

CHAPTER 2

Solving Linear Equations

BIG Ideas

- Translate verbal sentences into equations and equations into verbal sentences.
- Solve equations and proportions.
- Find percents of change.
- Solve equations for given variables.
- Solve mixture problems and uniform motion problems.

Key Vocabulary

equivalent equations (p. 79)

identity (p. 99)

proportion (p. 105)

percent of change (p. 111)

Real-World Link

Baseball Linear equations can be used to solve problems in every facet of life. For example, a baseball player's slugging percentage is found by using an equation based on a mixture, or weighted average, of five factors, including at bats and hits.

FOLDABLES™ Study Organizer

Solving Linear Equations Make this Foldable to help you organize information about solving linear equations. Begin with 5 sheets of plain $8\frac{1}{2}$ " by 11" paper.

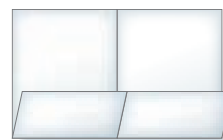
- 1 **Fold** in half along the width.



- 3 **Repeat** four times and glue all five pieces together.



- 2 **Fold** the bottom to form a pocket. Glue edges.



- 4 **Label** each pocket. Place an index card in each pocket.



GET READY for Chapter 2

Diagnose Readiness You have two options for checking Prerequisite Skills.

Option 2



Take the Online Readiness Quiz at algebra1.com

Option 1

Take the Quick Check below. Refer to the Quick Review for help.

QUICK Check

Write an algebraic expression for each verbal expression. (Lesson 1-1)

- five greater than half a number t
- the product of seven and s divided by the product of eight and y
- the sum of three times a and the square of b
- w to the fifth power decreased by 37

Evaluate each expression. (Lesson 1-2)

- $3 \cdot 6 - \frac{12}{4}$
- $5(13 - 7) - 22$
- $5(7 - 2) - 3^2$
- $\frac{2 \cdot 6 - 4}{2}$
- $(25 - 4)(2^2 - 1)$
- $36 \div 4 - 2 + 3$
- $\frac{19 - 5}{7} + 3$
- $\frac{1}{4}(24) - \frac{1}{2}(12)$
- MONEY** For his birthday, Brad received \$25 from each of his four aunts. After putting 10% of this money in the bank, Brad bought 5 CDs at \$10 each. How much money does he have left?

Find each percent. (Prerequisite Skill)

- Five is what percent of 20?
- What percent of 5 is 15?
- What percent of 300 is 21?
- ELECTIONS** In an exit poll, 456 of 900 voters said that they voted for Dunlap. Approximately what percentage of voters polled voted for Dunlap?

QUICK Review

EXAMPLE 1

Write an algebraic expression for the phrase *two more than five times a number t* .

two more than five times a number t
2 + 5 × t

The expression is $2 + 5t$.

EXAMPLE 2

Evaluate $7 - \frac{6}{3}[5(17 - 6) + 2] + 108$.

$$\begin{aligned} &= 7 - \frac{6}{3}[5(17 - 6) + 2] + 108 \\ &= 7 - \frac{6}{3}[5(11) + 2] + 108 && \text{Evaluate inside the parentheses.} \\ &= 7 - \frac{6}{3}[55 + 2] + 108 && \text{Multiply.} \\ &= 7 - \frac{6}{3}[57] + 108 && \text{Add inside the brackets.} \\ &= 7 - 2[57] + 108 && \text{Simplify.} \\ &= 7 - 114 + 108 && \text{Multiply.} \\ &= 1 && \text{Simplify.} \end{aligned}$$

EXAMPLE 3

What percent of 64 is 6?

$$\begin{aligned} \frac{a}{b} &= \frac{p}{100} && \text{Use the percent proportion.} \\ \frac{6}{64} &= \frac{p}{100} && \text{Replace } a \text{ with 6 and } b \text{ with 64.} \\ 6(100) &= 64p && \text{Find the cross products.} \\ 600 &= 64p && \text{Multiply.} \\ 9.375 &= p && \text{Divide each side by 64.} \\ 6 &\text{ is } 9.375\% \text{ of } 64. \end{aligned}$$

Writing Equations

Main Ideas

- Translate verbal sentences into equations.
- Translate equations into verbal sentences.

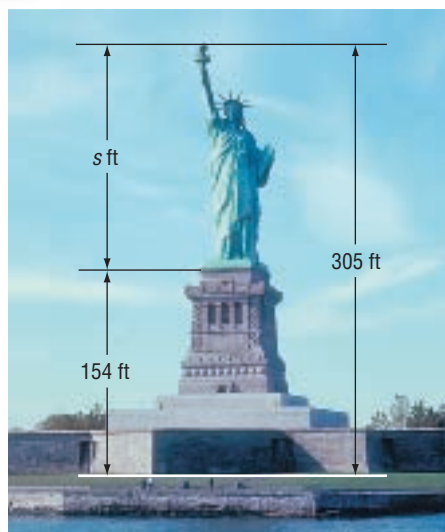
New Vocabulary

four-step problem-solving plan
defining a variable
formula

GET READY for the Lesson

The Statue of Liberty stands on a pedestal that is 154 feet high. The height of the pedestal and the statue is 305 feet. You can write an equation to represent this situation.

Words	The height of the pedestal and the statue is 305 feet.
Variable	Let s represent the height of the statue.
Equation	The height of the pedestal and the statue is 305 feet. $154 + s = 305$



Source: World Book Encyclopedia

Write Equations When writing equations, use variables to represent the unspecified numbers or measures. Then write the verbal expressions as algebraic expressions. Some verbal expressions that suggest the *equals sign* are listed below.

is is as much as is the same as is identical to

Study Tip

Look Back

To review translating verbal expressions to algebraic expressions, see Lesson 1-1.

EXAMPLE Translate Sentences into Equations

1 Translate each sentence into an equation.

a. Five times the number a squared is three times the sum of b and c .

$$\begin{array}{ccccccccccc} \text{Five} & \text{times} & \text{a} & \text{is} & \text{three} & \text{times} & \text{the sum of} \\ 5 & \times & \text{squared} & = & 3 & \times & \text{b and c.} \\ & & a^2 & & & & (b + c) \end{array}$$

The equation is $5a^2 = 3(b + c)$.

b. Nine times a number subtracted from 95 equals 37.

Rewrite the sentence so it is easier to translate. Let n = the number.

$$\begin{array}{ccccccc} 95 & \text{less} & \text{nine times } n & \text{equals} & 37. \\ 95 & - & 9n & = & 37 \end{array} \quad \text{The equation is } 95 - 9n = 37.$$

CHECK Your Progress

- 1A. Two plus the quotient of a number and 8 is the same as 16.
1B. Twenty-seven times k is h squared decreased by 9.

Using the **four-step problem-solving plan** can help you solve any word problem.

KEY CONCEPT

Four-Step Problem-Solving Plan

- Step 1** Explore the problem.
- Step 2** Plan the solution.
- Step 3** Solve the problem.
- Step 4** Check the solution.

Reading Math

Verbal Problems

In a verbal problem, the sentence that tells what you are asked to find usually contains *find*, *what*, *when*, or *how*.

Step 1 Explore the Problem

To solve a verbal problem, first read the problem carefully and explore what the problem is about.

- Identify what information is given.
- Identify what you are asked to find.

Step 2 Plan the Solution

One strategy you can use to solve a problem is to write an equation. Choose a variable to represent one of the unspecified numbers in the problem. This is called **defining a variable**. Then use the variable to write expressions for the other unspecified numbers.

Step 3 Solve the Problem

Use the strategy you chose in Step 2 to solve the problem.

Step 4 Check the Solution

Check your answer in the context of the original problem.

- Does your answer make sense?
- Does it fit the information in the problem?



Real-World Link

The first ice cream plant was established in 1851 by Jacob Fussell. Today, 2,000,000 gallons of ice cream are produced in the United States each day.

Source: World Book Encyclopedia

Real-World EXAMPLE Use the Four-Step Plan

2 ICE CREAM Use the information at the left. In how many days can 40,000,000 gallons of ice cream be produced in the United States?

Explore You know that 2,000,000 gallons of ice cream are produced in the United States each day. You want to know how many days it will take to produce 40,000,000 gallons of ice cream.

Plan Write an equation. Let d represent the number of days.

$$\begin{array}{ccccccc} & & & \text{the number} & & & \\ & & & \text{of days} & \text{equals} & & \\ 2,000,000 & \text{times} & & & & 40,000,000. \\ 2,000,000 & \times & d & = & & 40,000,000 \end{array}$$

Solve $2,000,000d = 40,000,000$ Find d mentally by asking, "What number times 2,000,000 equals 40,000,000?"
 $d = 20$

It will take 20 days to produce 40,000,000 gallons of ice cream.

Check If 2,000,000 gallons of ice cream are produced in one day, 2,000,000 \times 20 or 40,000,000 gallons are produced in 20 days. The answer makes sense.



CHECK Your Progress

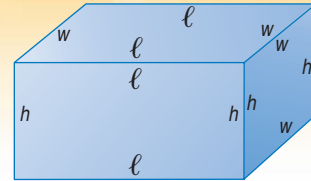
2. **GOVERNMENT** There are 50 members in the North Carolina Senate. This is 70 fewer than the number in the North Carolina House of Representatives. How many members are in the North Carolina House of Representatives?

A **formula** is an equation that states a rule for the relationship between certain quantities.

ALGEBRA LAB

Surface Area

- Mark each side of a rectangular box as the length ℓ , the width w , or the height h .
- Use scissors to cut the box so that each surface or face of the box is a separate piece.



ANALYZE

Write an expression for the area of each side of the box.

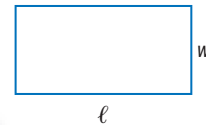
- front
 - back
 - left side
 - right side
 - top
 - bottom
7. The surface area S of a rectangular box is the sum of all the areas of the faces of the box. Write a formula for the surface area of a rectangular box.

MAKE A CONJECTURE

8. If s represents the length of the side of a cube, write a formula for the surface area S of a cube.

EXAMPLE Write a Formula

- 3 Translate the sentence into a formula. *The perimeter of a rectangle equals two times the length plus two times the width.*



Words Perimeter equals two times the length plus two times the width.

Variables Let P = perimeter, ℓ = length, and w = width.

Formula

Perimeter	equals	two times	plus	two times
P	=	the length	+	the width.
		2ℓ		$2w$

The formula for the perimeter of a rectangle is $P = 2\ell + 2w$.

CHECK Your Progress

3. **GEOMETRY** Translate the sentence into a formula. *In a right triangle, the square of the measure of the hypotenuse c is equal to the sum of the squares of the measures of the legs, a and b .*

Online Personal Tutor at algebra1.com

Write Verbal Sentences You can also translate equations into verbal sentences or make up your own verbal problem if you are given an equation.

EXAMPLE Translate Equations into Sentences

4 Translate each equation into a verbal sentence.

a. $3m + 5 = 14$

$3m$ + 5 = 14
Three times m plus five equals fourteen.

b. $w + v = y^2$

$w + v$ = y^2
The sum of w and v equals the square of y .

CHECK Your Progress

4A. $13 = 2 + 6t$

4B. $\frac{1}{4}n + 5 = n - 7$

EXAMPLE Write a Problem

5 Write a problem based on the given information.

a = Rafael's age $a + 5$ = Tierra's age $a + 2(a + 5) = 46$

The equation adds Rafael's age a plus twice Tierra's age ($a + 5$) to get 46.

Sample problem:

Tierra is 5 years older than Rafael. The sum of Rafael's age and twice Tierra's age equals 46. How old is Rafael?

CHECK Your Progress

5. p = price of jeans $0.2p$ = discount $p - 0.2p = 31.20$

CHECK Your Understanding

Example 1
(p. 70)

Translate each sentence into an equation.

- Twice a number t decreased by eight equals seventy.
- Five times the sum of m and n is the same as seven times n .
- Half of p is the same as p minus three.
- A number squared is as much as twelve more than the number.

Example 2
(p. 71)

- SAVINGS** Misae has \$1900 in the bank. She wishes to increase her account to a total of \$2500 by depositing \$30 per week from her paycheck. Write and use an equation to find how many weeks she needs to reach her goal.

Example 3
(p. 72)

Translate each sentence into a formula.

- The area A of a triangle equals one half times the base b times the height h .
- The circumference C of a circle equals the product of 2, π , and the radius r .

Example 4
(p. 72)

Translate each equation into a verbal sentence.

8. $1 = 7 + \frac{z}{5}$

9. $14 + d = 6d$

10. $\frac{1}{3}b - 4 = 2a$

Example 5
(p. 73)

Write a problem based on the given information.

11. c = cost of a suit
 $c - 25 = 150$

12. p = price of a new backpack
 $0.055p$ = tax; $p + 0.055p = 31.65$

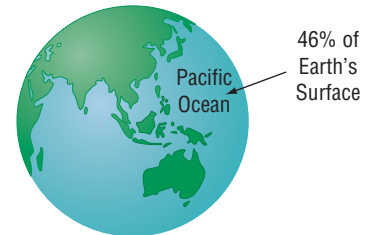
Exercises

HOMEWORK HELP	
For Exercises	See Examples
13–20	1
21–22	2
23–26	3
27–32	4
33–34	5

Translate each sentence into an equation.

- The sum of twice r and three times s is identical to thirteen.
- The quotient of t and forty is the same as twelve minus half of s .
- Two hundred minus three times a number is equal to nine.
- The sum of one-third a number and 25 is as much as twice the number.
- The square of m minus the cube of n is sixteen.
- Two times z is equal to two times the sum of v and w .

- GEOGRAPHY** The Pacific Ocean covers about 46% of Earth. If P represents the area of the Pacific Ocean and E represents the area of Earth, write an equation for this situation.



Source: World Book Encyclopedia

- GARDENING** Mrs. Patton is planning to place a fence around her vegetable garden. The fencing costs \$1.75 per yard. She buys f yards of fencing and pays \$3.50 in tax. If the total cost is \$73.50, write an equation to represent the situation.



- LITERATURE** Edgar Rice Burroughs published his first *Tarzan of the Apes* story in 1912. In 1928, the California town where he lived was named Tarzana. Let y represent the number of years after 1912 that the town was named. Write and use an equation to determine the number of years between the first Tarzan story and the naming of the town.

- WRESTLING** Darius weighs 155 pounds. He wants to start the wrestling season weighing 160 pounds. If g represents the number of pounds he wants to gain, write an equation to represent this situation. Then use the equation to find the number of pounds Darius needs to gain.

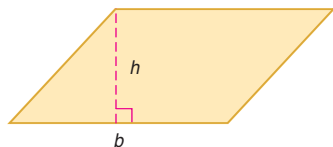
Real-World Link

More than 50 movies featuring Tarzan have been made. The first, *Tarzan of the Apes*, in 1918, was among the first movies to gross over \$1 million.

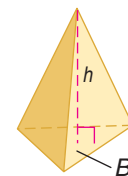
Source: www.tarzan.org

Translate each sentence into a formula.

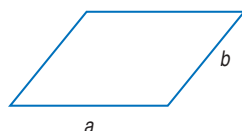
- The area A of a parallelogram is the base b times the height h .



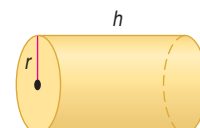
- The volume V of a pyramid is one-third times the product of the area of the base B and its height h .



- The perimeter P of a parallelogram is twice the sum of the lengths of the two adjacent sides, a and b .



- The volume V of a cylinder equals the product of π , the square of the radius r of the base, and the height.



Translate each equation into a verbal sentence.

27. $d - 14 = 5$

28. $2f + 6 = 19$

29. $k^2 + 17 = 53 - j$

30. $2a = 7a - b$

31. $\frac{3}{4}p + \frac{1}{2} = p$

32. $\frac{2}{5}w = \frac{1}{2}w + 3$

Write a problem based on the given information.

33. y = Yolanda's height in inches
 $y + 7$ = Lindsey's height in inches
 $2y + (y + 7) = 193$

34. b = price of a book
 $0.065b$ = tax
 $2(b + 0.065b) = 42.49$

Translate each sentence into an equation.

35. Half the sum of nine and a number is the same as the number minus three.
 36. The quotient of g and h is the same as seven more than twice the sum of g and h .

GEOMETRY For Exercises 37 and 38, use the following information.

The volume V of a cone equals one third times the product of π , the square of the radius r of the base, and the height h .

37. Write the formula for the volume of a cone.
 38. Find the volume of a cone if r is 10 centimeters and h is 30 centimeters.

GEOMETRY For Exercises 39 and 40, use the following information.

The volume V of a sphere is four thirds times π times the radius r of the sphere cubed.

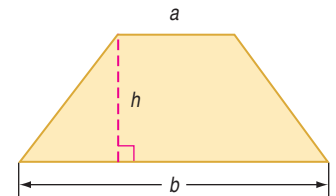
39. Write a formula for the volume of a sphere.
 40. Find the volume of a sphere if r is 4 inches.

TELEVISION For Exercises 41–43, use the following information.

During a highly rated one-hour television show, the entertainment portion lasted 15 minutes longer than 4 times the advertising portion a .

41. Write an expression for the entertainment portion.
 42. Write an equation to represent the situation.
 43. Use the guess-and-check strategy to determine the number of minutes spent on advertising.

44. **GEOMETRY** If a and b represent the lengths of the bases of a trapezoid and h represents its height, then the formula for the area A of the trapezoid is $A = \frac{1}{2}h(a + b)$. Write the formula in words.

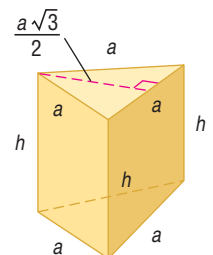


Translate each equation into a verbal sentence.

45. $4(t - s) = 5s + 12$

46. $7(x + y) = 35$

47. **CHALLENGE** The surface area of a prism is the sum of the areas of the faces of the prism. Write a formula for the surface area of the triangular prism. Explain how you organized the parts into a simplified equation.



48. **OPEN ENDED** Apply what you know about writing equations to write a problem about a school activity that can be answered by solving $x + 16 = 30$.



Real-World Link

A typical half-hour show runs for 22 minutes and has 8 minutes of commercials.

Source: gaebler.com

EXTRA PRACTICE
 See pages 720, 745.
Math online
 Self-Check Quiz at algebra1.com

H.O.T. Problems

49. *Writing in Math* Use the information about the Statue of Liberty on page 70 to explain how equations are used to describe heights. Include an equation relating the heights of the Sears Tower, which is 1454 feet tall, the two antenna towers on top of the building, which are a feet tall, and the total height, which is 1707 feet.

STANDARDIZED TEST PRACTICE

50. Which equation *best* represents the relationship between the number of hours an electrician works h and the total charges c ?

Cost of Electrician	
emergency house call	\$30 one-time fee
rate	\$55/hour

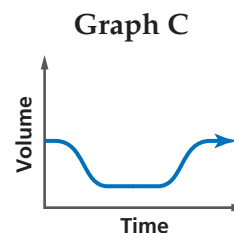
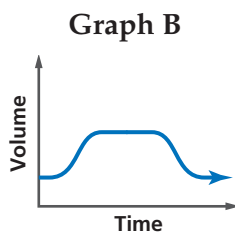
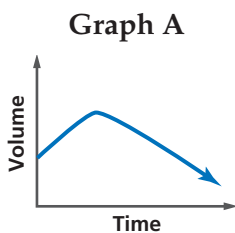
- A $c = 30 + 55$ C $c = 30 + 55h$
 B $c = 30h + 55$ D $c = 30h + 55h$

51. **REVIEW** A car traveled at 55 miles per hour for 2.5 hours and then at 65 miles per hour for 3 hours. How far did the car travel in all?

- F 300.5 mi
 G 305 mi
 H 330 mi
 J 332.5 mi

Spiral Review

52. **ENTERTAINMENT** Juanita has the volume on her stereo turned up. When her telephone rings, she turns the volume down. After she gets off the phone, she returns the volume to its previous level. Identify which graph shows the volume of Juanita's stereo during this time. (Lesson 1-9)



Find each square root. If necessary, round to the nearest hundredth. (Lesson 1-8)

53. $\sqrt{8100}$

54. $-\sqrt{\frac{25}{36}}$

55. $\sqrt{90}$

56. $-\sqrt{55}$

Simplify each expression. (Lesson 1-5)

57. $12d + 3 - 4d$

58. $7t^2 + t + 8t$

59. $3(a + 2b) + 5a$

Evaluate each expression. (Lesson 1-2)

60. $5(8 - 3) + 7 \cdot 2$

61. $6(4^3 + 2)$

62. $7(0.2 + 0.5) - 0.6$

GET READY for the Next Lesson

PREREQUISITE SKILL Find each sum or difference. (Pages 694–697)

63. $0.57 + 2.8$

64. $5.28 - 3.4$

65. $9 - 7.35$

66. $\frac{2}{3} + \frac{1}{5}$

67. $\frac{1}{6} + \frac{2}{3}$

68. $\frac{7}{9} - \frac{2}{3}$

Algebra Lab



Solving Addition and Subtraction Equations

You can use algebra tiles to solve equations. To solve an equation means to find the value of the variable that makes the equation true. After you model the equation, the goal is to get the x -tile by itself on one side of the mat using the rules stated below.

Reading Math

Variables You *isolate* the variable by getting it by itself on one side of the equation.

