4 dramas, and 3 musicals.
- 12. BOOKS** Marquis buys 4 books, one from each category. He can choose from 12 mystery, 8 science fiction, 10 classics, and 5 biographies.
- 13. SOCCER** The Jackson Jackals and the Westfield Tigers are going to play a best three-out-of-five games soccer tournament.

Example 2 When Jerri packs her lunch, she can choose to make a turkey (T) or roast beef (R) sandwich on French (F) or sourdough bread (S). She also can pack an apple (A) or an orange (O). Draw a tree diagram to show the number of different ways Jerri can select these items.



There are 8 different ways for Jerri to select these items.

12-3

Permutations and Combinations (pp. 653–662)

Evaluate each expression.

14. ${}_4P_2$ 15. ${}_8C_3$
 16. ${}_4C_4$ 17. $({}_7C_1)({}_6C_3)$
 18. $({}_7P_3)({}_7P_2)$ 19. $({}_3C_2)({}_4P_1)$

CLASS PHOTO For Exercises 20 and 21, use the following information.

The French teacher at East High School wants to arrange the 7 students who joined the French club for a yearbook photo.

20. Does this situation involve a permutation or a combination?
 21. How many different ways can the 7 students be arranged?

Example 3 Find ${}_{12}C_8$.

$$\begin{aligned} {}_{12}C_8 &= \frac{12!}{(12-8)!8!} \\ &= \frac{12!}{4!8!} \\ &= \frac{12 \cdot 11 \cdot 10 \cdot 9}{4!} \\ &= 495 \end{aligned}$$

Example 4 Find ${}_9P_4$.

$$\begin{aligned} {}_9P_4 &= \frac{9!}{(9-4)!} \\ &= \frac{9!}{5!} \\ &= \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} \\ &= 3024 \end{aligned}$$

12-4

Probability of Compound Events (pp. 663–670)

A bag of colored paper clips contains 30 red clips, 22 blue clips, and 22 green clips. Find each probability if three clips are drawn randomly from the bag and are not replaced.

22. $P(\text{blue, red, green})$ 23. $P(\text{red, red, blue})$

One card is randomly drawn from a standard deck of 52 cards. Find each probability.

24. $P(\text{heart or red})$ 25. $P(10 \text{ or spade})$
 26. **BASEBALL** Travis Hafner of the Cleveland Indians has a batting average of .391, which means he has gotten a hit 39.1% of the time. Victor Martinez bats directly after Hafner and has a batting average of .375. What are the chances that both men will get hits their first time up to bat?

Example 5 A box contains 8 red chips, 6 blue chips, and 12 white chips. Three chips are randomly drawn from the box and not replaced. Find $P(\text{red, white, blue})$.

$$\text{First chip: } P(\text{red}) = \frac{8}{26} \quad \frac{\text{red chips}}{\text{total chips}}$$

$$\text{Second chip: } P(\text{white}) = \frac{12}{25} \quad \frac{\text{white chips}}{\text{chips remaining}}$$

$$\text{Third chip: } P(\text{blue}) = \frac{6}{24} \quad \frac{\text{blue chips}}{\text{chips remaining}}$$

$$\begin{aligned} P(\text{red, white, blue}) &= P(\text{red}) \cdot P(\text{white}) \cdot P(\text{blue}) \\ &= \frac{8}{26} \cdot \frac{12}{25} \cdot \frac{6}{24} \\ &= \frac{576}{15600} \text{ or } \frac{12}{325} \end{aligned}$$

12-5 Probability Distributions (pp. 672–676)

ACTIVITIES The table shows the probability distribution for the number of extracurricular activities in which students at Boardwalk High School participate.

27. Show that the probability distribution is valid.
28. If a student is chosen at random, what is the probability that the student participates in 1 to 3 activities?
29. Make a probability histogram of the data.

Extracurricular Activities	
$X =$ Number of Activities	Probability
0	0.04
1	0.12
2	0.37
3	0.30
4+	0.17

Example 6 A local cable provider asked its subscribers how many television sets they had in their homes. The results of their survey are shown in the probability distribution.

Televisions per Household	
$X =$ Number of Televisions	Probability
1	0.18
2	0.36
3	0.34
4	0.08
5+	0.04

- a. Show that the probability distribution is valid.

For each value of X , the probability is greater than or equal to 0 and less than or equal to 1. $0.18 + 0.36 + 0.34 + 0.08 + 0.04 = 1$, so the sum of the probabilities is 1.

- b. If a household is selected at random, what is the probability that it has fewer than 4 televisions?

$$\begin{aligned}
 P(X < 4) &= P(X = 1) + P(X = 2) + P(X = 3) \\
 &= 0.18 + 0.36 + 0.34 \\
 &= 0.88
 \end{aligned}$$

The probability that a randomly selected household has fewer than 4 televisions is 88%.

