UNIT 2
Linear Functions

Focus

Use linear functions and inequalities to represent and model real-world situations.

CHAPTER 3
Functions and Patterns

BIG Idea Understand the concepts of a relation and a function, and determine whether a given relation defined by a graph, a set of ordered pairs, or a symbolic expression is a function.

CHAPTER 4
Analyzing Linear Equations

BIG Idea Graph a linear equation, compute an equation's *x*- and *y*-intercepts, and derive linear equations by using the point-slope formula.

CHAPTER 5
Solving Systems of Linear Equations

BIG Idea Solve a system of two linear equations in two variables algebraically and interpret the answer graphically.

CHAPTER 6
Solving Linear Inequalities

BIG Idea Solve absolute value inequalities, multistep problems involving linear inequalities in one variable, and solve a system of two linear inequalities in two variables.

Cross-Curricular Project

Algebra and Sports

The Spirit of the Games The first Olympic Games featured only one event–a foot race. In 2004, the Olympic Games featured thousands of competitors in about 300 events. The 2004 summer games were held in Athens, Greece. In this project, you will explore how linear functions can be used to represent times in Olympic events.

MathOnline **Log on to algebra1.com to begin.**

CHAPTER 3

Functions and Patterns

BIG Ideas

- Understand the skills required to manipulate symbols to solve problems and simplify expressions.
- Understand the meaning of the slope and intercepts of the graphs of linear functions.

Key Vocabulary

arithmetic sequence (p. 165)
function (p. 149)
inverse (p. 145)
y-intercept (p. 156)

Real-World Link

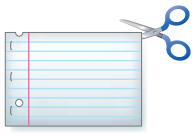
Currency A function is a rule or a formula. You can use a function to describe real-world situations like converting between currencies. For example, in Japan, an item that costs 10,000 yen is equivalent to about 87 U.S. dollars.

FOLDABLES™ Study Organizer

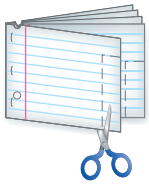
Functions and Patterns Make this Foldable to help you organize your notes about graphing relations and functions. Begin with three sheets of notebook paper.

1. **Fold** each sheet of paper in half from top to bottom.

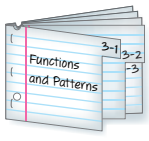
2. **Cut** along fold. Staple the six half-sheets together to form a booklet.



3. **Cut** tabs into margin. The top tab is 2 lines deep, the next tab is 6 lines deep, and so on.



4. **Label** each of the tabs with a lesson number. Use the last page for vocabulary.



GET READY for Chapter 3

Diagnose Readiness You have two options for checking Prerequisite Skills.

Option 1

Take the Quick Check below. Refer to the Quick Review for help.

QUICK Check

Evaluate each expression if $a = -1$, $b = 4$, and $c = -3$. (Lesson 1-2)

1. $a + b - c$

2. $2c - b$

3. $3a - 6b - 2c$

4. $6a + 8b + \frac{2}{3}c$

5. FOOD Noah is buying a sandwich with 1 type of meat, 2 types of cheese, and 2 types of vegetable. Each topping costs \$1.55, \$0.65, and \$0.85 respectively. How much will Noah spend on the sandwich?

Solve each equation for y. (Lesson 2-8)

6. $2x + y = 1$

7. $x = 8 - y$

8. $6x - 3y = 12$

9. $2x + 3y = 9$

10. $9 - \frac{1}{2}y = 4x$

11. $\frac{y + 5}{3} = x + 2$

Graph each ordered pair on a coordinate grid. (Lesson 1-9)

12. (3, 0)

13. (−2, 1)

14. (−3, 3)

15. (−5, 5)

16. (0, 6)

17. (2, −1)

18. MAPS Taylor is looking at a map and needs to go 3 blocks east and 2 blocks south from where he is standing now. If he is standing at (0, 0), what will his coordinates be when he arrives at his destination?

QUICK Review

EXAMPLE 1

Evaluate $a + 2b + 3c$ if $a = -1$, $b = 4$, and $c = -3$.

$a + 2b + 3c$	Original expression
$= (-1) + 2(4) + 3(-3)$	Substitute -1 for a, 4 for b, and -3 for c.
$= -1 + 8 - 9$	Multiply.
$= -2$	Simplify.

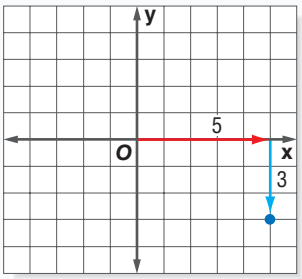
EXAMPLE 2

Solve $x - 2 = \frac{y}{3}$ for y.

$x - 2 = \frac{y}{3}$	Original equation
$3(x - 2) = (3)\frac{y}{3}$	Multiply each side by 3.
$3x - 6 = y$	Simplify.