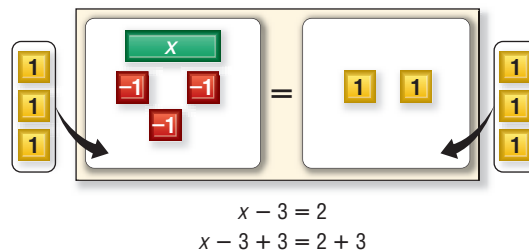
Concepts in Motion
Animation algebra1.com

Rules for Equation Models	
You can remove or add the same number of identical algebra tiles to each side of the mat without changing the equation.	
One positive tile and one negative tile of the same unit are a zero pair . Since $1 + (-1) = 0$, you can remove or add zero pairs to the equation mat without changing the equation.	

ACTIVITY

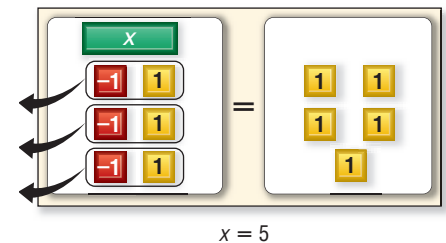
Use an equation model to solve $x - 3 = 2$.

Step 1 Model the equation.



Place 1 x -tile and 3 negative 1-tiles on one side of the mat. Place 2 positive 1-tiles on the other side of the mat. Then add 3 positive 1-tiles to each side.

Step 2 Isolate the x -term.



Group the tiles to form zero pairs. Then remove all the zero pairs. The resulting equation is $x = 5$.

MODEL AND ANALYZE

Use algebra tiles to solve each equation.

- $x + 5 = 7$
- $x + (-2) = 8$
- $x + 4 = 5$
- $x + (-3) = 4$
- $x + 3 = -4$
- $x + 7 = 2$

MAKE A CONJECTURE

- If $a = b$, what can you say about $a + c$ and $b + c$?
- If $a = b$, what can you say about $a - c$ and $b - c$?

Solving Equations by Using Addition and Subtraction

GET READY for the Lesson

The graph shows some of the fastest-growing occupations from 1992 to 2005.



Source: Bureau of Labor Statistics

The percent of growth for travel agents is 5 less than the percent of growth for medical assistants. An equation can be used to find the percent of growth expected for medical assistants. If m is the percent of growth for medical assistants, then $66 = m - 5$. You can use a property of equality to find the value of m .

Solve Using Addition Suppose the boys' soccer team has 15 members and the girls' soccer team has 15 members. If each team adds 3 new players, the number of members on the boys' and girls' teams would still be equal.

$$15 = 15 \quad \text{Each team has 15 members before adding the new players.}$$

$$15 + 3 = 15 + 3 \quad \text{Each team adds 3 new members.}$$

$$18 = 18 \quad \text{Each team has 18 members after adding the new members.}$$

This example illustrates the **Addition Property of Equality**.

KEY CONCEPT

Addition Property of Equality

Words If an equation is true and the same number is added to each side, the resulting equation is true.

Symbols For any numbers a , b , and c , if $a = b$, then $a + c = b + c$.

Examples

$7 = 7$	$14 = 14$
$7 + 3 = 7 + 3$	$14 + (-6) = 14 + (-6)$
$10 = 10$	$8 = 8$

Main Ideas

- Solve equations by using addition.
- Solve equations by using subtraction.

New Vocabulary

equivalent equations
solve an equation

Study Tip

Look Back

To review **writing algebraic expressions**, see Lesson 1-1.

If the same number is added to each side of an equation, then the result is an equivalent equation. **Equivalent equations** have the same solution.

$$t + 3 = 5 \quad \text{The solution of this equation is 2.}$$

$$t + 3 + 4 = 5 + 4 \quad \text{Using the Addition Property of Equality, add 4 to each side.}$$

$$t + 7 = 9 \quad \text{The solution of this equation is also 2.}$$

Study Tip

Coefficients

Remember that x means $1 \cdot x$. The coefficient of x is 1.

To **solve an equation** means to find all values of the variable that make the equation a true statement. One way to do this is to isolate the variable having a coefficient of 1 on one side of the equation. You can sometimes do this by using the Addition Property of Equality.

EXAMPLE Solve by Adding

1 Solve each equation. Check your solution.

a. $m - 48 = 29$

$$m - 48 = 29 \quad \text{Original equation}$$

$$m - 48 + 48 = 29 + 48 \quad \text{Add 48 to each side.}$$

$$m = 77 \quad -48 + 48 = 0 \text{ and } 29 + 48 = 77$$

To check that 77 is the solution, substitute 77 for m in the original equation.

CHECK $m - 48 = 29$ Original equation

$$77 - 48 \stackrel{?}{=} 29 \quad \text{Substitute 77 for } m.$$

$$29 = 29 \quad \checkmark \quad \text{Subtract.}$$

The solution 77 is correct.

b. $21 + q = -18$

$$21 + q = -18 \quad \text{Original equation}$$

$$21 + q + (-21) = -18 + (-21) \quad \text{Add } -21 \text{ to each side.}$$

$$q = -39 \quad 21 + (-21) = 0 \text{ and } -18 + (-21) = -39$$

The solution is -39 . To check, substitute -39 for q in the original equation.

CHECK Your Progress

1A. $32 = r - 8$

1B. $7 = 42 + t$

Solve Using Subtraction Similar to the Addition Property of Equality, the **Subtraction Property of Equality** can also be used to solve equations.

KEY CONCEPT

Subtraction Property of Equality

Words If an equation is true and the same number is subtracted from each side, the resulting equation is true.

Symbols For any numbers a , b , and c , if $a = b$, then $a - c = b - c$.

Examples

$17 = 17$	$3 = 3$
$17 - 9 = 17 - 9$	$3 - 8 = 3 - 8$
$8 = 8$	$-5 = -5$



EXAMPLE Solve by Subtracting

- 2 Solve $142 + d = 97$. Check your solution.

$$142 + d = 97 \quad \text{Original equation}$$

$$142 + d - 142 = 97 - 142 \quad \text{Subtract 142 from each side.}$$

$$d = -45 \quad 142 - 142 = 0 \text{ and } 97 - 142 = -45$$

The solution is -45 . To check, substitute -45 for d in the original equation.

CHECK Your Progress Solve each equation. Check your solution.

2A. $27 + k = 30$

2B. $-12 = p + 16$

Remember that subtracting a number is the same as adding its inverse.

EXAMPLE Solve by Adding or Subtracting

- 3 Solve $g + \frac{3}{4} = -\frac{1}{8}$ in two ways.

Method 1 Use the Subtraction Property of Equality.

$$g + \frac{3}{4} = -\frac{1}{8} \quad \text{Original equation}$$

$$g + \frac{3}{4} - \frac{3}{4} = -\frac{1}{8} - \frac{3}{4} \quad \text{Subtract } \frac{3}{4} \text{ from each side.}$$

$$g = -\frac{7}{8} \quad \frac{3}{4} - \frac{3}{4} = 0 \text{ and } -\frac{1}{8} - \frac{3}{4} = -\frac{1}{8} - \frac{6}{8} \text{ or } -\frac{7}{8}$$

Method 2 Use the Addition Property of Equality.

$$g + \frac{3}{4} = -\frac{1}{8} \quad \text{Original equation}$$

$$g + \frac{3}{4} + \left(-\frac{3}{4}\right) = -\frac{1}{8} + \left(-\frac{3}{4}\right) \quad \text{Add } -\frac{3}{4} \text{ to each side.}$$

$$g = -\frac{7}{8} \quad \frac{3}{4} + \left(-\frac{3}{4}\right) = 0 \text{ and } -\frac{1}{8} + \left(-\frac{3}{4}\right) = -\frac{1}{8} + \left(-\frac{6}{8}\right) \text{ or } -\frac{7}{8}$$

CHECK Your Progress

3. Solve $t + 10 = 55$ in two ways.

EXAMPLE Write and Solve an Equation

- 4 Write an equation for the problem. Then solve the equation.

A number increased by 5 is equal to 42. Find the number.

A number	increased by	5	is equal to	42.
n	+	5	=	42

$$n + 5 = 42 \quad \text{Original equation}$$

$$n + 5 - 5 = 42 - 5 \quad \text{Subtract 5 from each side.}$$

$$n = 37$$

The solution is 37.

CHECK Your Progress

4. Twenty-five is 3 less than a number. Find the number.

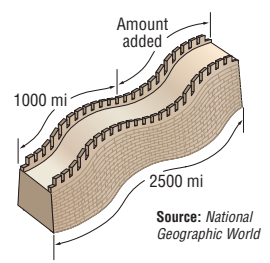
Study Tip**Checking Solutions**

You should always check your solution in the context of the original problem. For instance, in Example 4, is 37 increased by 5 equal to 42? The solution checks.



Real-World EXAMPLE

5 HISTORY In the fourteenth century, part of the Great Wall of China was repaired and the wall was extended. When the wall was completed, it was 2500 miles long. How much of the wall was added during the 1300s?



Real-World Link

The first emperor of China, Qui Shi Huangdi, ordered the building of the Great Wall of China to protect his people from nomadic tribes that attacked and looted villages. By 204 B.C., this wall guarded 1000 miles of China's border.

Source: National Geographic World

Words	The original length plus the additional length is 2500.				
Variable	Let a = the additional length.				
Equation	The original length	plus	the additional length	is	2500.
	1000	+	a	=	2500

$$1000 + a = 2500 \quad \text{Original equation}$$

$$1000 + a - 1000 = 2500 - 1000 \quad \text{Subtract 1000 from each side.}$$

$$a = 1500 \quad \text{1000 - 1000 = 0 and 2500 - 1000 = 1500}$$

The Great Wall of China was extended 1500 miles in the 1300s.

CHECK Your Progress

5. DEER In a recent year, 1286 female deer were born in Lewis County. That was 93 fewer than the number of male deer born. How many male deer were born that year?

online Personal Tutor at algebra1.com

CHECK Your Understanding

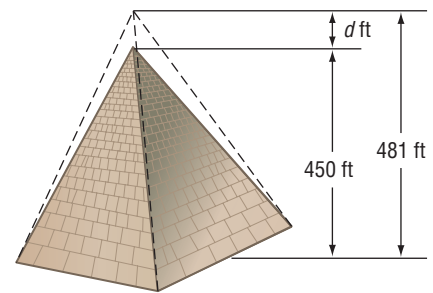
Solve each equation. Check your solution.

- | | | | |
|------------------------------------|-------------------------------------|---------------------------|-------------------------------------|
| Examples 1–3
(pp. 79–80) | 1. $n - 20 = 5$ | 2. $104 = y - 67$ | 3. $-4 + t = -7$ |
| | 4. $g + 5 = 33$ | 5. $19 + p = 6$ | 6. $15 = b - (-65)$ |
| | 7. $h + \frac{2}{5} = \frac{7}{10}$ | 8. $-6 = \frac{1}{4} + m$ | 9. $\frac{2}{3} + w = 1\frac{1}{2}$ |

Example 4 (p. 80) Write an equation for each problem. Then solve the equation and check your solution.

10. Twenty-one subtracted from a number is -8 . Find the number.
 11. A number increased by 91 is 37. Find the number.

Example 5 (p. 81) **12. HISTORY** Over the years, the height of the Great Pyramid at Giza, Egypt, has decreased. Use the figure to write an equation to represent the situation. Then find the decrease in the height of the pyramid.



Source: World Book Encyclopedia

Exercises

HOMEWORK HELP	
For Exercises	See Examples
13–26	1–3
27–30	4
31, 32	5

Solve each equation. Check your solution.

13. $v - 9 = 14$

14. $44 = t - 72$

15. $-61 = d + (-18)$

16. $p + (-26) = 16$

17. $18 + z = 40$

18. $19 = c + 12$

19. $n + 23 = 4$

20. $-67 = 11 + k$

21. $18 - (-f) = 91$

22. $88 = 125 - (-u)$

23. $\frac{2}{3} + r = -\frac{4}{9}$

24. $\frac{3}{4} = w + \frac{2}{5}$

25. $-\frac{1}{2} + a = \frac{5}{8}$

26. $-\frac{7}{10} = y - \frac{3}{5}$

Write an equation for each problem. Then solve the equation and check your solution.

27. Eighteen subtracted from a number equals 31. Find the number.

28. What number decreased by 77 equals -18 ?

29. A number increased by -16 is -21 . Find the number.

30. The sum of a number and -43 is 102. What is the number?

For Exercises 31 and 32, write an equation for each situation. Then solve the equation.

31. **GAS MILEAGE** A midsize car with a 4-cylinder engine goes 34 miles on a gallon of gasoline. This is 10 miles more than a luxury car with an 8-cylinder engine goes on a gallon of gasoline. How many miles does a luxury car travel on a gallon of gasoline?

32. **IN THE MEDIA** The world's biggest-ever passenger plane, the *Airbus A380*, was first used by Singapore Airlines in 2005. The following description appeared on a news Web site after the plane was introduced.

"That airline will see the A380 transporting some 555 passengers, 139 more than a similarly set-up 747." Source: cnn.com

How many passengers does a similarly set-up 747 transport?

Solve each equation. Then check your solution.

33. $k + 0.6 = -3.8$

34. $8.5 + t = 7.1$

35. $4.2 = q - 3.5$

36. $q - 2.78 = 4.2$

37. $6.2 = -4.83 + y$

38. $-6 = m + (-3.42)$

Write an equation for each problem. Then solve the equation and check your solution.

39. What number minus one half is equal to negative three fourths?

40. The sum of 19 and 42 and a number is equal to 87. What is the number?



Real-World Link

The Airbus A380 is 239 feet 6 inches long, 79 feet 1 inch high, and has a wingspan of 261 feet 10 inches.

Source: nationalgeographic.com

41. **CARS** The average time t it takes to manufacture a car in the United States is 24.9 hours. This is 8.1 hours longer than the average time it takes to manufacture a car in Japan. Write and solve an addition equation to find the average time to manufacture a car in Japan.

42. If $x - 7 = 14$, what is the value of $x - 2$?

43. If $t + 8 = -12$, what is the value of $t + 1$?

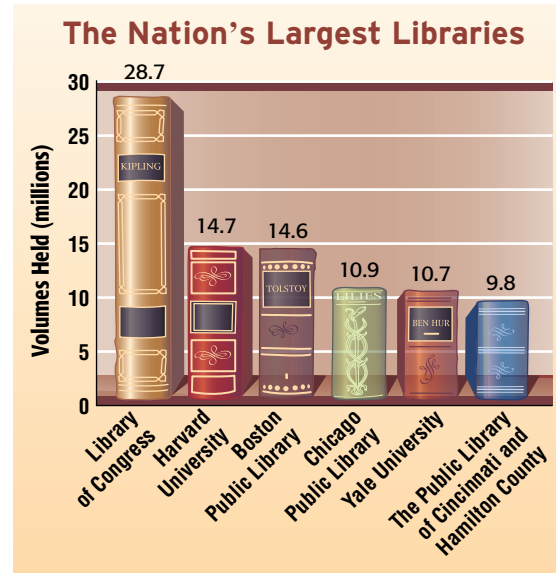
ANALYZE GRAPHS For Exercises 44–47, use the graph at the right to write an equation for each situation. Then solve the equation.

44. How many more volumes does the Library of Congress have than the Harvard University Library?

45. How many more volumes does the Harvard University Library have than the Boston Public Library?

46. How many more volumes does the Library of Congress have than the Boston Public Library?

47. What is the total number of volumes in the three largest U.S. libraries?



Source: American Library Association

EXTRA PRACTICE
See pages 720, 745.
Math online
Self-Check Quiz at algebra1.com

H.O.T. Problems

48. **Which One Doesn't Belong?** Identify the equation that does not belong with the other three. Explain your reasoning.

$n + 14 = 27$

$12 + n = 25$

$n - 16 = 29$

$n - 4 = 9$

49. **OPEN ENDED** Write an equation involving addition and demonstrate two ways to solve it.

50. **CHALLENGE** If $a - b = x$, what values of a , b , and x would make the equation $a + x = b + x$ true? Explain your reasoning.

51. **CHALLENGE** Determine whether each sentence is *sometimes*, *always*, or *never* true. Explain your reasoning.

a. $x + x = x$

b. $x + 0 = x$

52. **Writing in Math** Use the data about occupations on page 78 to explain how equations can be used to compare data. Include a sample problem and related equation using the information in the graph and an explanation of how to solve the equation.

STANDARDIZED TEST PRACTICE

53. Which problem is best represented by the equation $w - 15 = 33$?
- A Jake added w ounces of water to his water bottle, which originally contained 33 ounces of water. How much water did he add?
- B Jake added 15 ounces of water to his water bottle, for a total of 33 ounces of water. How much water w was originally in the bottle?
- C Jake drank 15 ounces of water from his water bottle and 33 ounces were left. How much water w was originally in the bottle?
- D Jake drank 15 ounces of water from his water bottle, which originally contained 33 ounces. How much water w was left?

54. **REVIEW** The table shows the results of a survey given to 500 international travelers. Based on the data, which statement is true?

Vacation Plans	
Destination	Percent
The Tropics	37
Europe	19
Asia	17
Other	17
No Vacation	10

- F Fifty international travelers have no vacation plans.
- G Fifteen international travelers are going to Asia.
- H One third of international travelers are going to the tropics.
- J One hundred international travelers are going to Europe.

Spiral Review

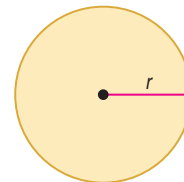
GEOMETRY For Exercises 55 and 56, use the following information. (Lesson 2-1)

The area of a circle is the product of π times the radius r squared.

55. Write the formula for the area of the circle.
56. If a circle has a radius of 16 inches, find its area.

Replace each \bullet with $>$, $<$, or $=$ to make the sentence true. (Lesson 1-8)

57. $\frac{1}{2} \bullet \sqrt{2}$ 58. $\frac{3}{4} \bullet \frac{2}{3}$ 59. $0.375 \bullet \frac{3}{8}$



Identify the hypothesis and conclusion of each statement. Then write the statement in if-then form. (Lesson 1-7)

60. There is a science quiz every Friday. 61. For $y = 2$, $4y - 6 = 2$.
62. **SHOPPING** Shawnel bought 8 bagels at \$0.95 each, 8 doughnuts at \$0.80 each and 8 small cartons of milk for \$1.00 each. Write and solve an expression to determine the total cost. (Lesson 1-5)

GET READY for the Next Lesson

PREREQUISITE SKILL Find each product or quotient. (Pages 698–701)

63. 6.5×2.8 64. $17.8 \div 2.5$ 65. $\frac{2}{3} \times \frac{5}{8}$ 66. $\frac{8}{9} \div \frac{4}{15}$

2-3

Solving Equations by Using Multiplication and Division

Main Ideas

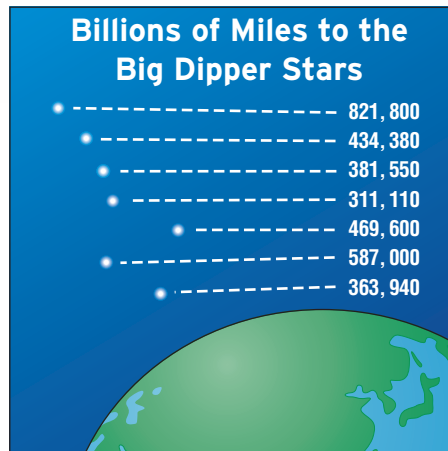
- Solve equations by using multiplication.
- Solve equations by using division.

GET READY for the Lesson

The diagram shows the distance between Earth and each star in the Big Dipper. Light travels at a rate of about 5,870 billion miles per year. The rate or speed at which something travels times the time equals the distance it travels. The following equation can be used to find the time it takes light from the closest star in the Big Dipper to reach Earth.

$$rt = d$$

$$5870t = 311,110$$



Source: National Geographic World

Solve Using Multiplication To solve equations such as the one above, you can use the **Multiplication Property of Equality**.

KEY CONCEPT

Multiplication Property of Equality

Words If an equation is true and each side is multiplied by the same number, the resulting equation is true.

Symbols For any numbers a , b , and c , if $a = b$, then $ac = bc$.

Examples

$6 = 6$	$9 = 9$	$10 = 10$
$6 \times 2 = 6 \times 2$	$9 \times (-3) = 9 \times (-3)$	$10 \times \frac{1}{2} = 10 \times \frac{1}{2}$
$12 = 12$	$-27 = -27$	$5 = 5$

EXAMPLE Solve Using Multiplication by a Positive Number

1 Solve $\frac{t}{3} = 7$. Check your solution.

$\frac{t}{3} = 7$	Original equation
$3\left(\frac{t}{3}\right) = 3(7)$	Multiply each side by 3.
$t = 21$	$\frac{t}{3}(3) = t$ and $7(3) = 21$

CHECK $\frac{t}{3} = 7$	Original equation
$\frac{21}{3} \stackrel{?}{=} 7$	Substitute 21 for t .
$7 = 7$ ✓	The solution is 21.

CHECK Your Progress

Solve each equation. Check your solution.

1A. $18 = \frac{w}{2}$

1B. $\frac{n}{3} = -\frac{2}{5}$

EXAMPLE Solve Using Multiplication by a Fraction**2** Solve each equation.

a. $(2\frac{1}{4})g = \frac{1}{2}$

$(2\frac{1}{4})g = \frac{1}{2}$ Original equation

$(\frac{9}{4})g = \frac{1}{2}$ Rewrite the mixed number as an improper fraction.

$\frac{4}{9}(\frac{9}{4})g = \frac{4}{9}(\frac{1}{2})$ Multiply each side by $\frac{4}{9}$, the reciprocal of $\frac{9}{4}$.

$g = \frac{2}{9}$ Check this result.

b. $42 = -6m$

$42 = -6m$ Original equation

$-\frac{1}{6}(42) = -\frac{1}{6}(-6m)$ Multiply each side by $-\frac{1}{6}$, the reciprocal of -6 .