12-6 Probability Simulations (pp. 677–683)

BIOLOGY While studying flower colors in biology class, students are given the Punnett square below. The Punnett square shows that red parent plant flowers (Rr) produce red flowers (RR and Rr) and pink flowers (rr).

	R	r
R	RR	Rr
r	Rr	rr

30. If 5 flowers are produced, find the theoretical probability that there will be 4 red flowers and 1 pink flower.
31. Describe items that the students could use to simulate the colors of 5 flowers.
32. The results of a simulation of flowers are shown in the table. What is the experimental probability that there will be 3 red flowers and 2 pink flowers?

Outcomes	Frequency
5 red, 0 pink	15
4 red, 1 pink	30
3 red, 2 pink	23
2 red, 3 pink	7
1 red, 4 pink	4
0 red, 5 pink	1

Example 7 A group of 3 coins are tossed.

- a. Find the theoretical probability that there will be 2 heads and 1 tail.

Each coin toss can be heads or tails, so there are $2 \cdot 2 \cdot 2$ or 8 possible outcomes. There are 3 possible combinations of 2 heads and one tail, HHT, HTH, or TTH. So, the theoretical probability is $\frac{3}{8}$.

- b. The results of a simulation in which three coins are tossed ten times are

Outcomes	Frequency
3 heads, 0 tails	1
2 heads, 1 tail	4
1 head, 2 tails	3
0 heads, 3 tails	2

shown in the table. What is the experimental probability that there will be 1 head and 2 tails?

Of the 10 trials, 3 resulted in 1 head and 2 tails, so the experimental probability is $\frac{3}{10}$ or 30%.

- c. Compare the theoretical probability of 2 heads and 1 tail and the experimental probability of 2 heads and 1 tail.

The theoretical probability is $\frac{3}{8}$ or 37.5%, while the experimental probability is $\frac{3}{10}$ or 30%. The probabilities are close.

Identify the sample, suggest a population from which it was selected, and state whether it is *unbiased* (random) or *biased*. If unbiased, classify the sample as *simple*, *stratified*, or *systematic*. If biased, classify as *convenience* or *voluntary response*.

- 1. DOGS** A veterinarian needs a sample of dogs in her kennel to be tested for fleas. She selects the first five dogs who run from the pen.
- 2. LIBRARIES** A librarian wants to sample book titles checked out on Wednesday. He randomly chooses a book checked out each hour that the library is open.

There are two roads from Ashville to Bakersville, four roads from Bakersville to Clifton, and two roads from Clifton to Derry.

3. Draw a tree diagram showing the possible routes from Ashville to Derry.
4. How many different routes are there from Ashville to Derry?

Determine whether each situation involves a *permutation* or a *combination*. Then determine the number of possible arrangements.

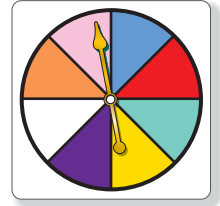
5. Six students in a class meet in a room that has nine chairs.
6. the top four finishers in a race with ten participants
7. A class has 15 girls and 19 boys. A committee is formed with two girls and two boys, each with a distinct responsibility.

A bag contains 4 red, 6 blue, 4 yellow, and 2 green marbles. Once a marble is selected, it is not replaced. Find each probability.

8. $P(\text{blue, green})$
9. $P(\text{yellow, yellow})$
10. $P(\text{red, blue, yellow})$
11. $P(\text{blue, red, not green})$

The spinner is spun, and a die is rolled. Find each probability.

12. $P(\text{yellow, 4})$
13. $P(\text{red, even})$
14. $P(\text{purple or white, not prime})$
15. $P(\text{green, even or less than 5})$



A magician randomly selects a card from a standard deck of 52 cards. Without replacing it, the magician has a member of the audience randomly select a card. Find each probability.

16. $P(\text{club, heart})$
17. $P(\text{jack or queen})$
18. $P(\text{black 7, diamond})$
19. $P(\text{queen or red, jack of spades})$
20. $P(\text{black 10, ace or heart})$

The table shows the number of ways the coins can land heads up when four coins are tossed at the same time. Find each probability.

Four Coins Tossed	
Number of Heads	Possible Outcomes
0	1
1	4
2	6
3	4
4	1

21. $P(\text{no heads})$
22. $P(\text{at least two heads})$
23. $P(\text{two tails})$
24. **MULTIPLE CHOICE** Two numbers a and b can be arranged in two different orders: a, b and b, a . In how many ways can three numbers be arranged?
A 3 B 4 C 5 D 6
25. **MULTIPLE CHOICE** If a coin is tossed three times, what is the probability that the results will be heads exactly one time?
F $\frac{2}{3}$ H $\frac{3}{8}$
G $\frac{1}{5}$ J $\frac{1}{8}$

Standards Practice

Cumulative, Chapters 1–12

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. The table shows the results of a survey given to 600 customers at a music store.