$-7 = m$ Check this result.

CHECK Your Progress

2A. $\frac{3}{5}k = 6$

2B. $-\frac{1}{4} = \frac{2}{3}b$

Real-World EXAMPLE**3** **SPACE TRAVEL** Refer to the information at the left. An item's weight on the Moon is about one sixth its weight on Earth. What was the weight of Neil Armstrong's suit and life-support backpacks on Earth?

Words	One sixth	times	the weight on Earth	equals	the weight on the Moon.
Variables	Let w = the weight on Earth.				
Equation	$\frac{1}{6}$	\cdot	w	$=$	33

$\frac{1}{6}w = 33$ Original equation

$6(\frac{1}{6}w) = 6(33)$ Multiply each side by 6.

$w = 198$ $\frac{1}{6}(6) = 1$ and $33(6) = 198$

Neil Armstrong's suit and backpacks were about 198 pounds on Earth.

CHECK Your Progress**3. SURVEYS** In a recent survey of 13- to 15-year-old girls, 225, or about $\frac{9}{20}$ of those surveyed, said they talk on the telephone while they watch television. About how many girls were surveyed?**online** Personal Tutor at algebra1.com**Real-World Link**

On July 20, 1969, Neil Armstrong stepped on the surface of the Moon. On the Moon, his suit and life-support backpacks weighed about 33 pounds.

Source: NASA

Solve Using Division In Example 2b, the equation $42 = -6m$ was solved by multiplying each side by $-\frac{1}{6}$. The same result could have been obtained by dividing each side by -6 . This method uses the **Division Property of Equality**.

KEY CONCEPT

Division Property of Equality

Words If an equation is true and each side is divided by the same non-zero number, the resulting equation is true.

Symbols For any numbers a , b , and c with $c \neq 0$, if $a = b$, then $\frac{a}{c} = \frac{b}{c}$.

Examples

$15 = 15$	$28 = 28$
$\frac{15}{3} = \frac{15}{3}$	$\frac{28}{-7} = \frac{28}{-7}$
$5 = 5$	$-4 = -4$

EXAMPLE Solve Using Division

4 Solve each equation. Check your solution.

a. $13s = 195$

$13s = 195$	Original equation
$\frac{13s}{13} = \frac{195}{13}$	Divide each side by 13.
$s = 15$	$\frac{13s}{13} = s$ and $\frac{195}{13} = 15$

CHECK $13s = 195$	Original equation
$13(15) \stackrel{?}{=} 195$	Substitute 15 for s .
$195 = 195$ ✓	Multiply.

b. $-3x = 12$

$-3x = 12$	Original equation
$\frac{-3x}{-3} = \frac{12}{-3}$	Divide each side by -3 .
$x = -4$	$\frac{-3x}{-3} = x$ and $\frac{12}{-3} = -4$

CHECK $-3x = 12$	Original equation
$-3(-4) \stackrel{?}{=} 12$	Substitute -4 for x .
$12 = 12$ ✓	Multiply.

Study Tip

Alternative Method

You can also solve equations like those in Examples 4 and 5 by using the Multiplication Property of Equality. For instance, in Example 4b, you could multiply each side by $-\frac{1}{3}$.

CHECK Your Progress

4A. $84 = 3b$

4B. $-42 = -3s$

EXAMPLE Write and Solve an Equation Using Division

5 Write an equation for the problem below. Then solve the equation.

Negative eighteen times a number equals -198 .

Negative eighteen	times	a number	equals	-198 .
-18	\times	n	$=$	-198
$-18n = -198$	Original equation			
$\frac{-18n}{-18} = \frac{-198}{-18}$	Divide each side by -18 .			
$n = 11$	Check this result.			

CHECK Your Progress

5. Write an equation for the following problem. Then solve the equation.
Negative forty-two equals the product of six and a number.

CHECK Your Understanding

Examples 1 and 2 (pp. 85–86)

Solve each equation. Check your solution.

1. $\frac{t}{7} = -5$

2. $\frac{a}{36} = \frac{4}{9}$

3. $\frac{2}{3}n = 10$

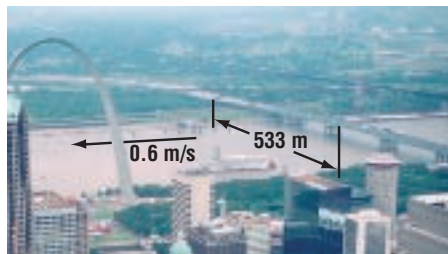
4. $\frac{8}{9} = \frac{4}{5}k$

5. $12 = \frac{x}{-3}$

6. $-\frac{r}{4} = \frac{1}{7}$

Example 3 (p. 86)

7. **GEOGRAPHY** The *discharge* of a river equals the product of its width, its average depth, and its speed. At one location in St. Louis, the Mississippi River is 533 meters wide, its speed is 0.6 meter per second, and its discharge is 3198 cubic meters per second. How deep is the Mississippi River at this location?



Example 4 (p. 87)

Solve each equation. Check your solution.

8. $8t = 72$

9. $20 = 4w$

10. $45 = -9a$

11. $-2g = -84$

Example 5 (p. 87)

Write an equation for each problem. Then solve the equation.

12. Five times a number is 120. What is the number?

13. One third equals negative seven times a number. What is the number?

Exercises

HOMEWORK HELP	
For Exercises	See Examples
14–25	1–2
26, 27	3
28–33	4
34–37	5

Solve each equation. Check your solution.

14. $\frac{x}{9} = 10$

15. $\frac{b}{7} = -11$

16. $\frac{3}{4} = \frac{c}{24}$

17. $\frac{2}{3} = \frac{1}{8}y$

18. $\frac{2}{3}n = 14$

19. $\frac{3}{5}g = -6$

20. $4\frac{1}{5} = 3p$

21. $-5 = 3\frac{1}{2}x$

22. $6 = -\frac{1}{2}n$

23. $-\frac{2}{5} = -\frac{z}{45}$

24. $-\frac{8}{24} = \frac{5}{12}$

25. $-\frac{v}{5} = -45$

26. **GENETICS** About two twenty-fifths of the male population in the world cannot distinguish red from green. If there are 14 boys in the ninth grade that cannot distinguish red from green, about how many ninth-grade boys are there in all? Write and solve an equation to find the answer.

27. **WORLD RECORDS** In 1998, Winchell's House of Donuts in Pasadena, California, created the world's largest doughnut. It weighed 2.5 tons and had a circumference of about 298.5 feet. What was its diameter?
(*Hint:* $C = \pi d$)

Solve each equation. Check your solution.

28. $8d = 48$

29. $-65 = 13t$

30. $-5r = 55$

31. $-252 = 36s$

32. $-58 = -29h$

33. $-26a = -364$



Real-World Link

Nolan Ryan pitched a fastball that was officially clocked by the *Guinness Book of World Records* at 100.9 miles per hour in a game played on August 20, 1974.

Source: www.baseball-almanac.com

EXTRA PRACTICE
See pages 720, 745.

Math online
Self-Check Quiz at algebra1.com

H.O.T. Problems

Write an equation for each problem. Then solve the equation.

- 34. Seven times a number equals -84 . What is the number?
- 35. Two fifths of a number equals -24 . Find the number.
- 36. Negative 117 is nine times a number. Find the number.
- 37. Twelve is one fifth of a number. What is the number?

Solve each equation. Check your solution.

- 38. $(3\frac{1}{4})p = 2\frac{1}{2}$
- 39. $-5h = -3\frac{2}{3}$
- 40. $(-2\frac{3}{5})t = -22$
- 41. $3.15 = 1.5y$
- 42. $-11.78 = 1.9f$
- 43. $-2.8m = 9.8$

Write an equation for each problem. Then solve the equation.

- 44. Negative three eighths times a number equals 12. What is the number?
- 45. Two and one half times a number equals one and one fifth. Find the number.
- 46. One and one third times a number is -4.82 . What is the number?

BASEBALL For Exercises 47 and 48, use the following information.

In baseball, if all other factors are the same, the speed of a four-seam fastball is faster than a two-seam fastball. The distance from the pitcher's mound to home plate is 60.5 feet.

- 47. How long does it take a two-seam fastball to go from the pitcher's mound to home plate? Round to the nearest hundredth. (*Hint: $d = rt$*)
- 48. How much longer does it take for a two-seam fastball to reach home plate than a four-seam fastball?

Two-Seam Fastball
126 ft/s



Four-Seam Fastball
132 ft/s

Source: *Baseball and Mathematics*

PHYSICAL SCIENCE For Exercises 49–51, use the following information.

For every 8 grams of oxygen in water, there is 1 gram of hydrogen. In science lab, Ayame and her classmates are asked to determine how many grams of hydrogen and oxygen are in 477 grams of water.

- 49. If x represents the number of grams of hydrogen, write an expression to represent the number of grams of oxygen.
- 50. Write an equation to represent the situation.
- 51. How many grams of hydrogen and oxygen are in 477 grams of water?
- 52. **OPEN ENDED** Write a multiplication equation that has a solution of -4 . Then relate the equation and solution to a real-life problem.
- 53. **REASONING** Compare and contrast the Multiplication Property of Equality and the Division Property of Equality and explain why they can be considered the same property.
- 54. **CHALLENGE** Discuss how you can use $6y - 7 = 4$ to find the value of $18y - 21$. Then find the value of y .

55. **FIND THE ERROR** Casey and Camila are solving $8n = -72$. Who is correct? Explain your reasoning.

Casey

$$8n = -72$$

$$8n(8) = -72(8)$$

$$n = -576$$

Camila

$$8n = -72$$

$$\frac{8n}{8} = \frac{-72}{8}$$

$$n = -9$$

56. **Writing in Math** Use the data about light speed on page 85 to explain how equations can be used to find how long it takes light to reach Earth. Include an explanation of how to find how long it takes light to reach Earth from the closest star in the Big Dipper and an equation describing the situation for the star in the Big Dipper farthest from Earth.

STANDARDIZED TEST PRACTICE

57. **REVIEW** Which is the *best* estimate for the number of minutes on the calling card advertised below?



- A 10 min C 50 min
B 20 min D 200 min

58. **REVIEW** Mr. Morisson is draining his cylindrical, above ground pool. The pool has a radius of 10 feet and a standard height of 4.5 feet. If the pool water is pumped out at a constant rate of 5 gallons per minute, about how long will it take to drain the pool? ($1 \text{ ft}^3 = 7.5 \text{ gal}$)

- F 37.8 min
G 7 h
H 25.4 h
J 35.3 h

Spiral Review

Solve each equation. Check your solution. (Lesson 2-2)

59. $m + 14 = 81$

60. $d - 27 = -14$

61. $17 - (-w) = -55$

62. Translate the following sentence into an equation. (Lesson 2-1)
Ten times a number a is equal to 5 times the sum of b and c .

63. **MUSIC** Ryan practiced playing his violin 40 minutes on Monday and n minutes each on Tuesday, Wednesday, and Thursday. Write an expression for the total amount of time that he practiced during those four days. (Lesson 1-1)

GET READY for the Next Lesson

PREREQUISITE SKILL Use the order of operations to find each value. (Lesson 1-2)

64. $9 + 2 \times 8$

65. $24 \div 3 - 8$

66. $\frac{3}{8}(17 + 7)$

67. $\frac{15 - 9}{26 + 12}$

Algebra Lab

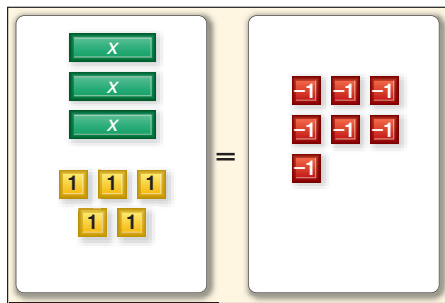
Solving Multi-Step Equations

You can use an equation model to solve multi-step equations.

ACTIVITY

Solve $3x + 5 = -7$.

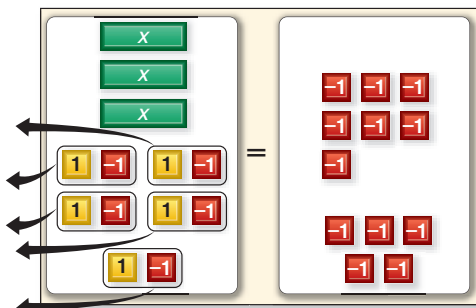
Step 1 Model the equation.



$$3x + 5 = -7$$

Place 3 x -tiles and 5 positive 1-tiles on one side of the mat. Place 7 negative 1-tiles on the other side of the mat.

Step 3 Remove zero pairs.



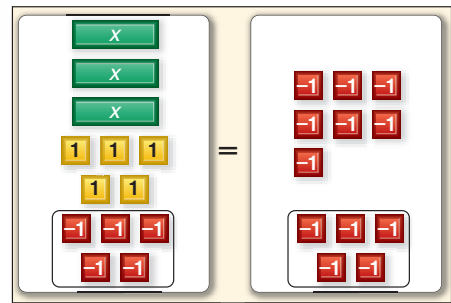
$$3x = -12$$

Group the tiles to form zero pairs and remove the zero pairs.

Concepts in Motion

Animation algebra1.com

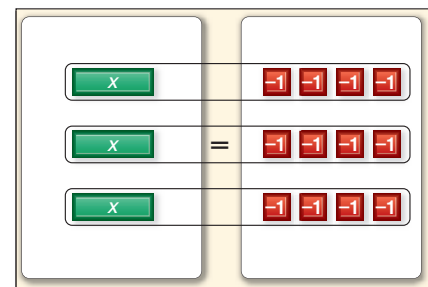
Step 2 Isolate the x -term.



$$3x + 5 - 5 = -7 - 5$$

Since there are 5 positive 1-tiles with the x -tiles, add 5 negative 1-tiles to each side to form zero pairs.

Step 4 Group the tiles.



$$\frac{3x}{3} = \frac{-12}{3}$$

Separate the tiles into 3 equal groups to match the 3 x -tiles. Each x tile is paired with 4 negative 1 tiles. Thus, $x = -4$.

MODEL Use algebra tiles to solve each equation.

1. $2x - 3 = -9$

2. $3x + 5 = 14$

3. $3x - 2 = 10$

4. $-8 = 2x + 4$

5. $3 + 4x = 11$

6. $2x + 7 = 1$

7. $9 = 4x - 7$

8. $7 + 3x = -8$

9. $3x - 1 = -10$

10. **MAKE A CONJECTURE** What steps would you use to solve $7x - 12 = -61$?

Solving Multi-Step Equations

Main Ideas

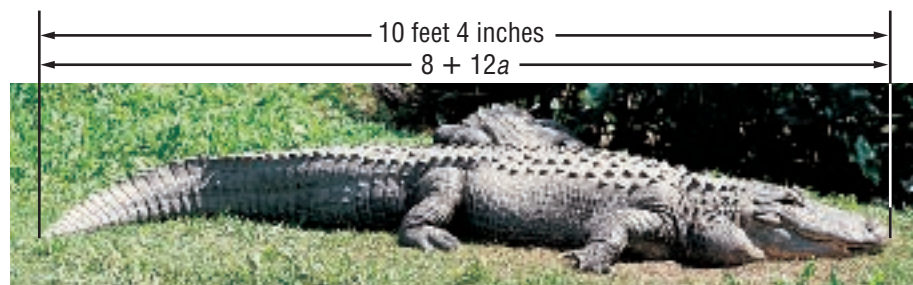
- Solve equations involving more than one operation.
- Solve consecutive integer problems.

New Vocabulary

multi-step equations
consecutive integers
number theory

GET READY for the Lesson

An alligator hatchling 8 inches long grows about 12 inches per year. The expression $8 + 12a$ represents the length in inches of an alligator that is a years old.



Since 10 feet 4 inches equals $10(12) + 4$ or 124 inches, the equation $8 + 12a = 124$ can be used to estimate the age of the alligator in the photograph. Notice that this equation involves more than one operation.

Solve Multi-Step Equations To solve equations with more than one operation, often called **multi-step equations**, undo operations by working backward.

EXAMPLE Solve Using Addition and Division

1 Solve $7m - 17 = 60$. Check your solution.

$$7m - 17 = 60 \quad \text{Original equation}$$

$$7m - 17 + 17 = 60 + 17 \quad \text{Add 17 to each side.}$$

$$7m = 77 \quad \text{Simplify.}$$

$$\frac{7m}{7} = \frac{77}{7} \quad \text{Divide each side by 7.}$$

$$m = 11 \quad \text{Simplify.}$$

CHECK $7m - 17 = 60$ Original equation

$$7(11) - 17 \stackrel{?}{=} 60 \quad \text{Substitute 11 for } m.$$

$$77 - 17 \stackrel{?}{=} 60 \quad \text{Multiply.}$$

$$60 = 60 \quad \checkmark$$

CHECK Your Progress

Solve each equation. Check your solution.

1A. $2a - 6 = 4$

1B. $8 = 3r + 7$

1C. $\frac{t}{8} + 21 = 14$

EXAMPLE Solve Using Multiplication and Addition

2 Solve $\frac{p-15}{9} = -6$.

$$\frac{p-15}{9} = -6 \quad \text{Original equation}$$

$$9\left(\frac{p-15}{9}\right) = 9(-6) \quad \text{Multiply each side by 9.}$$

$$p-15 = -54 \quad \text{Simplify.}$$

$$p-15 + 15 = -54 + 15 \quad \text{Add 15 to each side.}$$

$$p = -39 \quad \text{Simplify.}$$

CHECK Your Progress

Solve each equation. Check your solution.

2A. $\frac{k-12}{5} = 4$

2B. $\frac{n+1}{-2} = 15$

Real-World EXAMPLE Write and Solve a Multi-Step Equation

- 3 SKIING** Hugo is buying a pair of water skis that are on sale for $\frac{2}{3}$ of the original price. After he uses a \$25 gift certificate, the total cost before taxes is \$115. What was the original price of the skis? Write an equation for the problem. Then solve the equation.

Words	Two-thirds	of	the price	minus	25	is	115.
Variable	Let p = original price of the skis.						
Equation	$\frac{2}{3}$	\cdot	p	$-$	25	$=$	115

$$\frac{2}{3}p - 25 = 115 \quad \text{Original equation}$$

$$\frac{2}{3}p - 25 + 25 = 115 + 25 \quad \text{Add 25 to each side.}$$

$$\frac{2}{3}p = 140 \quad \text{Simplify.}$$

$$\frac{3}{2}\left(\frac{2}{3}p\right) = \frac{3}{2}(140) \quad \text{Multiply each side by } \frac{3}{2}.$$

$$p = 210 \quad \text{Simplify.}$$

The original price of the skis was \$210.

CHECK Your Progress

- 3.** Write an equation for the following problem. Then solve the equation. *Sixteen is equal to 7 increased by the product of 3 and a number.*

Study Tip**Leading coefficients**

Use the same steps to solve a multi-step equation if the leading coefficient is a fraction or an integer.

Solve Consecutive Integer Problems Consecutive integers are integers in counting order, such as 7, 8, and 9. Beginning with an even integer and counting by two will result in *consecutive even integers*. Beginning with an odd integer and counting by two will result in *consecutive odd integers*.

Consecutive Even Integers

$$-4, -2, 0, 2, 4$$

Consecutive Odd Integers

$$-3, -1, 1, 3, 5$$

The study of numbers and the relationships between them is called **number theory**.

EXAMPLE Solve a Consecutive Integer Problem

- 4 NUMBER THEORY** Write an equation for the problem below. Then solve the equation and answer the problem.

Find three consecutive even integers whose sum is -42 .

Let n = the least even integer.

Then $n + 2$ = the next greater even integer, and $n + 4$ = the greatest of the three even integers.

Words The sum of three consecutive even integers is -42 .

Equation $n + (n + 2) + (n + 4) = -42$

$$n + (n + 2) + (n + 4) = -42 \quad \text{Original equation}$$

$$3n + 6 = -42 \quad \text{Simplify.}$$

$$3n + 6 - 6 = -42 - 6 \quad \text{Subtract 6 from each side.}$$

$$3n = -48 \quad \text{Simplify.}$$

$$\frac{3n}{3} = \frac{-48}{3} \quad \text{Divide each side by 3.}$$

$$n = -16 \quad \text{Simplify.}$$

$$n + 2 = -16 + 2 \text{ or } -14 \quad n + 4 = -16 + 4 \text{ or } -12$$

The consecutive even integers are -16 , -14 , and -12 .

CHECK -16 , -14 , and -12 are consecutive even integers.

$$-16 + (-14) + (-12) = -42 \quad \checkmark$$

CHECK Your Progress

- 4.** Write an equation for the following problem. Then solve the equation and answer the problem.
Find three consecutive integers whose sum is 21.

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Study Tip

Representing Consecutive Integers

You can use the same expressions to represent either consecutive even integers or consecutive odd integers. It is the value of n (odd or even) that differs between the two expressions.

CHECK Your Understanding

Examples 1–2
(pp. 92–93)

Solve each equation. Check your solution.

1. $4g - 2 = -6$

2. $18 = 5p + 3$

3. $9 = 1 + \frac{m}{7}$

4. $\frac{3}{2}a - 8 = 11$

5. $20 = \frac{n-3}{8}$

6. $\frac{b+4}{-2} = -17$

Example 3
(p. 93)

7. **NUMBER THEORY** Twelve decreased by twice a number equals -34 . Write an equation for this situation and then find the number.

8. **WORLD CULTURES** The English alphabet contains 2 more than twice as many letters as the Hawaiian alphabet. How many letters are there in the Hawaiian alphabet?

Example 4
(p. 94)

Write an equation and solve each problem.

9. Find three consecutive integers with a sum of 42.

10. Find three consecutive even integers with a sum of -12 .

Exercises

HOMEWORK HELP	
For Exercises	See Examples
11–22	1–2
23, 24	3
25–28	4

Solve each equation. Check your solution.

11. $5n + 6 = -4$

12. $-11 = 7 + 3c$

13. $15 = 4a - 5$

14. $7g - 14 = -63$

15. $\frac{a}{7} - 3 = -2$

16. $\frac{c}{-3} + 5 = 7$

17. $9 + \frac{y}{5} = 6$

18. $\frac{t}{8} - 6 = -12$

19. $\frac{r+1}{3} = 8$

20. $11 = \frac{5+m}{-2}$

21. $14 = \frac{d-6}{2}$

22. $\frac{17-s}{4} = -10$

Write an equation and solve each problem.

23. Six less than two thirds of a number is negative ten. Find the number.

24. Twenty-nine is thirteen added to four times a number. What is the number?

25. Find three consecutive odd integers with a sum of 51.

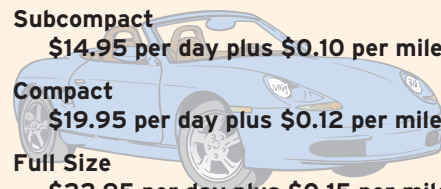
26. Find three consecutive even integers with a sum of -30 .

27. Find four consecutive integers with a sum of 94.

28. Find four consecutive integers with a sum of 26.

29. **ANALYZE TABLES** Adele Jones is on a business trip and plans to rent a subcompact car from Speedy Rent-A-Car. Her company has given her a budget of \$60 per day for car rental. What is the maximum distance Ms. Jones can drive in one day and still stay within her budget?

Speedy Rent-A-Car Price List	
Subcompact	\$14.95 per day plus \$0.10 per mile
Compact	\$19.95 per day plus \$0.12 per mile
Full Size	\$22.95 per day plus \$0.15 per mile





Real-World Link

Many mountain climbers experience altitude sickness caused by a decrease in oxygen. Climbers can acclimate themselves to these higher altitudes by camping for one or two weeks at various altitudes as they ascend the mountain.

Source: *Shape*

Write an equation and solve each problem.

30. NUMBER THEORY Maggie was thinking of a number. If she multiplied the number by 3, subtracted 8, added 2 times the original number, added -4 , and then subtracted the original number, the result was 48. Write an equation that the number satisfies. Then solve the equation.

31. MOUNTAIN CLIMBING A general rule for those climbing more than 7000 feet above sea level is to allow a total of $\left(\frac{a - 7000}{2000} + 2\right)$ weeks of camping during the ascension. In this expression, a represents the altitude in feet. If a group of mountain climbers have allowed for 9 weeks of camping, how high can they climb without worrying about altitude sickness?

Solve each equation. Check your solution.

32. $-3d - 1.2 = 0.9$

33. $-2.5r - 32.7 = 74.1$

34. $0.2n + 3 = 8.6$

35. $-9 - \frac{p}{4} = 5$

36. $\frac{-3j - (-4)}{-6} = 12$

37. $3.5x + 5 - 1.5x = 8$

38. If $3a - 9 = 6$, what is the value of $5a + 2$?

39. If $2x + 1 = 5$, what is the value of $3x - 4$?

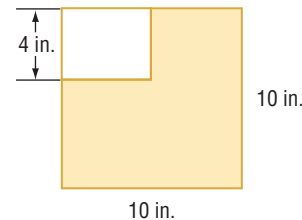
SHOE SIZE For Exercises 40 and 41, use the following information.

If ℓ represents the length of a person's foot in inches, the expression $2\ell - 12$ can be used to estimate his or her shoe size.

40. What is the approximate length of the foot of a person who wears size 8?

41. Measure your foot and use the expression to determine your shoe size. How does this number compare to the size of shoe you are wearing?

42. GEOMETRY A rectangle is cut from the corner of a 10-inch by 10-inch piece of paper. The area of the remaining piece of paper is $\frac{4}{5}$ of the area of the original piece of paper. If the width of the rectangle removed from the paper is 4 inches, what is the length of the rectangle?



43. OPEN ENDED Write a problem that can be modeled by the equation $2x + 40 = 60$. Then solve the equation and explain the solution in the context of the problem.

44. REASONING Describe the steps used to solve $\frac{w + 3}{5} - 4 = 6$.

45. CHALLENGE Determine whether the following statement is *sometimes*, *always*, or *never* true. Explain your reasoning.
The sum of two consecutive even numbers equals the sum of two consecutive odd numbers.

46. Writing in Math Use the information about alligators on page 92 to explain how equations can be used to estimate the age of an animal. Include an explanation of how to solve the equation representing the alligator and an estimate of the age of the alligator.

EXTRA PRACTICE
See pages 721, 745.
Math online
Self-Check Quiz at algebra1.com

H.O.T. Problems

47. **REVIEW** A hang glider 25 meters above the ground started to descend at a constant rate of 2 meters per second. Which equation could be used to determine h , the hang glider's height after t seconds of descent?

- A $h = 25t + 2$
- B $h = -25t + 2$
- C $h = 2t + 25$
- D $h = -2t + 25$

48. **REVIEW** Two rectangular walls each with a length of 12 feet and a width of 23 feet need to be painted. It costs \$0.08 per square foot for paint. How much money will it cost to paint the two walls?

- F \$22.08
- G \$23.04
- H \$34.50
- J \$44.16

49. Maddie works at Game Exchange. They are having a sale on video games and DVDs.

Item	Price	Special
Video Games	\$20	Buy 2 Get 1 Free
DVDs	\$15	Buy 1 Get 1 Free

She purchases four video games and uses her employee discount of 15%. If sales tax is 7.25%, how much does she spend on the games?

- A \$54.70
- B \$55.35
- C \$60
- D \$64.35

Spiral Review

Solve each equation. Check your solution. (Lesson 2-3)

50. $-7t = 91$

51. $\frac{r}{15} = -8$

52. $26 = \frac{2}{3}b$

53. **TRANSPORTATION** In 2005, there were 9 more models of sport utility vehicles than there were in 2000. Write and solve an equation to find how many models of sport utility vehicles there were in 2000. (Lesson 2-2)

Sport Utility Vehicles	
Year	Number of Models
2000	m
2005	56

Use the Distributive Property to rewrite each expression. Then find the product. (Lesson 1-5)

54. $17 \cdot 9$

55. $13(101)$

56. $18 \cdot 2\frac{1}{9}$

Write an algebraic expression for each verbal expression. (Lesson 1-1)

57. the product of 5 and m plus half of n

58. the sum of 3 times a and the square of b

GET READY for the Next Lesson

PREREQUISITE SKILL Simplify each expression. (Lesson 1-5)

59. $5d - 2d$

60. $11m - 5m$

61. $8t + 6t$

62. $7g - 15g$

63. $-9f + 6f$

64. $-3m + (-7m)$

Solving Equations with the Variable on Each Side

Main Ideas

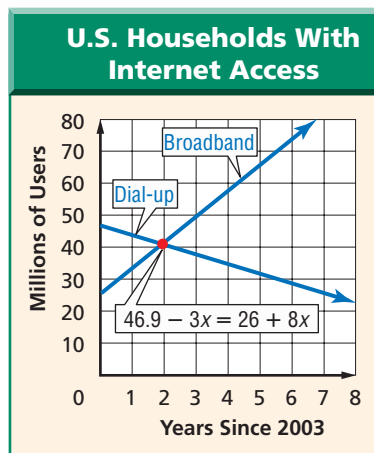
- Solve equations with the variable on each side.
- Solve equations involving grouping symbols.

New Vocabulary

identity

GET READY for the Lesson

In 2003, about 46.9 million U.S. households had dial-up Internet service and about 26 million had broadband service. During the next five years, it was projected that the number of dial-up users would decrease an average of 3 million per year and the number of broadband users would increase an average of 8 million per year. The following expressions represent the number of dial-up and broadband Internet users x years after 2003.



Source: Strategy Analytics

Dial-Up Internet Users: $46.9 - 3x$

Broadband Internet Users: $26 + 8x$

The equation $46.9 - 3x = 26 + 8x$ represents the time at which the number of dial-up and broadband Internet users are equal. Notice that this equation has the variable x on each side.

Variables On Each Side To solve equations with variables on each side, first use the Addition or Subtraction Property of Equality to write an equivalent equation that has all of the variables on one side.

EXAMPLE Solve an Equation with Variables on Each Side

- 1 Solve $-2 + 10p = 8p - 1$. Check your solution.

$$-2 + 10p = 8p - 1 \quad \text{Original equation}$$

$$-2 + 10p - 8p = 8p - 1 - 8p \quad \text{Subtract } 8p \text{ from each side.}$$

$$-2 + 2p = -1 \quad \text{Simplify.}$$

$$-2 + 2p + 2 = -1 + 2 \quad \text{Add 2 to each side.}$$

$$2p = 1 \quad \text{Simplify.}$$

$$\frac{2p}{2} = \frac{1}{2} \quad \text{Divide each side by 2.}$$

$$p = \frac{1}{2} \text{ or } 0.5 \quad \text{Simplify.}$$

The solution is $\frac{1}{2}$ or 0.5. Check by substituting into the original equation.

CHECK Your Progress Solve each equation. Check your solution.

1A. $3w + 2 = 7w$

1B. $\frac{s}{2} + 1 = \frac{1}{4}s - 6$

Study Tip

Solving Equations

The equation in Example 1 can also be solved by first subtracting $10p$ from each side.

Review Vocabulary

Distributive Property:

For any numbers a , b , and c , $a(b + c) = ab + ac$ and $a(b - c) = ab - ac$. (Lesson 1-5)

Grouping Symbols When solving equations that contain grouping symbols, first use the Distributive Property to remove the grouping symbols.

EXAMPLE Solve an Equation with Grouping Symbols

2 Solve $4(2r - 8) = \frac{1}{7}(49r + 70)$. Check your solution.

$$4(2r - 8) = \frac{1}{7}(49r + 70) \quad \text{Original equation}$$

$$8r - 32 = 7r + 10 \quad \text{Distributive Property}$$

$$8r - 32 - 7r = 7r + 10 - 7r \quad \text{Subtract } 7r \text{ from each side.}$$

$$r - 32 = 10 \quad \text{Simplify.}$$

$$r - 32 + 32 = 10 + 32 \quad \text{Add 32 to each side.}$$

$$r = 42 \quad \text{Simplify.}$$

CHECK $4(2r - 8) = \frac{1}{7}(49r + 70) \quad \text{Original equation}$

$$4[2(42) - 8] \stackrel{?}{=} \frac{1}{7}[49(42) + 70] \quad \text{Substitute 42 for } r.$$

$$4(84 - 8) \stackrel{?}{=} \frac{1}{7}(2058 + 70) \quad \text{Multiply.}$$

$$4(76) \stackrel{?}{=} \frac{1}{7}(2128) \quad \text{Add and subtract.}$$

$$304 = 304 \quad \checkmark$$



Check Your Progress

Solve each equation. Check your solution.

2A. $8s - 10 = 3(6 - 2s)$

2B. $7(n - 1) = -2(3 + n)$

Some equations may have no solution. That is, there is no value of the variable that will result in a true equation. Other equations are true for all values of the variables. An equation like this is called an **identity**.

EXAMPLE No Solutions or Identity

3 Solve each equation.

a. $2m + 5 = 5(m - 7) - 3m$

$$2m + 5 = 5(m - 7) - 3m \quad \text{Original equation}$$

$$2m + 5 = 5m - 35 - 3m \quad \text{Distributive Property}$$

$$2m + 5 = 2m - 35 \quad \text{Simplify.}$$

$$2m + 5 - 2m = 2m - 35 - 2m \quad \text{Subtract } 2m \text{ from each side.}$$

$$5 = -35 \quad \text{This statement is false.}$$

Since $5 = -35$ is a false statement, this equation has no solution.

b. $3(r + 1) - 5 = 3r - 2$

$$3(r + 1) - 5 = 3r - 2 \quad \text{Original equation}$$

$$3r + 3 - 5 = 3r - 2 \quad \text{Distributive Property}$$

$$3r - 2 = 3r - 2 \quad \text{Reflexive Property of Equality}$$

Since the expressions on each side of the equation are the same, this equation is an identity. It is true for all values of r .



CHECK Your Progress

Solve each equation.

3A. $7x + 5(x - 1) = -5 + 12x$ **3B.** $6(y - 5) = 2(10 + 3y)$

CONCEPT SUMMARY

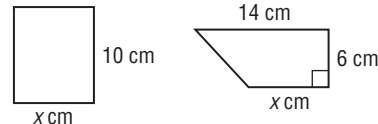
Steps for Solving Equations

- Step 1** Simplify the expressions on each side. Use the Distributive Property as needed.
- Step 2** Use the Addition and/or Subtraction Properties of Equality to get the variables on one side and the numbers without variables on the other side. Simplify.
- Step 3** Use the Multiplication or Division Property of Equality to solve.

STANDARDIZED TEST EXAMPLE

- 4** Find the value of x so that the figures have the same area.

- A 5
B 6
C 7
D 8



Read the Test Item

The equation $10x = \frac{1}{2}(14 + x)(6)$ represents this situation.

Solve the Test Item

You can solve the equation or substitute each value into the equation and see if it makes the equation true. We will solve by substitution.

A $10x = \frac{1}{2}(14 + x)(6)$

B $10x = \frac{1}{2}(14 + x)(6)$

$10(5) \stackrel{?}{=} \frac{1}{2}(14 + 5)(6)$

$10(6) \stackrel{?}{=} \frac{1}{2}(14 + 6)(6)$

$50 \stackrel{?}{=} \frac{1}{2}(19)(6)$

$60 \stackrel{?}{=} \frac{1}{2}(20)(6)$

$50 \neq 57$

$60 = 60 \checkmark$

Since the value 6 results in a true statement, you do not need to check 7 and 8. The answer is B.

CHECK Your Progress

- 4. SHOPPING** A purse is on sale for one fourth off the original price, or \$12 off. What was the original price of the purse?

F \$12

G \$36

H \$48

J \$60

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Test-Taking Tip

Substitution If you are asked to solve a complicated equation, it sometimes takes less time to check each possible answer rather than to actually solve the equation.

CHECK Your Understanding

Examples 1 and 2
(pp. 98–99)

Solve each equation. Check your solution.

1. $20c + 5 = 5c + 65$

2. $\frac{3}{8} - \frac{1}{4}t = \frac{1}{2}t - \frac{3}{4}$

3. $3(a - 5) = -6$

4. $6 = 3 + 5(d - 2)$

5. **NUMBER THEORY** Four times the greater of two consecutive integers is 1 more than five times the lesser number. Find the integers.

Example 3
(pp. 99–100)

Solve each equation. Check your solution.

6. $5 + 2(n + 1) = 2n$

7. $7 - 3r = r - 4(2 + r)$

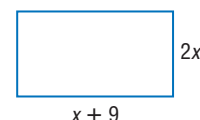
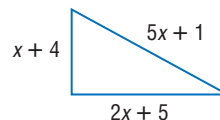
8. $14v + 6 = 2(5 + 7v) - 4$

9. $5h - 7 = 5(h - 2) + 3$

Example 4
(p. 100)

10. **STANDARDIZED TEST EXAMPLE**

Find the value of x so that the figures have the same perimeter.



- A 4 C 6
B 5 D 7

Exercises

HOMEWORK HELP	
For Exercises	See Examples
11–18, 25–28	1
19–24, 29, 30	2
31–34	3
43, 44	4

Solve each equation. Check your solution.

11. $3k - 5 = 7k - 21$

12. $5t - 9 = -3t + 7$

13. $8s + 9 = 7s + 6$

14. $3 - 4q = 10q + 10$

15. $\frac{3}{4}n + 16 = 2 - \frac{1}{8}n$

16. $\frac{1}{4} - \frac{2}{3}y = \frac{3}{4} - \frac{1}{3}y$

17. $\frac{c + 1}{8} = \frac{c}{4}$

18. $\frac{3m - 2}{5} = \frac{7}{10}$

19. $8 = 4(3c + 5)$

20. $7(m - 3) = 7$

21. $6(r + 2) - 4 = -10$

22. $5 - \frac{1}{2}(x - 6) = 4$

23. $4(2a - 1) = -10(a - 5)$

24. $2(w - 3) + 5 = 3(w - 1)$

25. One half of a number increased by 16 is four less than two thirds of the number. Find the number.

26. The sum of one half of a number and 6 equals one third of the number. What is the number?

27. Two less than one third of a number equals 3 more than one fourth of the number. Find the number.

28. Two times a number plus 6 is three less than one fifth of the number. What is the number?

29. **NUMBER THEORY** Twice the greater of two consecutive odd integers is 13 less than three times the lesser number. Find the integers.

30. **NUMBER THEORY** Three times the greatest of three consecutive even integers exceeds twice the least by 38. What are the integers?

Solve each equation. Check your solution.

31. $4(f - 2) = 4f$

32. $\frac{3}{2}y - y = 4 + \frac{1}{2}y$

33. $3(1 + d) - 5 = 3d - 2$

34. $-3(2n - 5) = 0.5(-12n + 30)$

EXTRA PRACTICE

See pages 721, 745.

Math online

Self-Check Quiz at algebra1.com



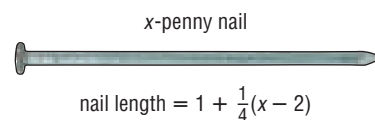
Real-World Link

The normal values for resting heart rates in beats per minute:

- infants: 100–160
- children 1–10 years: 70–120
- children over 10 and adults: 60–80
- well-trained athletes: 40–60

Source: MedlinePlus.com

- 35. HEALTH** When exercising, a person's pulse rate should not exceed a certain limit. This maximum rate is represented by the expression $0.8(220 - a)$, where a is age in years. Find the age of a person whose maximum pulse is 152.
- 36. HARDWARE** Traditionally, nails are given names such as 2-penny, 3-penny, and so on. These names describe the lengths of the nails. Use the diagram to find the name of a nail that is $2\frac{1}{2}$ inches long.



Source: World Book Encyclopedia

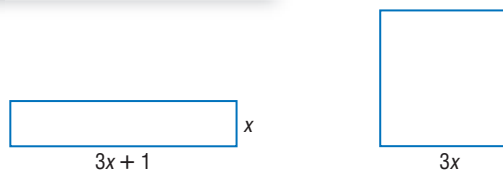
Solve each equation. Check your solution.

- 37.** $\frac{1}{4}(7 + 3g) = -\frac{8}{8}$ **38.** $\frac{1}{6}(a - 4) = \frac{1}{3}(2a + 4)$
- 39.** $1.03p - 4 = -2.15p + 8.72$ **40.** $18 - 3.8t = 7.36 - 1.9t$
- 41.** $5.4w + 8.2 = 9.8w - 2.8$ **42.** $2[s + 3(s - 1)] = 18$

- 43. ANALYZE TABLES** The table shows the households that had Brand A and Brand B of personal computers in a recent year and the average growth rates. How long will it take for the two brands to be in the same number of households?

Brand of Computer	Millions of Households with Computer	Growth Rate (million households per year)
A	4.9	0.275
B	2.5	0.7

- 44. GEOMETRY** The rectangle and square shown at right have the same perimeter. Find the dimensions of each figure.



H.O.T. Problems

- 45. CHALLENGE** Write an equation that has one or more grouping symbols, the variable on each side of the equals sign, and a solution of -2 . Discuss the steps you used to write the equation.
- 46. OPEN ENDED** Find a counterexample to the statement *All equations have a solution*. Explain your reasoning.
- 47. REASONING** Determine whether each solution is correct. If the solution is not correct, describe the error and give the correct solution.
- a.** $2(g + 5) = 22$
 $2g + 5 = 22$
 $2g + 5 - 5 = 22 - 5$
 $2g = 17$
 $g = 8.5$
- b.** $5d = 2d - 18$
 $5d - 2d = 2d - 18 - 2d$
 $3d = -18$
 $d = -6$
- 48. Writing in Math** Use the information about Internet users on page 98 to explain how an equation can be used to determine when two populations are equal. Include the steps for solving the equation and the year when the number of dial-up Internet users will equal the number of broadband Internet users according to the model. Explain why this method can be used to predict events.

STANDARDIZED TEST PRACTICE

49. Which equation represents the second step of the solution process?

Step 1 $4(2x + 7) - 6 = 3x$

Step 2

Step 3 $5x + 28 - 6 = 0$

Step 4 $5x = -22$

Step 5 $x = -4.4$

- A $4(2x - 6) + 7 = 3x$
 B $4(2x + 1) = 3x$
 C $8x + 7 - 6 = 3x$
 D $8x + 28 - 6 = 3x$

50. **REVIEW** Tanya sells cosmetics door-to-door. She makes \$5 an hour and 15% commission on the total dollar value on whatever she sells. If Tanya's commission is increased to 17%, how much money will she make if she sells \$300 dollars worth of product and works 30 hours?

- F \$201
 G \$226
 H \$255
 J \$283

Spiral Review

Solve each equation. Check your solution. (Lesson 2-4)

51. $\frac{2}{9}v - 6 = 14$

52. $\frac{x-3}{7} = -2$

53. $5 - 9w = 23$

54. **HEALTH** A female burns 4.5 Calories per minute pushing a lawn mower. Write an equation to represent the number of Calories C burned if Ebony pushes a lawn mower for m minutes. How long will it take Ebony to burn 150 Calories mowing the lawn? (Lesson 2-3)

55. A teacher took a survey of his students to find out how many televisions they had in their homes. The results are shown in the table. Write a set of ordered pairs representing the data in the table and draw a graph showing the relationship between students and the number of televisions in their homes. (Lesson 1-9)

Televisions	Number of Students
1	8
2	12
3	4
4	2

Write an algebraic expression for each verbal expression.