Favorite Music	Percent
Jazz	12
Pop	58
Classical	14
Other	16

Based on these data, which of the following statements is true?

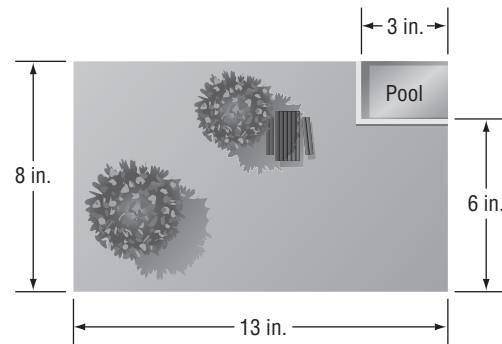
- A More than half of the customers' favorite music is classical or jazz.
- B More customers' favorite is pop music than all other types of music.
- C More customers' favorite music is something other than jazz, pop, or classical.
- D The number of customers whose favorite music is pop is more than five times the number of customers whose favorite music is jazz.
2. Which equation describes a line that has a y -intercept of -3 and a slope of 6 ?
- F $y = -3x + 6$
- G $y = (-3 + x)6$
- H $y = (-3x + 1)6$
- J $y = 6x - 3$

TEST-TAKING TIP

Question 2 Know the slope-intercept form of linear equations, $y = mx + b$, and understand the definition of slope.

3. **GRIDDABLE** Hailey is driving her car at a rate of 65 miles per hour. What is her rate in miles per second? Round to the nearest hundredth.

4. The diagram shown below is a scale drawing of Sally's backyard. She is ordering a cover for her swimming pool that costs \$5.00 per square foot.



What information must be provided in order to find the total cost of the cover?

- A The width of the swimming pool.
- B The thickness of the cover.
- C The scale of inches to feet on the drawing.
- D The amount that Sally has budgeted for the cover.
5. At Marvin's Pizza Place, 30% of the customers order pepperoni pizza. Also, 65% of the customers order a cola to drink. What is the probability that a customer selected at random orders a pepperoni pizza and a cola?
- F $\frac{95}{100}$ H $\frac{35}{100}$
- G $\frac{1}{3}$ J $\frac{39}{200}$
6. When graphed, which function would appear to be shifted 3 units down from the graph of $f(x) = x^2 + 2$?
- A $f(x) = x^2 + 5$
- B $f(x) = x^2$
- C $f(x) = x^2 - 3$
- D $f(x) = x^2 - 1$

7. **GRIDDABLE** Laura's Pizza Shop has your choice of 5 meats, 3 cheeses, and 4 vegetables. How many different combinations are there if you choose 1 meat, 1 cheese, and 1 vegetable?



8. Carlos rolled a 6 sided die 60 times. The results of his rolls are shown in the table.

Number	Frequency
1	12
2	9
3	14
4	10
5	7
6	8

What is the difference between the theoretical probability and the experimental probability for rolling a 5?

- F 5%
G 12%
H 17%
J 30%
9. Miguel rolled a six-sided die 60 times. The results are shown in the table below.

Side	Number of Times Landed
1	12
2	10
3	8
4	15
5	7
6	8

Which number has the same experimental probability as theoretical probability?

- A 1
B 2
C 3
D 4

10. Milla sold T-shirts for 10 days in a row for her band fund-raiser. In those 10 days, she sold 120 T-shirts for an average of 12 T-shirts per day. For how many days must she sell 20 T-shirts to bring her average to 18 T-shirts per day?

- F 10
G 20
H 30
J 40

11. Maryn is filling up a cylindrical water can. How many times more water could she fit in the can if the radius was doubled?

- A 2 times
B 3 times
C 4 times
D 8 times

Pre-AP

**Record your answers on a sheet of paper.
Show your work.**

12. At WackyWorld Pizza, the Random Special is a random selection of two different toppings on a large cheese pizza. The available toppings are pepperoni, sausage, onion, mushrooms, and green peppers.

- a. How many different Random Specials are possible? Show how you found your answer.
b. If you order the Random Special, what is the probability that it will have onions?
c. If you order the Random Special, what is the probability that it will have neither onion nor green peppers?

NEED EXTRA HELP?												
If You Missed Question...	1	2	3	4	5	6	7	8	9	10	11	12
Go to Lesson or Page...	12-6	4-3	11-4	2-4	12-4	9-1	12-4	12-6	12-6	11-9	708	12-2 and 12-3