Then simplify, indicating the properties used. (Lesson 1-6)

56. twice the product of p and q increased by the product of p and q

57. three times the square of x plus the sum of x squared and seven times x

Find the solution set for each inequality, given the replacement set. (Lesson 1-3)

58. $3x + 2 > 2$; $\{0, 1, 2\}$

59. $2y^2 - 1 > 0$; $\{1, 3, 5\}$

Evaluate each expression when $a = 5$, $b = 8$, and $c = 1$. (Lesson 1-2)

60. $5(b - a)$

61. $\frac{3a^2}{b + c}$

62. $(a + 2b) - c$

GET READY for the Next Lesson

PREREQUISITE SKILL Simplify each fraction. (Pages 694–695)

63. $\frac{28}{49}$

64. $\frac{36}{60}$

65. $\frac{8}{120}$

66. $\frac{108}{9}$

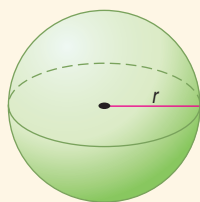
Translate each equation into a verbal sentence.

(Lesson 2-1)

- $2 = x - 9$
- $8 + 7t = 22$
- $a = 1 + \frac{3}{5}b$
- $n^2 - 6 = 5n$

GEOMETRY For Exercises 5 and 6, use the following information.

The surface area S of a sphere equals four times π times the square of the radius r . (Lesson 2-1)



- Write the formula for the surface area of a sphere.
- What is the surface area of a sphere if the radius is 7 centimeters?

Solve each equation. Check your solution.

(Lesson 2-2)

- $d + 18 = -27$
- $m - 77 = -61$
- $-12 + a = -36$
- $t - (-16) = 9$

PALEONTOLOGY For Exercises 11 and 12, use the following information. (Lesson 2-2)

The skeleton of a juvenile dinosaur was recently found in Illinois. If the dinosaur had been fully grown, it would have been $4\frac{1}{2}$ feet taller.

- Write an equation to find the height x of the juvenile dinosaur if a fully grown dinosaur is 12 feet tall.
- Solve the equation to find the height of the dinosaur.

Solve each equation. Check your solution. (Lesson 2-3)

- $\frac{2}{3}p = 18$
- $-17y = 391$
- $5x = -45$
- $-\frac{2}{5}d = -10$

- TECHNOLOGY** In a phone survey of teens who have Internet access, three fourths, or 825 of those surveyed, said they use instant messaging. How many teens were surveyed? (Lesson 2-3)

Solve each equation. Check your solution. (Lesson 2-4)

- $-3x - 7 = 18$
- $5 = \frac{m-5}{4}$
- $4h + 5 = 11$
- $5d - 6 = 3d + 9$

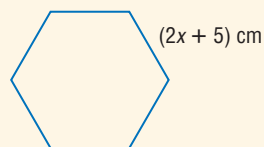
- MULTIPLE CHOICE** Coach Bronson recorded the heights of 130 freshmen and 95 seniors. Which expression could be used to find the average height of these freshmen and seniors? (Lesson 2-4)

Students	Average Height
freshmen	5 feet 5 inches
seniors	5 feet 8 inches

- $\frac{65(130) + 68(95)}{225}$
- $[65(130) + 68(95)] \cdot 225$
- $65(130) + 68(95) \cdot \frac{1}{225}$
- $\frac{68(130) + 65(95)}{225}$

Solve each equation. Then check your solution. (Lesson 2-5)

- $7 + 2(w + 1) = 2w + 9$
- $-8(4 + 9r) = 7(-2 - 11r)$
- NUMBER THEORY** Two thirds of a number equals 3 increased by one half of the number. Find the number. (Lesson 2-5)
- MULTIPLE CHOICE** The sides of the hexagon are the same length. If the perimeter of the hexagon is $18x + 9$ square centimeters, what is the length of each side? (Lesson 2-5)



- 3.5 cm
- 7 cm
- 12 cm
- 33.5 cm

Main Ideas

- Determine whether two ratios form a proportion.
- Solve proportions.

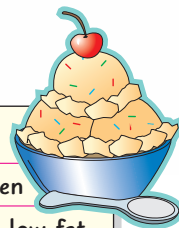
New Vocabulary

ratio
proportion
extremes
means
rate

GET READY for the Lesson

The ingredients in the recipe will make 4 servings of honey frozen yogurt. Keri can use ratios and equations to find the amount of each ingredient needed to make enough yogurt for her club meeting.

Honey Frozen Yogurt	
2 cups 2% milk	2 eggs, beaten
$\frac{3}{4}$ cup honey	2 cups plain low-fat yogurt
1 dash salt	1 tablespoon vanilla



Ratios and Proportions A **ratio** is a comparison of two numbers by division. The ratio of x to y can be expressed in the following ways.

$$x \text{ to } y \quad x:y \quad \frac{x}{y}$$

The recipe above states that for 4 servings you need 2 cups of milk.

The ratio of servings to milk may be written as 4 to 2, 4:2, or $\frac{4}{2}$.
In simplest form, the ratio is written as 2 to 1, 2:1, or $\frac{2}{1}$.

Suppose you wanted to double the recipe to have 8 servings. The amount of milk required would be 4 cups. The ratio of servings to milk is $\frac{8}{4}$. When

this ratio is simplified, the ratio is $\frac{2}{1}$. Notice that this ratio is equal to the original ratio. An equation stating that two ratios are equal is called a **proportion**.

So, we can state that $\frac{4}{2} = \frac{8}{4}$ is a proportion.

$$\begin{array}{l} \left[\begin{array}{c} \div 2 \\ \downarrow \\ \frac{4}{2} = \frac{2}{1} \\ \uparrow \\ \div 2 \end{array} \right] \quad \left[\begin{array}{c} \div 4 \\ \downarrow \\ \frac{8}{4} = \frac{2}{1} \\ \uparrow \\ \div 4 \end{array} \right] \end{array}$$

Study Tip**Whole Numbers**

A ratio that is equivalent to a whole number is written with a denominator of 1.

EXAMPLE Determine Whether Ratios Form a Proportion

- 1** Determine whether the ratios $\frac{4}{5}$ and $\frac{24}{30}$ form a proportion.

$$\begin{array}{l} \left[\begin{array}{c} \div 1 \\ \downarrow \\ \frac{4}{5} = \frac{4}{5} \\ \uparrow \\ \div 1 \end{array} \right] \quad \left[\begin{array}{c} \div 6 \\ \downarrow \\ \frac{24}{30} = \frac{4}{5} \\ \uparrow \\ \div 6 \end{array} \right] \end{array}$$

The ratios are equal. Therefore, they form a proportion.

CHECK Your Progress

Determine whether each pair of ratios forms a proportion. Write *yes* or *no*.

1A. $\frac{6}{10}$, $\frac{2}{5}$

1B. $\frac{1}{6}$, $\frac{5}{30}$

There are special names for the terms in a proportion.

0.8 and 0.7 are called the **means**. They are the middle terms of the proportion.

$$0.4 : 0.8 = 0.7 : 1.4$$

0.4 and 1.4 are called the **extremes**. They are the first and last terms of the proportion.

 **Vocabulary Link**

Extremes

Everyday Use

something at one end or the other of a range, as in extremes of heat and cold

Math Use the first and last terms of a proportion

KEY CONCEPT

Means–Extremes Property of Proportion

Words In a proportion, the product of the extremes is equal to the product of the means.

Symbols If $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$.

Examples Since $\frac{2}{4} = \frac{1}{2}$, $2(2) = 4(1)$ or $4 = 4$.

Another way to determine whether two ratios form a proportion is to use cross products. If the cross products are equal, then the ratios form a proportion.

EXAMPLE Use Cross Products

Study Tip

Cross Products

When you find cross products, you are said to be *cross multiplying*.

2 Use cross products to determine whether each pair of ratios forms a proportion.

a. $\frac{0.4}{0.8}, \frac{0.7}{1.4}$

$$\frac{0.4}{0.8} \stackrel{?}{=} \frac{0.7}{1.4} \quad \text{Write the equation.}$$

$$0.4(1.4) \stackrel{?}{=} 0.8(0.7) \quad \text{Find the cross products.}$$

$$0.56 = 0.56 \quad \text{Simplify.}$$

The cross products are equal, so the ratios form a proportion.

b. $\frac{6}{8}, \frac{24}{28}$

$$\frac{6}{8} \stackrel{?}{=} \frac{24}{28} \quad \text{Write the equation.}$$

$$6(28) \stackrel{?}{=} 8(24) \quad \text{Find the cross products.}$$

$$168 \neq 192 \quad \text{Simplify.}$$

The cross products are not equal, so the ratios do not form a proportion.

 **CHECK Your Progress**

2A. $\frac{0.2}{1.8}, \frac{1}{0.9}$

2B. $\frac{15}{36}, \frac{35}{42}$

Solve Proportions To solve proportions that involve a variable, use cross products and the techniques used to solve other equations.

EXAMPLE Solve a Proportion

3 Solve the proportion $\frac{n}{15} = \frac{24}{16}$.

$$\frac{n}{15} = \frac{24}{16} \quad \text{Original equation}$$

$$16(n) = 15(24) \quad \text{Find the cross products.}$$

$$16n = 360 \quad \text{Simplify.}$$

$$\frac{16n}{16} = \frac{360}{16} \quad \text{Divide each side by 16.}$$

$$n = 22.5 \quad \text{Simplify.}$$

Concepts in Motion
Interactive Lab algebra1.com

CHECK Your Progress

Solve each proportion. If necessary, round to the nearest hundredth.

3A. $\frac{r}{8} = \frac{25}{40}$

3B. $\frac{3.2}{4} = \frac{2.6}{n}$

The ratio of two measurements having different units of measure is called a **rate**. For example, a price of \$1.99 per dozen eggs, a speed of 55 miles per hour, and a salary of \$30,000 per year are all rates. Proportions are often used to solve problems involving rates.

Study Tip

To ensure that the proportion is set up correctly, you should label the units. For example,

$$\frac{15 \text{ mi}}{2 \text{ hrs}} = \frac{25 \text{ mi}}{x \text{ hrs}}$$

Real-World EXAMPLE

4 **BICYCLING** Trent goes on a 30-mile bike ride every Saturday. He rides the distance in 4 hours. At this rate, how far can he ride in 6 hours?

Explore Let m represent the number of miles Trent can ride in 6 hours.

Plan Write a proportion for the problem using rates.

$$\begin{array}{l} \text{miles} \rightarrow \frac{30}{4} = \frac{m}{6} \leftarrow \text{miles} \\ \text{hours} \rightarrow \frac{4}{4} = \frac{6}{6} \leftarrow \text{hours} \end{array}$$

Solve Estimate: If he rides 30 miles in 4 hours, then he would ride 60 miles in 8 hours. So, in 6 hours, he would ride between 30 and 60 miles.

$$\frac{30}{4} = \frac{m}{6} \quad \text{Original proportion}$$

$$30(6) = 4(m) \quad \text{Find the cross products.}$$

$$180 = 4m \quad \text{Simplify.}$$

$$\frac{180}{4} = \frac{4m}{4} \quad \text{Divide each side by 4.}$$

$$45 = m \quad \text{Simplify.}$$

Check Check the reasonableness of the solution. If Trent rides 30 miles in 4 hours, he rides 7.5 miles in 1 hour. So, in 6 hours, Trent can ride 6×7.5 or 45 miles. The answer is correct.



CHECK Your Progress

4. **EXERCISE** It takes 7 minutes for Isabella to walk around the track twice. At this rate, how many times can she walk around the track in a half hour?

Online Personal Tutor at algebra1.com

A ratio or rate called a **scale** compares the size of a model to the actual size of the object using a proportion. Maps and blueprints are two common scale drawings.

Real-World EXAMPLE

5. **CRATER LAKE** The scale of a map for Crater Lake National Park is 2 inches = 9 miles. The distance between Discovery Point and Phantom Ship Overlook on the map is about $1\frac{3}{4}$ inches. What is the distance d between these two places?

$$\begin{array}{l} \text{scale} \rightarrow \frac{2}{9} = \frac{1\frac{3}{4}}{d} \quad \leftarrow \text{scale} \\ \text{actual} \rightarrow \end{array}$$

$$2(d) = 9\left(1\frac{3}{4}\right) \quad \text{Find the cross products.}$$

$$2d = \frac{63}{4} \quad \text{Simplify.}$$

$$2d \div 2 = \frac{63}{4} \div 2 \quad \text{Divide each side by 2.}$$

$$d = \frac{63}{8} \text{ or } 7\frac{7}{8} \quad \text{Simplify.}$$

The actual distance is about $7\frac{7}{8}$ miles.

CHECK Your Progress

5. **AIRPLANES** On a model airplane, the scale is 5 centimeters = 2 meters. If the wingspan of the model is 28.5 centimeters, what is the wingspan of the actual airplane?



Real-World Link

Crater Lake is a volcanic crater in Oregon that was formed by an explosion 42 times the blast of Mount St. Helens.

Source: travel.excite.com

CONcepts
in MOTION

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CHECK Your Understanding

Examples 1, 2
(pp. 105–106)

Determine whether each pair of ratios forms a proportion. Write *yes* or *no*.

1. $\frac{4}{11}, \frac{12}{33}$

2. $\frac{16}{17}, \frac{8}{9}$

3. $\frac{2.1}{3.5}, \frac{0.5}{0.7}$

Example 3
(p. 107)

Solve each proportion. If necessary, round to the nearest hundredth.

4. $\frac{3}{4} = \frac{6}{x}$

5. $\frac{a}{45} = \frac{5}{15}$

6. $\frac{0.6}{1.1} = \frac{n}{8.47}$

Example 4
(p. 107)

7. **TRAVEL** The Lehmans' minivan requires 5 gallons of gasoline to travel 120 miles. How much gasoline will they need for a 350-mile trip?

Example 5
(p. 108)

8. **BLUEPRINTS** On a blueprint for a house, 2.5 inches equals 10 feet. If the length of a wall is 12 feet, how long is the wall on the blueprint?

Exercises

HOMEWORK HELP	
For Exercises	See Examples
9–14	1, 2
15–26	3
27, 28	4
29, 30	5

Determine whether each pair of ratios forms a proportion. Write yes or no.

9. $\frac{3}{2}, \frac{21}{14}$

10. $\frac{8}{9}, \frac{12}{18}$

11. $\frac{2.3}{3.4}, \frac{3.0}{3.6}$

12. $\frac{5}{2}, \frac{4}{1.6}$

13. $\frac{21.1}{14.4}, \frac{1.1}{1.2}$

14. $\frac{4.2}{5.6}, \frac{1.68}{2.24}$

Solve each proportion. If necessary, round to the nearest hundredth.

15. $\frac{4}{x} = \frac{2}{10}$

16. $\frac{1}{y} = \frac{3}{15}$

17. $\frac{6}{5} = \frac{x}{15}$

18. $\frac{20}{28} = \frac{n}{21}$

19. $\frac{6}{8} = \frac{7}{a}$

20. $\frac{16}{7} = \frac{9}{b}$

21. $\frac{w}{2} = \frac{4.5}{6.8}$

22. $\frac{t}{0.3} = \frac{1.7}{0.9}$

23. $\frac{2}{0.21} = \frac{8}{n}$

24. $\frac{2.4}{3.6} = \frac{s}{1.8}$

25. $\frac{1}{0.19} = \frac{12}{n}$

26. $\frac{7}{1.066} = \frac{z}{9.65}$

27. **WORK** Jun earns \$152 in 4 days. At that rate, how many days will it take him to earn \$532?

28. **DRIVING** Lanette drove 248 miles in 4 hours. At that rate, how long will it take her to drive an additional 93 miles?

29. **MODELS** A collector's model racecar is scaled so that 1 inch on the model equals $6\frac{1}{4}$ feet on the actual car. If the model is $\frac{2}{3}$ inch high, how high is the actual car?

30. **GEOGRAPHY** On a map of Illinois, the distance between Chicago and Algonquin is 3.2 centimeters. If 2 centimeters = 40 kilometers, what is the approximate distance between the two cities?

Solve each proportion. If necessary, round to the nearest hundredth.

31. $\frac{6}{14} = \frac{7}{x-3}$

32. $\frac{5}{3} = \frac{6}{x+2}$

33. $\frac{3-y}{4} = \frac{1}{9}$

34. **PETS** A research study shows that three out of every twenty pet owners bought their pets from breeders. Of the 122 animals cared for by a veterinarian, how many would you expect to have been bought from breeders?

ANALYZE TABLES For Exercises 35 and 36, use the table.

35. Write a ratio of the number of gold medals won to the total number of medals won for each country.

All-Time Summer Olympic Medal Standings, 1896–2004				
Country	Gold	Silver	Bronze	Total
United States	907	697	615	2219
USSR/UT/Russia	525	436	409	1370
Germany/E. Ger/W. Ger	388	408	434	1230
Great Britain	189	242	237	668
France	199	202	230	631
Italy	189	154	168	511
Sweden	140	157	179	476

Source: infoplease.com and athens2004.com

36. Do any two of the ratios you wrote for Exercise 35 form a proportion? If so, explain the real-world meaning of the proportion.

EXTRA PRACTICE
See pages 721, 745.

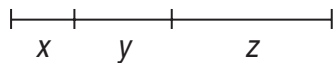
Math online
Self-Check Quiz at algebra1.com

H.O.T. Problems

- 37. OPEN ENDED** Find an example of ratios used in an advertisement. Analyze the ratios and describe how they are used to sell the product.
- 38. REASONING** Explain the difference between a ratio and a proportion.
- 39. CHALLENGE** Consider the proportion $a:b:c = 3:1:5$. What is the value of $\frac{2a + 3b}{4b + 3c}$? (Hint: Choose different values of a , b , and c for which the proportion is true and evaluate the expression.)
- 40. Writing in Math** Use the information about recipes on page 105 to explain how ratios are used in recipes. Include an explanation of how to use a proportion to determine how much honey is needed if you use 3 eggs, and a description of how to alter the recipe to get 5 servings.

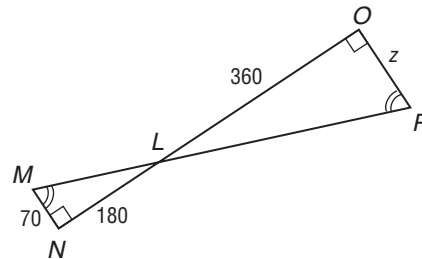
STANDARDIZED TEST PRACTICE

- 41.** In the figure, $x:y = 2:3$ and $y:z = 3:5$. If $x = 10$, find the value of z .



- A 15
- B 20
- C 25
- D 30

- 42. REVIEW** If $\triangle LMN$ is similar to $\triangle LPO$, what is the length of side z ?



- F 240 units
- G 140 units
- H 120 units
- J 70 units

Spiral Review

Solve each equation. Then check your solution. (Lessons 2-4 and 2-5)

- 43. $8y - 10 = -3y + 2$
- 44. $17 + 2n = 21 + 2n$
- 45. $-6(d - 3) = -d$
- 46. $5 - 9w = 23$
- 47. $\frac{m}{-5} + 6 = 31$
- 48. $\frac{z - 7}{5} = -3$

- 49.** Sketch a reasonable graph for the temperature in the following statement.
In August, Joel enters his house and turns on the air conditioner. (Lesson 1-9)

Evaluate each expression. (Lesson 1-2)

- 50. $12(5) - 6(4)$
- 51. $7(0.2 + 0.5) - 0.6$
- 52. $[6^2 - 3(2 + 5)] \div 5$

GET READY for the Next Lesson

PREREQUISITE SKILL Find each percent. (Pages 702–703)

- 53. Eighteen is what percent of 60?
- 54. What percent of 14 is 4.34?
- 55. Six is what percent of 15?
- 56. What percent of 2 is 8?

Main Ideas

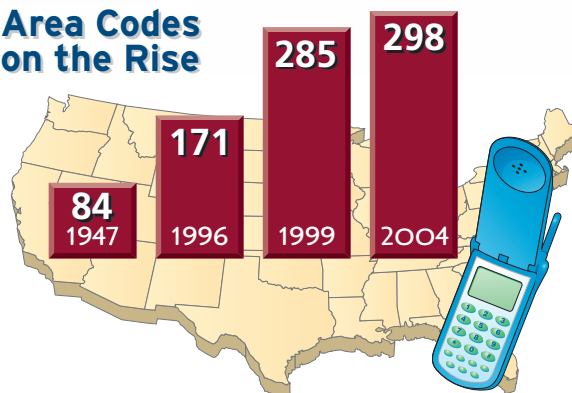
- Find percents of increase and decrease.
- Solve problems involving percents of change.

New Vocabulary

percent of change
percent of increase

GET READY for the Lesson

Phone companies began using area codes in 1947. The graph shows the number of area codes in use in different years. The growth in the number of area codes can be described by using a percent of change.

Area Codes on the Rise

Source: Associated Press

Percent of Change When an increase or decrease is expressed as a percent, the percent is called the **percent of change**. If the new number is greater than the original number, the percent of change is a **percent of increase**. If the new number is less than the original, the percent of change is a **percent of decrease**.

EXAMPLE Find Percent of Change

1 State whether each percent of change is a percent of increase or a percent of decrease. Then find each percent of change.

a. original: \$25
new: \$28

Since the new amount is greater than the original, this is a percent of increase. Find the *amount* of change.

$$28 - 25 = 3$$

Use the original number, 25, as the base.

$$\begin{aligned} \text{change} &\rightarrow \frac{3}{25} = \frac{r}{100} \\ \text{original amount} &\rightarrow \frac{3}{25} = \frac{r}{100} \\ 3(100) &= 25(r) \\ 300 &= 25r \\ \frac{300}{25} &= \frac{25r}{25} \\ 12 &= r \end{aligned}$$

The percent of increase is 12%.

b. original: 30
new: 12

This is a percent of decrease because the new amount is less than the original. Find the amount of change.

$$30 - 12 = 18$$

Use the original number, 30, as the base.

$$\begin{aligned} \text{change} &\rightarrow \frac{18}{30} = \frac{r}{100} \\ \text{original amount} &\rightarrow \frac{18}{30} = \frac{r}{100} \\ 18(100) &= 30(r) \\ 1800 &= 30r \\ \frac{1800}{30} &= \frac{30r}{30} \\ 60 &= r \end{aligned}$$

The percent of decrease is 60%.

Review Vocabulary

Percent Proportion
percent proportion:

$$\frac{\text{part}}{\text{base}} = \frac{\text{percent}}{100}$$

(page 702)



Real-World Link

On November 12, 1892, the Allegheny Athletic Association paid William "Pudge" Heffelfinger \$500 to play football. This game is considered the start of professional football.

Source: *World Book Encyclopedia*

CHECK Your Progress

1A. original: 66; new: 30

1B. original: 9.8; new: 12.1

Real-World EXAMPLE Find the Missing Value

- 2 **FOOTBALL** The National Football League's (NFL) fields are 120 yards long. The Canadian Football League's (CFL) fields are 25% longer. How long is a CFL field?

Let ℓ = the length of a CFL field. Since 25% is a percent of increase, an NFL field is shorter than a CFL field. Therefore, $\ell - 120$ represents the change.

$\frac{\text{change}}{\text{original amount}} \rightarrow \frac{\ell - 120}{120} = \frac{25}{100}$	Percent proportion
$(\ell - 120)(100) = 120(25)$	Find the cross products.
$100\ell - 12,000 = 3000$	Distributive Property
$100\ell - 12,000 + 12,000 = 3000 + 12,000$	Add 12,000 to each side.
$100\ell = 15,000$	Simplify.
$\frac{100\ell}{100} = \frac{15,000}{100}$	Divide each side by 100.
$\ell = 150$	Simplify.

The length of the field used by the CFL is 150 yards.

CHECK Your Progress

2. **TUITION** A recent percent of increase in tuition at Northwestern University was 5.4%. If the new cost is \$29,940 per year, find the original cost per year.

online Personal Tutor at algebra1.com

Solve Problems Sales tax is a tax that is added to the cost of the item. It is an example of a percent of increase.

Real-World EXAMPLE

- 3 **SALES TAX** A concert ticket costs \$45. If the sales tax is 6.25%, what is the total price of the ticket?

The tax is 6.25% of the price of the ticket.

$$6.25\% \text{ of } \$45 = 0.0625 \times 45 \quad 6.25\% = 0.0625$$

$$= 2.8125 \quad \text{Use a calculator.}$$

Round \$2.8125 to \$2.81. Add this amount to the original price.

$$\$45.00 + \$2.81 = \$47.81 \quad \text{The total price of the ticket is } \$47.81.$$

CHECK Your Progress

3. **TAXES** A new DVD costs \$24.99. If the sales tax is 7.25%, what is the total cost?

Discount is the amount by which the regular price of an item is reduced. It is an example of a percent of decrease.

Cross-Curricular Project

Math online A percent of increase or decrease can be used to describe trends in populations. Visit algebra1.com to continue work on your project.

Real-World EXAMPLE

- 4 DISCOUNT** A sweater is on sale for 35% off the original price. If the original price of the sweater is \$38, what is the discounted price?

The discount is 35% of the original price.

$$\begin{aligned} 35\% \text{ of } \$38 &= 0.35 \times 38 && 35\% = 0.35 \\ &= 13.30 && \text{Use a calculator.} \end{aligned}$$

Subtract \$13.30 from the original price.

$$\$38.00 - \$13.30 = \$24.70 \quad \text{The discounted price of the sweater is } \$24.70.$$

CHECK Your Progress

- 4. SALES** A picture frame originally priced at \$14.89 is on sale for 40% off. What is the discounted price?

CHECK Your Understanding

State whether each percent of change is a percent of increase or a percent of decrease. Then find each percent of change. Round to the nearest whole percent.

Example 1
(p. 111)

- original: 72
new: 36
- original: 45
new: 50
- original: 14 books
new: 16 books
- original: 150 T-shirts
new: 120 T-shirts

Example 2
(p. 112)

- 5. GEOGRAPHY** The distance from El Monte to Fresno is 211 miles. The distance from El Monte to Oakland is about 64.5% longer. To the nearest mile, what is the distance from El Monte to Oakland?

Find the total price of each item.

Example 3
(p. 112)

- software: \$39.50
sales tax: 6.5%
- compact disc: \$15.99
sales tax: 5.75%

Example 4
(p. 113)

Find the discounted price of each item.

- jeans: \$45.00
discount: 25%
- book: \$19.95
discount: 33%

Exercises

HOMEWORK	HELP
For Exercises	See Examples
10–17	1
18, 19	2
20–25	3
26–31	4

State whether each percent of change is a percent of increase or a percent of decrease. Then find each percent of change. Round to the nearest whole percent.

- original: 50
new: 70
- original: 25
new: 18
- original: 58
new: 152
- original: 13.7
new: 40.2
- original: 15.6 meters
new: 11.4 meters
- original: 132 students
new: 150 students
- original: \$85
new: \$90
- original: 40 hours
new: 32.5 hours





Real-World Career

Military

A military career can involve many different duties like working in a hospital, programming computers, or repairing helicopters. The military provides training and work in these fields and others.



For more information, go to algebra1.com.

18. EDUCATION According to the Census Bureau, the average income of a person with a high school diploma is \$27,351. The income of a person with a bachelor's degree is about 57% higher. What is the average income of a person with a bachelor's degree?

19. BOATS A 36-foot sailboat that is new costs 86% more than the same boat in good used condition. What is the cost of a new 36-foot sailboat?

Buying a Sailboat	
Type of Boat	Cost (\$)
Used	70,000
New	x

Find the total price of each item.

- 20. umbrella: \$14.00 tax: 5.5%
- 21. backpack: \$35.00 tax: 7%
- 22. candle: \$7.50 tax: 5.75%
- 23. hat: \$18.50 tax: 6.25%
- 24. clock radio: \$39.99 tax: 6.75%
- 25. sandals: \$29.99 tax: 5.75%

Find the discounted price of each item.

- 26. shirt: \$45.00 discount: 40%
- 27. socks: \$6.00 discount: 20%
- 28. watch: \$37.55 discount: 35%
- 29. gloves: \$24.25 discount: 33%
- 30. suit: \$175.95 discount: 45%
- 31. coat: \$79.99 discount: 30%

Find the final price of each item.

- 32. lamp: \$120.00 discount: 20% tax: 6%
- 33. dress: \$70.00 discount: 30% tax: 7%
- 34. camera: \$58.00 discount: 25% tax: 6.5%

35. MILITARY In 2000, the United States had 2.65 million active-duty military personnel. In 2004, there were 1.41 million active-duty military personnel. What was the percent of decrease? Round to the nearest whole percent.

36. THEME PARKS In 2003, 162.3 million people visited theme parks in the United States. In 2004, the number of visitors increased by about 4%. About how many people visited theme parks in the United States in 2004?

37. ANALYZE TABLES What are the projected 2050 populations for each country in the table? Which is projected to be the most populous?

Country	1997 Population (billions)	Projected Percent of Increase for 2050
China	1.24	22.6%
India	0.97	57.8%
United States	0.27	44.4%

Source: USA TODAY

38. RESEARCH Use the Internet or other reference to find the tuition for the last several years at a college of your choice. Find the percent of change for the tuition during these years. Predict the tuition for the year you plan to graduate from high school.

39. CHALLENGE Is the following equation *sometimes*, *always*, or *never* true? Explain your reasoning.

$$x\% \text{ of } y = y\% \text{ of } x$$

40. OPEN ENDED Give a counterexample to the statement *The percent of change must always be less than 100%*.

EXTRA PRACTICE
See pages 722, 745.
Math online
Self-Check Quiz at algebra1.com

H.O.T. Problems

41. **FIND THE ERROR** Laura and Cory are writing proportions to find the percent of change if the original number is 20 and the new number is 30. Who is correct? Explain your reasoning.

Laura

Amount of change: $30 - 20 = 10$

$$\frac{10}{20} = \frac{r}{100}$$

Cory

Amount of change: $30 - 20 = 10$

$$\frac{10}{30} = \frac{r}{100}$$

42. **Writing in Math** Use the data on page 111 to find the percent of increase in the number of area codes from 1999 to 2004. Explain why knowing a percent of change can be more informative than knowing how much the quantity changed.

STANDARDIZED TEST PRACTICE

43. The number of students at Franklin High School increased from 840 to 910 over a 5-year period. What was the percent of increase?

- A 8.3% C 18.5%
B 14.0% D 92.3%

44. **REVIEW** The rectangle has a perimeter of P centimeters. Which equation could be used to find the length of the rectangle?



- F $P = 2.4\ell$ H $P = 2.4 + 2\ell$
G $P = 4.8 + \ell$ J $P = 4.8 + 2\ell$

Spiral Review

Solve each proportion. (Lesson 2-6)

45. $\frac{a}{45} = \frac{3}{15}$

46. $\frac{2}{3} = \frac{8}{d}$

47. $\frac{5.2}{10.4} = \frac{t}{48}$

Solve each equation. Check your solution. (Lesson 2-5)

48. $6n + 3 = -3$

49. $7 + 5c = -23$

50. $18 = 4a - 2$

51. **SALES** As a salesperson, Mr. Goetz is paid a monthly salary and a commission on sales, as shown in the table. How much must Mr. Goetz sell to earn \$2000 this month?
(Lesson 2-4)

Mr. Goetz's Income	
Monthly salary	\$500
Commission on sales	2%

Evaluate each expression. (Lesson 1-2)

52. $(3 + 6) \div 3^2$

53. $6(12 - 7.5) - 7$

54. $20 \div 4 \cdot 8 \div 10$

GET READY for the Next Lesson

PREREQUISITE SKILL Solve each equation. Check your solution. (Lesson 2-5)

55. $7y + 7 = 3y - 5$

56. $7(d - 3) - 2 = 5$

57. $-8 = 4 - 2(a - 5)$

READING MATH

Sentence Method and Proportion Method

Recall that you can solve percent problems using two different methods. With either method, it is helpful to use “clue” words such as *is* and *of*. In the sentence method, *is* means equals and *of* means multiply. With the proportion method, the “clue” words indicate where to place the numbers in the proportion.

Sentence Method

15% of 40 is what number?

$$0.15 \cdot 40 = ?$$

Proportion Method

15% of 40 is what number?

$$\frac{\text{(is) } P}{\text{(of) } B} = \frac{R(\text{percent})}{100} \rightarrow \frac{P}{40} = \frac{15}{100}$$

You can use the proportion method to solve percent of change problems. In this case, use the proportion $\frac{\text{difference}}{\text{original}} = \frac{\%}{100}$. When reading a percent of change problem, or any other word problem, look for the important numerical information.

Example In life skills class, Kishi heated 20 milliliters of water. She let the water boil for 10 minutes. Afterward, only 17 milliliters of water remained, due to evaporation. What is the percent of decrease in the amount of water?

$$\frac{\text{difference}}{\text{original}} = \frac{\%}{100} \rightarrow \frac{20 - 17}{20} = \frac{r}{100} \quad \text{Percent proportion}$$

$$\frac{3}{20} = \frac{r}{100} \quad \text{Simplify.}$$

$$3(100) = 20(r) \quad \text{Find the cross products.}$$

$$300 = 20r \quad \text{Simplify.}$$

$$\frac{300}{20} = \frac{20r}{20} \quad \text{Divide each side by 20.}$$

$$15 = r \quad \text{Simplify.}$$

There was a 15% decrease in the amount of water.



Reading to Learn

Give the original number and the amount of change. Then write and solve a percent proportion.

1. Monsa needed to lose weight for wrestling. At the start of the season, he weighed 166 pounds. By the end of the season, he weighed 158 pounds. What is the percent of decrease in Monsa’s weight?
2. On Carla’s last Algebra test, she scored 94 points out of 100. On her first Algebra test, she scored 75 points out of 100. What is the percent of increase in her score?
3. An online bookstore tracks daily book sales. A certain book sold 12,476 copies on Monday. After the book was mentioned on a national news program, sales were 37,884 on Tuesday. What is the percent of increase from Monday to Tuesday?

Solving for a Specific Variable

Main Ideas

- Solve equations for given variables.
- Use formulas to solve real-world problems.

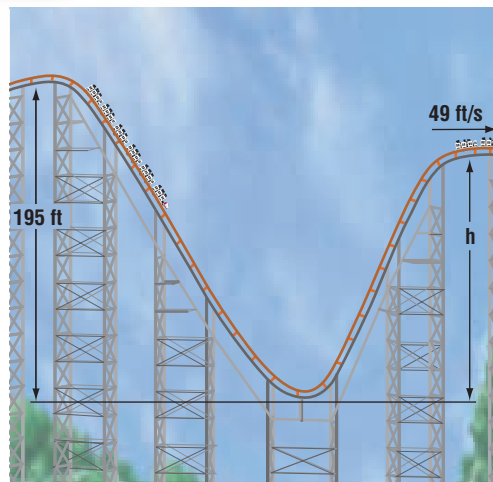
New Vocabulary

dimensional analysis

GET READY for the Lesson

Suppose the designer of the Magnum XL-200 decided to adjust the height of the second hill so that the coaster would have a speed of 49 feet per second when it reached the top.

If we ignore friction, the equation $g(195 - h) = \frac{1}{2}v^2$ can be used to find the height of the second hill. In this equation, g represents the acceleration due to gravity (32 feet per second squared), h is the height of the second hill, and v is the velocity of the coaster when it reaches the top of the second hill.



Solve for Variables Some equations such as the one above contain more than one variable. It is often useful to solve these equations for one of the variables.

EXAMPLE Solve an Equation for a Specific Variable

1 Solve $3x - 4y = 7$ for y .

$$3x - 4y = 7$$

Original equation

$$3x - 4y - 3x = 7 - 3x$$

Subtract $3x$ from each side.

$$-4y = 7 - 3x$$

Simplify.

$$\frac{-4y}{-4} = \frac{7 - 3x}{-4}$$

Divide each side by -4 .

$$y = \frac{7 - 3x}{-4} \text{ or } \frac{3x - 7}{4}$$

Simplify.

The value of y is $\frac{3x - 7}{4}$.

CHECK Your Progress

Solve each equation for the variable indicated.

1A. $15 = 3n + 6p$, for n

1B. $\frac{k - 2}{5} = 11j$, for k

It is sometimes helpful to use the Distributive Property to isolate the variable for which you are solving an equation or formula.

EXAMPLE Solve an Equation for a Specific Variable**2** Solve $2m - t = sm + 5$ for m .

$$2m - t = sm + 5 \quad \text{Original equation}$$

$$2m - t - sm = sm + 5 - sm \quad \text{Subtract } sm \text{ from each side.}$$

$$2m - t - sm = 5 \quad \text{Simplify.}$$

$$2m - t - sm + t = 5 + t \quad \text{Add } t \text{ to each side.}$$

$$2m - sm = 5 + t \quad \text{Simplify.}$$

$$m(2 - s) = 5 + t \quad \text{Use the Distributive Property.}$$

$$\frac{m(2 - s)}{2 - s} = \frac{5 + t}{2 - s} \quad \text{Divide each side by } 2 - s.$$

$$m = \frac{5 + t}{2 - s} \quad \text{Simplify.}$$


The value of m is $\frac{5 + t}{2 - s}$. Since division by 0 is undefined, $2 - s \neq 0$ or $s \neq 2$.

CHECK Your Progress

Solve each equation for the variable indicated.

2A. $d + 5c = 3d - 1$, for d

2B. $6q - 18 = qr + s$, for q

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Use Formulas Sometimes solving a formula for a specific variable will help you solve the problem.

Real-World EXAMPLE

5 YO-YOS Use the information about the largest yo-yo at the left. The formula for the circumference of a circle is $C = 2\pi r$, where C represents circumference and r represents radius.

a. Solve the formula for r .

$$C = 2\pi r \quad \text{Circumference formula}$$

$$\frac{C}{2\pi} = \frac{2\pi r}{2\pi} \quad \text{Divide each side by } 2\pi.$$

$$\frac{C}{2\pi} = r \quad \text{Simplify.}$$

b. Find the radius of the yo-yo.

$$\frac{C}{2\pi} = r \quad \text{Formula for radius}$$

$$\frac{32.7}{2\pi} = r \quad C = 32.7$$

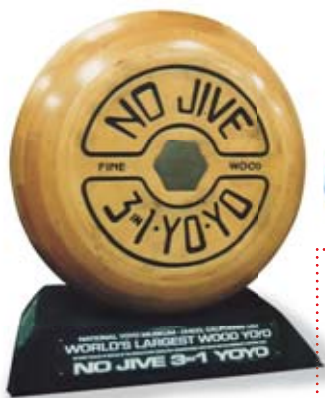
$$5.2 \approx r \quad \text{The yo-yo has a radius of about 5.2 feet.}$$

CHECK Your Progress

The formula for the volume of a rectangular prism is $V = \ell wh$, where ℓ is the length, w is the width, and h is the height.

3A. Solve the formula for w .

3B. Find the width of a rectangular prism that has a volume of 79.04 cubic centimeters, a length of 5.2 centimeters, and a height of 4 centimeters.

**Real-World Link**

The largest yo-yo in the world is 32.7 feet in circumference. It was launched by crane from a height of 189 feet.

Source: www.guinnessworldrecords.com

When using formulas, you may want to use **dimensional analysis**, which is the process of carrying units throughout a computation.

Real-World EXAMPLE

4 PHYSICAL SCIENCE The formula $s = \frac{1}{2}at^2$ represents the distance s that a free-falling object will fall near a planet or the Moon in a given time t . In the formula, a represents the acceleration due to gravity.

a. Solve the formula for a .

$$s = \frac{1}{2}at^2 \quad \text{Original formula}$$

$$\frac{2}{t^2}(s) = \frac{2}{t^2}\left(\frac{1}{2}at^2\right) \quad \text{Multiply each side by } \frac{2}{t^2}.$$

$$\frac{2s}{t^2} = a \quad \text{Simplify.}$$

b. A free-falling object near the Moon drops 20.5 meters in 5 seconds. What is the value of a for the Moon?

$$a = \frac{2s}{t^2} \quad \text{Formula for } a$$

$$= \frac{2(20.5 \text{ m})}{(5 \text{ s})^2} \quad s = 20.5 \text{ m and } t = 5 \text{ s.}$$

$$= \frac{1.64 \text{ m}}{\text{s}^2} \text{ or } 1.64 \text{ m/s}^2 \quad \text{Use a calculator.}$$

Acceleration due to gravity on the Moon is 1.64 meters per second squared.

CHECK Your Progress

The formula $s = vt + \frac{1}{2}at^2$ represents the distance s an object travels with an initial velocity v , time t , and constant rate of acceleration a .

4A. Solve the formula for v .

4B. A sports car accelerates at a rate of 8 ft/s^2 and travels 100 feet in about 2.8 seconds. What is the initial velocity to the nearest tenth?

CHECK Your Understanding

Examples 1, 2
(pp. 117–118)

Solve each equation or formula for the variable specified.

1. $-3x + b = 6x$, for x

2. $4z + b = 2z + c$, for z

3. $\frac{y+a}{3} = c$, for y

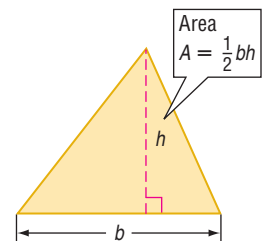
4. $p = a(b+c)$, for a

Example 3
(p. 118)

GEOMETRY For Exercises 5 and 6, use the formula for the area of a triangle.

5. Solve the formula for h .

6. What is the height of a triangle with an area of 28 square feet and a base of 8 feet?



Example 4
(p. 119)

7. SWIMMING A swimmer swims about one third of a lap per minute. At this rate, how many minutes would it take to swim 8 laps? (*Hint: Use $d = rt$.*)

Exercises

HOMEWORK HELP

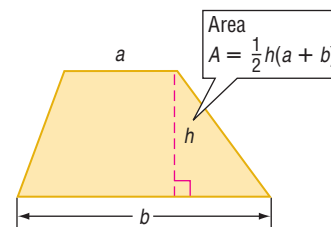
For Exercises	See Examples
8–15	1
16–21	2
22–25	3, 4

Solve each equation or formula for the variable specified.

8. $y = mx + b$, for m
9. $v = r + at$, for a
10. $km + 5x = 6y$, for m
11. $4b - 5 = -t$, for b
12. $c = \frac{3}{4}y + b$, for y
13. $\frac{3}{5}m + a = b$, for m
14. $\frac{3ax - n}{5} = -4$, for x
15. $\frac{by + 2}{3} = c$, for y
16. $5g + h = g$, for g
17. $8t - r = 12t$, for t
18. $3y + z = am - 4y$, for y
19. $9a - 2b = c + 4a$, for a
20. $at + b = ar - c$, for a
21. $2g - m = 5 - gh$, for g

GEOMETRY For Exercises 22 and 23, use the formula for the area of a trapezoid.

22. Solve the formula for h .
23. What is the height of a trapezoid with an area of 60 square meters and bases of 8 meters and 12 meters?



WORK For Exercises 24 and 25, use the following information.

The formula $s = \frac{w - 10e}{m}$ is often used by placement services to find keyboarding speeds. In the formula, s represents the speed in words per minute, w represents the number of words typed, e represents the number of errors, and m represents the number of minutes typed.

24. Solve the formula for e .
25. If Mateo typed 410 words in 5 minutes and received a keyboard speed of 76 words per minute, how many errors did he make?

Solve each equation or formula for the variable specified.

26. $S = \frac{n}{2}(A + t)$, for A
27. $p(t + 1) = -2$, for t
28. $\frac{5x + y}{a} = 2$, for a

Write an equation and solve for the variable specified.

29. Seven less than a number t equals another number r plus 6. Solve for t .
30. Five minus twice a number p equals 6 times another number q plus 1. Solve for p .
31. Five eighths of a number x is 3 more than one half of another number y . Solve for y .

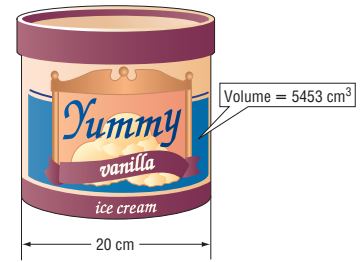
DANCING For Exercises 32 and 33, use the following information.

The formula $P = \frac{1.2W}{H^2}$ represents the amount of pressure exerted on the floor by a dancer's heel. In this formula, P is the pressure in pounds per square inch, W is the weight of a person wearing the shoe in pounds, and H is the width of the heel of the shoe in inches.

32. Solve the formula for W .
33. Find the weight of the dancer if the heel is 3 inches wide and the pressure exerted is 30 pounds per square inch.

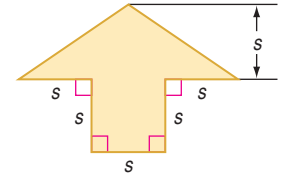
EXTRA PRACTICE
See pages 722, 745.
Math online
Self-Check Quiz at algebra1.com

- 34. PACKAGING** The Yummy Ice Cream Company wants to package ice cream in cylindrical containers that have a volume of 5453 cubic centimeters. The marketing department decides the diameter of the base of the containers should be 20 centimeters. How tall should the containers be? (Hint: $V = \pi r^2 h$)



H.O.T. Problems

- 35. CHALLENGE** Write a formula for the area of the arrow. Describe how you found it.
- 36. REASONING** Describe the possible values of t if $s = \frac{r}{t-2}$. Explain your reasoning.
- 37. OPEN ENDED** Write a formula for A , the area of a geometric figure such as a triangle or rectangle. Then solve the formula for a variable other than A .
- 38. Writing in Math** Use the information on page 117 to explain how equations are used to design roller coasters. Include a list of steps you could use to solve the equation for h , and the height of the second hill of the roller coaster.



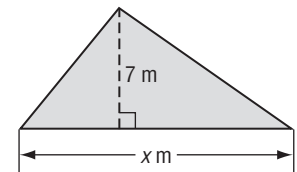
STANDARDIZED TEST PRACTICE

- 39.** If $2x + y = 5$, what is the value of $4x$?

- A $10 - y$
B $10 - 2y$
C $\frac{5 - y}{2}$
D $\frac{10 - y}{2}$

- 40. REVIEW** What is the base of the triangle if the area is 56 meters squared?

- F 4 m
G 8 m
H 16 m
J 28 m



Spiral Review

- 41. FOOD** In order for a food to be marked “reduced fat,” it must have at least 25% less fat than the same full-fat food. If one ounce of reduced-fat cheese has 8 grams of fat, what is the least amount of fat in one ounce of regular cheese? (Lesson 2-7)

Solve each proportion. (Lesson 2-6)

42. $\frac{2}{9} = \frac{5}{a}$

43. $\frac{15}{32} = \frac{t}{8}$

44. $\frac{x+1}{8} = \frac{3}{4}$

GET READY for the Next Lesson

PREREQUISITE SKILL Use the Distributive Property to rewrite each expression without parentheses. (Lesson 1-5)

45. $6(2 - t)$

46. $(5 + 2m)3$

47. $-7(3a + b)$

48. $\frac{2}{3}(6h - 9)$

Weighted Averages

Main Ideas

- Solve mixture problems.
- Solve uniform motion problems.

New Vocabulary

weighted average
mixture problem
uniform motion problem

GET READY for the Lesson

In an individual figure skating competition, the score for the long program is worth twice the score for the short program. Suppose a skater scores 5.5 in the short program and 5.8 in the long program. The final score is determined using a weighted average.

$$\begin{aligned}\frac{5.5(1) + 5.8(2)}{1 + 2} &= \frac{5.5 + 11.6}{3} \\ &= \frac{17.1}{3} \text{ or } 5.7\end{aligned}$$

The final score would be 5.7.

Mixture Problems The skater's average score is an example of a weighted average. The **weighted average** M of a set of data is the sum of the product of the number of units and the value per unit divided by the sum of the number of units. **Mixture problems**, in which two or more parts are combined into a whole, are solved using weighted averages.



Real-World EXAMPLE

Prices

1 TRAIL MIX How many pounds of mixed nuts selling for \$4.75 per pound should be mixed with 10 pounds of dried fruit selling for \$5.50 per pound to obtain a trail mix that sells for \$4.95 per pound?

Let w = the number of pounds of mixed nuts. Make a table.

	Units (lb)	Price per Unit (lb)	Total Price
Dried Fruit	10	\$5.50	$5.50(10)$
Mixed Nuts	w	\$4.75	$4.75w$
Trail Mix	$10 + w$	\$4.95	$4.95(10 + w)$

$$\begin{array}{lclcl} \text{Price of dried fruit} & \text{plus} & \text{price of nuts} & \text{equals} & \text{price of trail mix.} \\ 5.50(10) & + & 4.75w & = & 4.95(10 + w) \end{array}$$

$$5.50(10) + 4.75w = 4.95(10 + w) \quad \text{Original equation}$$

$$55.00 + 4.75w = 49.50 + 4.95w \quad \text{Distributive Property}$$

$$55.00 + 4.75w - 4.75w = 49.50 + 4.95w - 4.75w \quad \text{Subtract } 4.75w \text{ from each side.}$$

$$55.00 = 49.50 + 0.20w \quad \text{Simplify.}$$

$$55.00 - 49.50 = 49.50 + 0.20w - 49.50 \quad \text{Subtract } 49.50 \text{ from each side.}$$

$$5.50 = 0.20w \quad \text{Simplify.}$$

$$\frac{5.50}{0.20} = \frac{0.20w}{0.20} \quad \text{Divide each side by } 0.20.$$

$$27.5 = w \quad \text{Simplify.}$$

27.5 pounds of mixed nuts should be used for the trail mix.

CHECK Your Progress

1. **COFFEE** How many pounds of coffee beans that sell for \$9.50 per pound should be mixed with 2 pounds of coffee beans that sell for \$11.75 per pound to obtain a mix that sells for \$10 per pound?

Percent problems can also be solved using weighted averages.

Real-World EXAMPLE

Percent Mixture Problems

Study Tip

Mixture Problems

When you organize the information in mixture problems, remember that the final mixture must contain the sum of the parts in the correct quantities and at the correct percents.

2. **SCIENCE** A chemistry experiment calls for a 30% solution of copper sulfate. Kendra has 40 milliliters of 25% solution. How many milliliters of 60% solution should she add to make a 30% solution?

Let x = the amount of 60% solution to be added. Make a table.

	Amount of Solution (mL)	Amount of Copper Sulfate
25% Solution	40	$0.25(40)$
60% Solution	x	$0.60x$
30% Solution	$40 + x$	$0.30(40 + x)$

Write and solve an equation using the information in the table.

Amount of copper sulfate in 25% solution	plus	amount of copper sulfate in 60% solution	equals	amount of copper sulfate in 30% solution.
$0.25(40)$	+	$0.60x$	=	$0.30(40 + x)$
$0.25(40) + 0.60x = 0.30(40 + x)$ Original equation				
$10 + 0.60x = 12 + 0.30x$ Distributive Property				
$10 + 0.60x - 0.30x = 12 + 0.30x - 0.30x$ Subtract $0.30x$ from each side.				
$10 + 0.30x = 12$ Simplify.				
$10 - 0.30x - 10 = 12 - 10$ Subtract 10 from each side.				
$0.30x = 2$ Simplify.				
$\frac{0.30x}{0.30} = \frac{2}{0.30}$ Divide each side by 0.30.				
$x \approx 6.67$ Simplify.				

Kendra should add 6.67 milliliters of the 60% solution to the 40 milliliters of the 25% solution. Check by substituting 6.67 for x in the original equation.

CHECK Your Progress

2. **ANTIFREEZE** One type of antifreeze is 40% glycol, and another type of antifreeze is 60% glycol. How much of each kind should be used to make 100 gallons of antifreeze that is 48% glycol?

Study Tip

Uniform Motion Problems

Uniform motion problems are sometimes referred to as **rate problems**.

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Uniform Motion Problems Motion problems are another application of weighted averages. **Uniform motion problems** are problems where an object moves at a certain speed, or rate. The formula $d = rt$ is used to solve these problems. In the formula, d represents distance, r represents rate, and t represents time.



Real-World EXAMPLE **Speed of One Vehicle**

3 TRAVEL On Alberto's drive to his aunt's house, the traffic was light and he drove the 45-mile trip in one hour. However, the return trip took him two hours. What was his average speed for the round trip?

To find the average speed for each leg of the trip, rewrite $d = rt$ as $r = \frac{d}{t}$.

Going

$$r = \frac{d}{t}$$

$$= \frac{45 \text{ miles}}{1 \text{ hour}} \text{ or } 45 \text{ miles per hour}$$

Returning

$$r = \frac{d}{t}$$

$$= \frac{45 \text{ miles}}{2 \text{ hours}} \text{ or } 22.5 \text{ miles per hour}$$

You may think that the average speed of the trip would be $\frac{45 + 22.5}{2}$ or 33.75 miles per hour. However, Alberto did not drive at these speeds for equal amounts of time. You must find the weighted average for the trip.

Round Trip

Let M = the average speed.

$$M = \frac{45(1) + 22.5(2)}{1 + 2}$$

Definition of weighted average

$$= \frac{90}{3} \text{ or } 30$$

Simplify.

Alberto's average speed was 30 miles per hour.

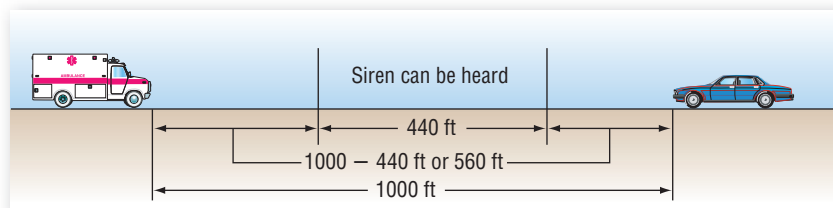
CHECK Your Progress

3. EXERCISE Austin jogged 2.5 miles in 16 minutes and then walked 1 mile in 10 minutes. What was his average speed?

Real-World EXAMPLE **Speeds of Two Vehicles**

4 SAFETY Use the information about sirens at the left. A car and an emergency vehicle are heading toward each other. The car is traveling at a speed of 30 miles per hour or about 44 feet per second. The emergency vehicle is traveling at a speed of 50 miles per hour or about 74 feet per second. If the vehicles are 1000 feet apart and the conditions are ideal, in how many seconds will the driver of the car first hear the siren?

Draw a diagram. The driver can hear the siren when the total distance traveled by the two vehicles equals $1000 - 440$ or 560 feet.



Let t = the number of seconds until the driver can hear the siren.

	r	t	$d = rt$
Car	44	t	$44t$
Emergency Vehicle	74	t	$74t$



Real-World Link

Under ideal conditions, a siren can be heard from up to 440 feet. However, under normal conditions, a siren can be heard from only 125 feet.

Source: U.S. Department of Transportation

Write and solve an equation.

Distance traveled by car	plus	distance traveled by emergency vehicle	equals	560 feet.
$44t$	+	$74t$	=	560

$44t + 74t = 560$ Original equation

$118t = 560$ Simplify.

$\frac{118t}{118} = \frac{560t}{118}$ Divide each side by 118.

$t \approx 4.75$ The driver will hear the siren in about 4.75 seconds.

CHECK Your Progress

4. **CYCLING** Two cyclists begin traveling in opposite directions on a circular bike trail that is 5 miles long. One cyclist travels 12 miles per hour, and the other travels 18 miles per hour. How long will it be before they meet?

CHECK Your Understanding

Example 1
(p. 122)

BUSINESS For Exercises 1–3, use the following information.

The Candle Supply Store sells votive wax for \$0.90 a pound and low-shrink wax for \$1.04 a pound. How many pounds of low-shrink wax should be mixed with 8 pounds of votive wax to obtain a blend that sells for \$0.98 a pound?

1. Copy and complete the table representing the problem.

	Number of Pounds	Price per Pound	Total Price
Votive Wax	8	\$0.90	0.90(8)
Low-Shrink Wax	p		
Blend	$8 + p$		

2. Write an equation to represent the problem.
3. How many pounds of low-shrink wax should be mixed with 8 pounds of votive wax?
4. **COFFEE** A specialty coffee store wants to create a special mix using two coffees, one priced at \$6.40 per pound and the other priced at \$7.28 per pound. How many pounds of the \$7.28 coffee should be mixed with 9 pounds of the \$6.40 coffee to sell the mixture for \$6.95 per pound?

Example 2
(p. 123)

FOOD For Exercises 5–7, use the following information.

How many quarts of pure pineapple juice should Theo add to a 20% pineapple drink to create 5 quarts of a 50% pineapple juice mixture?

5. Copy and complete the table representing the problem.

	Quarts	Total Amount of Juice
20% Juice	$5 - n$	
100% Juice	n	
50% Juice		

6. Write an equation to represent the problem.
7. How much pure pineapple juice and 20% juice should Theo use?

8. **METALS** An alloy of metals is 25% copper. Another alloy is 50% copper. How much of each should be used to make 1000 grams of an alloy that is 45% copper?
9. **TRAVEL** A boat travels 16 miles due north in 2 hours and 24 miles due west in 2 hours. What is the average speed of the boat?
10. **EXERCISE** Felisa jogged 3 miles in 25 minutes and then jogged 3 more miles in 30 minutes. What was her average speed in miles per minute?
11. **CYCLING** A cyclist begins traveling 18 miles per hour. At the same time and place, an in-line skater follows the cyclist's path and begins traveling 6 miles per hour. After how long will they be 24 miles apart?
12. **RESCUE** A fishing boat radioed the Coast Guard for a helicopter to pick up a sick crew member. At the time of the message, the boat is 660 kilometers from the helicopter and heading toward it. The average speed of the boat is 30 kilometers per hour, and the average speed of the helicopter is 300 kilometers per hour. How long will it take the helicopter to reach the boat?

Example 3
(p. 124)

Example 4
(pp. 124–125)

Exercises

HOMEWORK HELP	
For Exercises	See Examples
13–24	1
25, 26	2
27, 28	3
29–34	4

13. **GRADES** In Ms. Martinez's science class, a test is worth three times as much as a quiz. If a student has test grades of 85 and 92 and quiz grades of 82, 75, and 95, what is the student's average grade?
14. **ANALYZE TABLES** At Westbridge High School, a student's grade point average (GPA) is based on the student's grade and the class credit rating. Brittany's grades for this quarter are shown. Find Brittany's GPA if a grade of A equals 4 and a B equals 3.

Class	Credit Rating	Grade
Algebra 1	1	A
Science	1	A
English	1	B
Spanish	1	A
Music	$\frac{1}{2}$	B

METALS For Exercises 15–18, use the following information.

In 2005, the international price of gold was \$432 per ounce, and the international price of silver was \$7.35 per ounce. Suppose gold and silver were mixed to obtain 15 ounces of an alloy worth \$177.21 per ounce.

15. Copy and complete the table representing the problem.

	Number of Ounces	Price per Ounce	Value
Gold	g		
Silver	$15 - g$		
Alloy			

16. Write an equation to represent the problem.
17. How much gold was used in the alloy?
18. How much silver was used in the alloy?
19. **FUND-RAISING** The Madison High School marching band sold solid-color gift wrap for \$4 per roll and print gift wrap for \$6 per roll. The total number of rolls sold was 480, and the total amount of money collected was \$2340. How many rolls of each kind of gift wrap were sold?

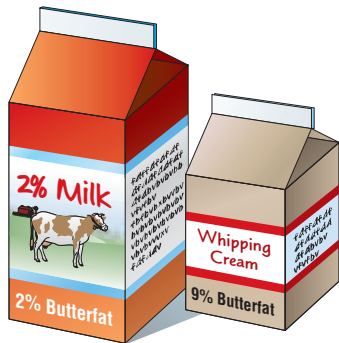
BUSINESS For Exercises 20–23, use the following information.

Party Supplies Inc. sells metallic balloons for \$2 each and helium balloons for \$3.50 per dozen. Yesterday, they sold 36 more metallic balloons than dozens of helium balloons. The total sales for both types of balloons were \$281.00.

20. Copy and complete the table representing the problem.

	Number	Price	Total Price
Metallic Balloons	b		
Dozens of Helium Balloons	$b - 36$		

21. Write an equation to represent the problem.
 22. How many metallic balloons were sold?
 23. How many dozen helium balloons were sold?



24. **MONEY** Lakeisha spent \$4.57 on color copies and black-and-white copies for her project. She made 7 more black-and-white copies than color copies. How many color copies did she make?

Type of Copy	Cost Per Page
color	\$0.44
black-and-white	\$0.07

25. **FOOD** Refer to the graphic at the left. How much whipping cream and 2% milk should be mixed to obtain 35 gallons of milk with 4% butterfat?
 26. **CHEMISTRY** Hector is performing a chemistry experiment that requires 140 milliliters of a 30% copper sulfate solution. He has a 25% copper sulfate solution and a 60% copper sulfate solution. How many milliliters of each solution should he mix to obtain the needed solution?
 27. **TRAVEL** A boat travels 36 miles in 1.5 hours and then 14 miles in 0.75 hour. What is the average speed of the boat?
 28. **EXERCISE** An inline skater skated 1.5 miles in 28 minutes and then 1.2 more miles in 10 minutes. What was the average speed in miles per minute?

TRAVEL For Exercises 29–31, use the following information.

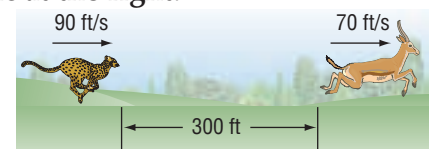
Two trains leave Smithville at the same time, one traveling east and the other west. The eastbound train travels at 40 miles per hour, and the westbound train travels at 30 miles per hour. Let h represent the hours since departure.

29. Copy and complete the table representing the situation.
 30. Write an equation to determine when the trains will be 245 miles apart.
 31. In how many hours will the trains be 245 miles apart?

	r	t	$d = rt$
Eastbound Train			
Westbound Train			

ANIMALS For Exercises 32–34, use the graphic at the right.

Let t represent the number of seconds until the cheetah catches its prey.



32. Copy and complete the table representing the situation.
 33. Write an equation to determine when the cheetah will catch its prey.
 34. When will the cheetah catch its prey?

	r	t	$d = rt$
Cheetah			
Prey			

EXTRA PRACTICE
See pages 722, 745.

Math online
Self-Check Quiz at algebra1.com

- 35. TRACK AND FIELD** A sprinter has a bad start, and his opponent is able to start 1 second before him. If the sprinter averages 8.2 meters per second and his opponent averages 8 meters per second, will he be able to catch his opponent before the end of the 200-meter race? Explain.
- 36. TRAVEL** A subway travels 60 miles per hour from Glendale to Midtown. Another subway, traveling at 45 miles per hour, takes 11 minutes longer for the same trip. How far apart are Glendale and Midtown?
- 37. OPEN ENDED** Describe a real-world example of a weighted average.
- 38. CHALLENGE** Write a mixture problem for $1.00x + 0.28(40) = 0.40(x + 40)$.
- 39. Writing in Math** Use the information on page 122 to explain how scores are calculated in a figure-skating competition. Include an explanation of how a weighted average can be used to find a skating score and a demonstration of how to find the weighted average of a skater who received a 4.9 in the short program and a 5.2 in the long program.

H.O.T. Problems

STANDARDIZED TEST PRACTICE

- 40. REVIEW** Eula Jones is investing \$6000, part at 4.5% interest and the rest at 6% interest. If d represents the amount invested at 4.5%, which expression represents the amount of interest earned in one year by the account paying 6%?
- A $0.06d$ C $0.06(d + 6000)$
B $0.06(d - 6000)$ D $0.06(6000 - d)$
- 41.** Todd drove from Boston to Cleveland, a distance of 616 miles. His breaks, gasoline, and food stops took 2 hours. If his trip took 16 hours altogether, what was his average speed?
- F 38.5 mph H 44 mph
G 40 mph J 47.5 mph

Spiral Review

Solve each equation for the variable specified. (Lesson 2-8)

42. $a + 6 = \frac{b-1}{4}$, for b

43. $3t - 4 = 6t - s$, for t

State whether each percent of change is a percent of increase or a percent of decrease. Then find the percent of change. Round to the nearest whole percent. (Lesson 2-7)

44. original: \$25
new: \$14

45. original: 35
new: 42

46. original: 244
new: 300

47. MONEY Tyler had \$80 in his savings account. After his mother made a deposit, his new balance was \$115. Write and solve an equation to find the amount of the deposit. (Lesson 2-2)

Cross-Curricular Project

Algebra and Social Studies

You're Only as Old as You Feel! It's time to complete your project. Use the information and data you have gathered about living to be 100 to prepare a portfolio or Web page. Be sure to include graphs and/or tables in the presentation.



Cross-Curricular Project at algebra1.com

Spreadsheet Lab

Finding a Weighted Average

You can use a spreadsheet to calculate weighted averages. It allows you to make calculations and print almost anything you can organize in a table.

The basic unit in a spreadsheet is called a cell. A cell may contain numbers, words, or a formula. Each cell is named by the column and row that describe its location. For example, cell B4 is in column B, row 4.

EXAMPLE

Greta Norris manages a sales firm. Each of her employees earns a different rate of commission. She has entered the commission rate and the sales for each employee for October in a spreadsheet. What was the average commission rate?

Commission				
	A	B	C	D
1	Employee	Rate	Sales	Commission
2	Mark	0.05	1588	=C2*B2
3	Andrea	0.075	1800	=C3*B3
4	Tamika	0.1	2008	=C4*B4
5	Daniel	0.1	2105	=C5*B5
6	Lasanda	0.05	1725	=C6*B6
7	Diego	0.06	1988	=C7*B7
8	Total		=SUM(C2:C7)	=SUM(D2:D7)
9	Weighted Average Commission Rate	=D8 / C8		
10				

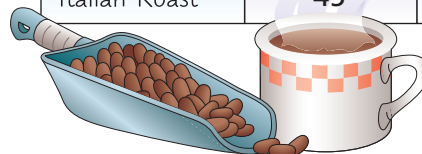
The spreadsheet shows the formula to calculate the weighted average. It multiplies the rate of commission by the amount of sales and finds the total amount of sales and amount of commission. Then it divides the amount of commission by the amount of sales. To the nearest tenth, the average rate of commission is 7.4%.

EXERCISES

For Exercises 1–3, use the table at the right.

1. What is the average price of a pound of coffee?
2. How does the November weighted average change if all of the coffee prices are increased by \$1.00? by 10%?
3. Find the weighted average of a pound of coffee if the shop sold 50 pounds of each type of coffee. How does the weighted average compare to the average of the per-pound coffee prices? Explain.

November Coffee Sales		
Product	Pounds Sold	Price Per Pound (\$)
Hawaiian Cafe	56	16.95
Mocha Java	97	12.59
House Blend	124	10.75
Decaf Espresso	71	10.15
Breakfast Blend	69	11.25
Italian Roast	45	9.95



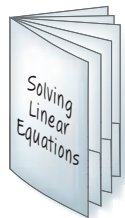


FOLDABLES™

Study Organizer

GET READY to Study

Be sure the following
Key Concepts are noted
in your Foldable.



Key Concepts

Writing Equations (Lesson 2-1)

- Four-Step Problem Solving Plan:
 - Step 1 Explore the problem.
 - Step 2 Plan the solution.
 - Step 3 Solve the problem.
 - Step 4 Check the solution.

Solving Equations (Lessons 2-2 to 2-5)

- Addition and Subtraction Properties of Equality: If an equation is true and the same number is added to or subtracted from each side, the resulting equation is true.
- Multiplication and Division Properties of Equality: If an equation is true and each side is multiplied by the same number or divided by the same non-zero number, the resulting equation is true.
- Steps for Solving Equations:
 - Step 1 Use the Distributive Property if necessary.
 - Step 2 Simplify expressions on each side.
 - Step 3 Use the Addition and/or Subtraction Properties of Equality to get the variables on one side and the numbers without variables on the other side.
 - Step 4 Simplify the expression on each side of the equals sign.
 - Step 5 Use the Multiplication or Division Property of Equality to solve.

Ratios and Proportions (Lesson 2-6)

- The Means-Extremes Property of Proportion states that in a proportion, the product of the extremes is equal to the product of the means.

Key Vocabulary

consecutive integers (p. 94)	multi-step equations (p. 92)
defining a variable (p. 71)	number theory (p. 94)
dimensional analysis (p. 119)	percent of change (p. 111)
equivalent equations (p. 79)	percent of decrease (p. 111)
extremes (p. 106)	percent of increase (p. 111)
formula (p. 72)	proportion (p. 105)
four-step problem solving plan (p. 71)	rate (p. 107)
identity (p. 99)	ratio (p. 105)
means (p. 106)	scale (p. 108)
mixture problem (p. 122)	solve an equation (p. 79)
	uniform motion problem (p. 123)
	weighted average (p. 122)

Vocabulary Check

State whether each sentence is *true* or *false*.
If *false*, replace the underlined word or
number to make a true sentence.

1. An example of consecutive integers is -10 and -9 .
2. A comparison of two numbers by division is called a ratio.
3. In the proportion $1:4 = x:2$, 4 and x are called the means.
4. Equations with more than one operation are called equivalent equations.
5. Dimensional analysis is the process of carrying units throughout a computation.
6. Mixture problems are solved using number theory.
7. An equation that is true for every value of the variable is called an identity.

Lesson-by-Lesson Review

2-1

Writing Equations (pp. 70-76)

Translate each sentence into an equation.

8. The sum of z and one fifth of w is ninety-six.
9. The product of m and n is as much as three times the sum of m and 8.
10. The difference of p and thirteen is identical to the square of p .

Translate each equation into a verbal sentence.

11. $\frac{56}{g} = 7 - 3g$
12. $8 + 2k = \frac{1}{4}k$
13. **DESERTS** One third of Earth's land surface is desert. There are 50 million square kilometers of desert. Define a variable and then write and solve an equation to find the total number of square kilometers of land surface.

Example 1 Translate the following sentence into an equation.

Forty-one increased by twice a number m is the same as three times the sum of a number m and seven.

$$41 + 2m = 3(m + 7)$$

Example 2 Translate $2y - 5 = \frac{1}{2}y$ into a verbal sentence.

Twice a number y minus five equals one half y .

2-2

Solving Equations by Using Addition and Subtraction (pp. 78-84)

Solve each equation.

14. $h - 15 = -22$
15. $\frac{3}{5} = \frac{2}{3} + a$
16. $16 - (-q) = 83$
17. $-55 = x + (-7)$

Write an equation for each problem. Then solve the equation and check your solution.

18. Nine subtracted from a number equals -15 . Find the number.
19. The sum of a number and -71 is 29. What is the number?
20. **CANS** The can opener was invented in 1858. That was 48 years after cans were first introduced. Write and solve an equation to determine in what year the can was introduced.

Example 3 Solve $13 + p = -5$.

$$13 + p = -5 \quad \text{Original equation}$$

$$13 + p - 13 = -5 - 13 \quad \text{Subtract 13 from each side.}$$

$$p = -18 \quad \text{Check this result.}$$

Example 4 Write an equation for the problem below. Then solve the equation.

The difference of a number and 62 is -47 . What is the number?

$$x - 62 = -47 \quad \text{Original equation}$$

$$x - 62 + 62 = -47 + 62 \quad \text{Add 62 to each side.}$$

$$x = 15 \quad \text{Check this result.}$$

2-3

Solving Equations by Using Multiplication and Division (pp. 85-90)

Solve each equation. Check your solution.

21. $\frac{a}{-7} = -29$

22. $-18 = \frac{3}{5}m$

23. $14p = 42$

24. $\frac{y}{32} = \frac{5}{-8}$

Write an equation for each problem. Then solve the equation.

25. Three eighths of a number equals 9. What is the number?

26. The quotient of a number and -4 is -72 . Find the number.

27. **DINING** China produces 45 billion chopsticks per year from 25 million trees. Write and solve an equation to find the number of chopsticks that can be produced from 1 tree.

Example 5 Solve $\frac{k}{-3} = -9$.

$$\frac{k}{-3} = -9 \quad \text{Original equation}$$

$$\frac{k}{-3}(-3) = -9(-3) \quad \text{Multiply each side by } -3.$$

$$k = 27 \quad \text{Check this result.}$$

Example 6 Write an equation for the problem below. Then solve the equation.

48 is negative twelve times a number. Find the number.

$$48 = -12n \quad \text{Original equation}$$

$$\frac{48}{-12} = \frac{-12n}{-12} \quad \text{Divide each side by } -12.$$

$$-4 = n \quad \text{Check this result.}$$

2-4

Solving Multi-Step Equations (pp. 92-97)

Solve each equation. Check your solution.

28. $5 = 4t - 7$

29. $6 + \frac{y}{3} = -45$

30. $9 = \frac{d+5}{8}$

31. $\frac{c}{-4} - 2 = -36$

Write an equation and solve each problem.

32. 22 increased by six times a number is -20 . What is the number?

33. Find three consecutive odd integers with a sum of 39.

34. **THEATER** Each row in a theater has eight more seats than the previous row. If the first row has 14 seats, which row has 46 seats?

Example 7 Write and solve an equation to find three consecutive even integers with a sum of 150.

Let n = the least even integer.
Then $n + 2$ = the next greater even integer
and $n + 4$ = the greatest of the three even integers.

$$n + (n + 2) + (n + 4) = 150 \quad \text{Original equation}$$

$$3n + 6 = 150 \quad \text{Simplify.}$$

$$3n = 144 \quad \text{Subtract 6 from each side.}$$

$$n = 48 \quad \text{Divide each side by 3.}$$

The integers are 48, 50, and 52.

2-5 Solving Equations with the Variable on Each Side (pp. 98-103)

Solve each equation. Check your solution.

35. $5b + 3 = 9b - 17$

36. $\frac{2}{3}n - 3 = \frac{1}{3}(2n - 9)$

37. **FUND-RAISING** The Band Boosters pay \$200 to rent a concession stand at a university football game. They purchase cans of soft drinks for \$0.25 each and sell them at the game for \$1.50 each. How many cans of soft drinks must they sell to break even?

Example 8 Solve $7x + 56 = 5x - 11$.

$7x + 56 = 5x - 11$ Original equation

$7x + 56 - 5x = 5x - 11 - 5x$ Subtract 5x from each side.

$2x + 56 = -11$ Simplify.

$2x + 56 - 56 = -11 - 56$ Subtract 56 from each side.

$2x = -67$ Simplify.

$\frac{2x}{2} = \frac{-67}{2}$ Divide each side by 2.

$x = -33.5$ Check this result.

2-6 Ratios and Proportions (pp. 105-110)

Solve each proportion. If necessary, round to the nearest hundredth.

38. $\frac{14}{x} = \frac{20}{8}$

39. $\frac{0.47}{6} = \frac{1.41}{m}$

40. **WORLD RECORDS** Dustin drank 14 ounces of tomato juice through a straw in 33 seconds. At this rate, approximately how long would it take him to drink a bottle of water that contains 16.9 ounces?

Example 9 Solve the proportion

$\frac{h}{15} = \frac{7}{21}$

$\frac{h}{15} = \frac{7}{21}$ Original equation

$h(21) = 15(7)$ Find the cross products.

$21h = 105$ Distributive Property

$\frac{21h}{21} = \frac{105}{21}$ Divide each side by 21.

$h = 5$ Simplify.

2-7 Percent of Change (pp. 111-115)

State whether each percent of change is a percent of increase or a percent of decrease. Then find each percent of change. Round to the nearest whole percent.

41. original: 54 42. original: 17
new: 46 new: 33

43. **TIPS** Felicia's meal cost \$23.74. How much money should she leave for a 15% tip?

Example 10 Find the total price of a CD that costs \$18.75 with 6.5% sales tax.

$6.5\% \text{ of } \$18.75 = 0.065 \times 18.75$ $6.5\% = 0.065$
 $= 1.21875$ Multiply.

Round \$1.21875 to \$1.22. Add this amount to the original price.

$\$18.75 + \$1.22 = \$19.97$

The total price of the CD is \$19.97.

2-8

Solving for a Specific Variable (pp. 117-121)

Solve each equation or formula for the variable specified.

44. $d = 5v + m$, for v

45. $\frac{7}{4}k - g = s$, for k

46. $\frac{ac - 3}{6} = w$, for c

47. $9h + z = pq + 2h$, for h

48. **TRAVEL** In the formula $d = rt$, d is the distance, r is the rate, and t is the time spent traveling. Solve this equation for r . Then find Aida's rate if she drove 219 miles in $3\frac{3}{4}$ hours.

Example 11 Solve $8q + 3b = 12$ for b .

$$8q + 3b = 12 \quad \text{Original equation}$$

$$8q + 3b - 8q = 12 - 8q \quad \text{Subtract } 8q \text{ from each side.}$$

$$3b = 12 - 8q \quad \text{Simplify.}$$

$$\frac{3b}{3} = \frac{12 - 8q}{3} \quad \text{Divide each side by 3.}$$

$$b = \frac{12 - 8q}{3} \quad \text{Simplify.}$$

The value of b is $\frac{12 - 8q}{3}$.

2-9

Weighted Averages (pp. 122-128)

49. **COFFEE** Jerome blends Brand A of coffee that sells for \$9.50 a pound with Brand B of coffee that sells for \$12.00 a pound. If the 45 pounds of blend sells for \$10.50 a pound, how many pounds of each type of coffee did Jerome use?
50. **PUNCH** Raquel is mixing lemon-lime soda and a fruit juice blend that is 45% juice. If she uses 3 quarts of soda, how many quarts of fruit juice must be added to produce punch that is 30% juice?
51. **JOGGING** Delmar ran 4 miles in 22 minutes, stopped and rested, and ran an additional 4 miles in 28 minutes. Find his average speed.
52. **TRAVEL** Connor is driving 65 miles per hour on the highway. Ed is 15 miles behind him driving at 70 miles per hour. After how many hours will Ed catch up to Connor?

Example 12 MANUFACTURING Percy mixes 750 liters of water with 250 liters of 2% bleach. What percent of bleach is in the resultant mixture?

Let x = the percent of bleach in the resultant mixture. Make a table.

	Amount of Solution (L)	Amount of Resultant Mixture
Water (0% bleach)	750	750(0)
Bleach (2% bleach)	250	250(0.02)
Resultant Mixture ($x\%$ bleach)	1000	1000(x)

Write and solve an equation using the information in the table.

$$750(0) + 250(0.02) = 1000(x) \quad \text{Original equation}$$

$$5 = 1000x \quad \text{Simplify.}$$

$$0.005 = x \quad \text{Divide each side by 1000.}$$

There is 0.5% bleach in the resultant mixture.

Translate each sentence into an equation.

- The sum of x and four times y is equal to twenty.
- Two thirds of n is negative eight fifths.

Solve each equation. Then check your solution.

- $-15 + k = 8$
- $k - 16 = -21$
- $-1.2x = 7.2$
- $5a = 125$
- $\frac{t-7}{4} = 11$
- $\frac{3}{4}y = 27$
- $-12 = 7 - \frac{y}{3}$
- $-\frac{2}{3}z = -\frac{4}{9}$

11. **MULTIPLE CHOICE** The perimeter of the larger square is 11.6 centimeters greater than the perimeter of the smaller square. What are the side lengths of the smaller square?



- A 1.1 cm C 4.5 cm
B 4.2 cm D 16.8 cm

Solve each equation. Then check your solution.

- $-3(x + 5) = 8x + 18$
- $2p + 1 = 5p - 11$

14. **POSTAGE** What was the percent of increase in the price of a first-class stamp from 2001 to 2006? Round to the nearest whole percent.

First-Class Stamps	
Year	Cost of Stamp
2001	34¢
2006	39¢

15. **MULTIPLE CHOICE** If $\frac{4}{5}$ of $\frac{3}{4} = \frac{2}{5}$ of $\frac{x}{4}$, find the value of x .

- F 12
G 6
H 3
J $\frac{3}{2}$

Solve each proportion.

- $\frac{2}{10} = \frac{1}{a}$
- $\frac{3}{5} = \frac{24}{x}$
- $\frac{n}{4} = \frac{3.25}{52}$
- $\frac{5}{12} = \frac{10}{x-1}$

State whether each percent of change is a percent of increase or a percent of decrease. Then find the percent of change. Round to the nearest whole percent.

20. original: 45 21. original: 12
new: 9 new: 20

Solve each equation or formula for the variable specified.

- $h = at - 0.25vt^2$, for a
- $a(y + 1) = b$, for y

24. **SALES** At The Central Perk coffee shop, Destiny sold 30 more cups of cappuccino than espresso, for a total of \$178.50 worth of espresso and cappuccino. How many cups of each were sold?

Coffee	Cost per Cup (\$)
espresso	2.00
cappuccino	2.50

25. **BOATING** The *Yankee Clipper* leaves the pier at 9:00 A.M. at 8 knots (nautical miles per hour). A half hour later, *The River Rover* leaves the same pier in the same direction traveling at 10 knots. At what time will *The River Rover* overtake *The Yankee Clipper*?

Standardized Test Practice

Cumulative, Chapters 1–2

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

- Martin's Car Rental rents cars for \$20 per 100 miles driven. How many miles could be driven for \$125?

A 6.25 C 625
B 100 D 6250
- Marcus's telephone company charges \$30 per month and \$0.05 per minute. How much did it cost him in January if he talked for 120 minutes?

F \$24
G \$36
H \$60
J \$90
- GRIDDABLE** Solve the equation.

$$\frac{1}{2}x + 12 = \frac{3}{4}x - 16$$
- Leland's lawn service charged Mr. Mackenzie \$65 to plant a new tree plus \$18.50 per hour to mow and fertilize his lawn. The total charge was \$157.50. For about how many hours did Leland's lawn service work on Mr. Mackenzie's lawn?

A 4
B 5
C 6
D 8
- Lauren has \$35 to spend on beads for making necklaces. If she spends \$15 for string and each bead costs \$2.50, solve the inequality $2.50n + 15 \leq 35$ to determine how many beads she can buy.

F more than 14 H more than 8
G less than 14 J less than 8

- Margot and Lainy are selling bracelets at the farmers' market. It costs \$25 to rent a booth. They sold each of their bracelets for \$15. If the profit was \$355, which equation represents the number of bracelets n they sold?

A $355 = 15n - 25$
B $355 = 25 + 15n$
C $25 = 355 + 15n$
D $355 = (25 + n)15$

TEST-TAKING TIP

Question 6 When you write an equation, check that the given values make a true statement. For example, in Question 6, substitute the values for the number of bracelets and the cost of the booth into your equation.

- GRIDDABLE** Brianne makes baby blankets for a baby store. She works on the blankets 30 hours per week. The store pays her \$9.50 per hour plus 30% of the profit. If her hourly rate is increased by \$0.75 and her commission is raised to 40%, how much will she earn in dollars if a total of \$300 in blankets is sold?

A 100
B 120
C 500
D 5400
- Maya is selling T-shirts to raise money for the student council. She makes \$6.75 for every T-shirt that she sells. If she wants to raise \$800 selling the T-shirts, what is a reasonable number of shirts she should sell?

F 100
G 120
H 500
J 5400
- GRIDDABLE** Mr. Hiskey is making a doll-sized model of his house for his daughter. The model is $\frac{1}{16}$ the size of the actual house. If the door in the model is 6 inches tall, how many inches tall is the actual door on the house?

A 6
B 16
C 96
D 100

10. Carter is riding his bicycle. He leaves his house and stops at a park that is 25 miles from his house. After the stop, he continues to ride his bicycle at a constant rate of 18 miles per hour away from his house. Which equation could be used to determine the time in hours t it will take him to reach a distance of 120 miles from his house?

- A $120 = 18t + 25$
- B $120 = t(18 + 25)$
- C $120 = 18(t + 25)$
- D $120 = 25t + 18$

11. The amount of water needed to fill a balloon best represents the balloon's _____.

- F volume
- G surface area
- H circumference
- J perimeter

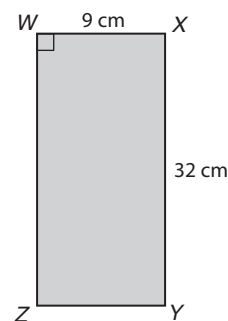
12. The scale factor of two similar triangles is 5:2. The perimeter of the smaller triangle is 108 inches. What is the perimeter of the larger triangle?

- A 540 in.
- B 270 in.
- C 216 in.
- D 43.2 in.

13. Bailey estimates that his income has gone up \$2000 each year from 1999 to 2005. What additional information is needed to calculate his income in 2005?

- F The range of his income from 1999 to 2005.
- G His income in 1999.
- H How much he thinks his income will go up in the next year.
- J The national average income.

14. Rectangle WXYZ is shown below.



If each dimension of the rectangle is doubled, what is the perimeter of WXYZ?

- A 82 cm
- B 100 cm
- C 164 cm
- D 288 cm

Pre-AP

Record your answers on a sheet of paper.
Show your work.

15. Latoya bought 48 one-foot-long sections of fencing. She plans to use the fencing to enclose a rectangular area for a garden.

- a. Using ℓ for the length and w for the width of the garden, write an equation for its perimeter.
- b. If the length ℓ in feet and width w in feet are positive integers, what is the greatest possible area of this garden?
- c. If the length and width in feet are positive integers, what is the least possible area of the garden?
- d. How do the shapes of the gardens with the greatest and least areas compare?

NEED EXTRA HELP?

If You Missed Question...	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
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