EXAMPLE 3

Graph (5, −3) on a coordinate grid.



Start at the origin. Since the x-coordinate is 5, move 5 units to the right.

Since the y-coordinate is −3, move down 3 units. Draw a dot.

Option 2

MathOnline Take the Online Readiness Quiz at algebra1.com.

EXPLORE 3-1

Algebra Lab

Modeling Relations

The observation of patterns is used in many disciplines such as science, history, economics, social studies, and mathematics. When a quantity depends on another, the pattern can be described in many ways.

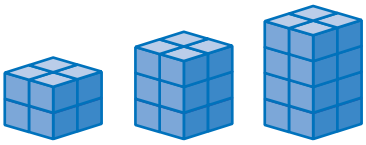
ACTIVITY

Step 1 Use centimeter cubes to build a tower similar to the one shown at the right.

Step 2 Copy the table below. Record the number of layers in the tower and the number of cubes used to build it in the table.

Layers	Cubes
1	4
2	
3	
4	
5	
6	
7	
8	

Step 3 Add layers to the tower. Record the number of layers and the number of cubes in each tower.



Analyze the Results

Study the data you recorded in the Activity.

1. As the number of layers in the tower increases, how does the number of cubes in the tower change?
2. If there are n layers in a tower, how many cubes are there in the tower? Explain.
3. Write the data in your table as ordered pairs (layers, cubes). Graph the ordered pairs.

Extension

4. Copy and complete the table at the right for the towers that you built. To determine the surface area, count the number of squares showing on each tower, including those on the base. (Hint: The surface area of the 1-layer tower above is 16.)
5. When a layer is added to the tower, what is the effect on the surface area of the tower? Explain.

Layers	Surface Area
1	16
2	
3	
4	
5	
6	
7	
8	

3-1 Representing Relations

Main Ideas

- Represent relations as sets of ordered pairs, tables, mappings, and graphs.
- Find the inverse of a relation.

New Vocabulary

mapping
inverse

GET READY for the Lesson

Ken Griffey, Jr.'s batting statistics for home runs and strikeouts can be represented as a set of ordered pairs. The number of home runs are the first coordinates, and the number of strikeouts are the second coordinates.

You can plot the ordered pairs on a graph to look for patterns.

Ken Griffey, Jr.		
Year	Home Runs	Strikeouts
1998	56	121
1999	48	108
2000	40	117
2001	22	72
2002	8	39
2003	13	44
2004	20	67

Source: baseball-reference.com

Study Tip

Look Back

To review **relations**, see Lesson 1-9.

Represent Relations Recall that a *relation* is a set of ordered pairs. A relation can also be represented by a table, a graph, or a mapping. A **mapping** illustrates how each element of the domain is paired with an element in the range.

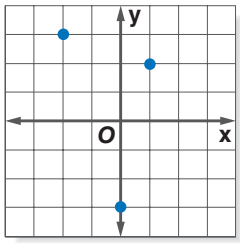
Ordered Pairs

(1, 2)
(−2, 3)
(0, −3)

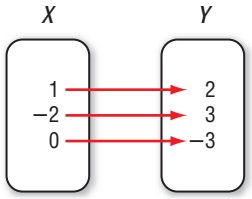
Table

x	y
1	2
−2	3
0	−3

Graph



Mapping



EXAMPLE 1 Represent a Relation

a. Express the relation {(3, 2), (−1, 2), (0, −3), (−2, −2)} as a table, a graph, and a mapping.

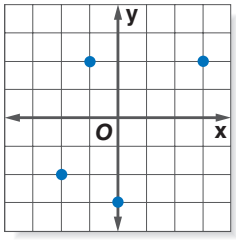
Table

List the x-coordinates in the first column and the corresponding y-coordinates in the second column.

x	y
3	2
−1	2
0	−3
−2	−2

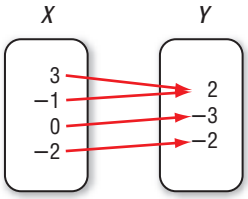
Graph

Graph each ordered pair on a coordinate plane.



Mapping

List the x-values in set X and the y-values in set Y. Draw arrows from the x-values in X to the corresponding y-values.



Concepts in MOtion
Animation algebra1.com

(continued on the next page)

Study Tip

Domain and Range

When writing the elements of the domain and range, if a value is repeated, you need to list it only once.

b. Determine the domain and range.

The domain for this relation is {−2, −1, 0, 3}.

The range is {−3, −2, 2}.

CHECK Your Progress

1A. Express the relation {(−4, 8), (−1, 9), (−4, 7), (6, 9)} as a table, a graph, and a mapping.

1B. Determine the domain and range of the relation.

Recall that the domain of a relation is the set of values of the independent variable and the range is the set of values of the dependent variable. This is useful when using relations that represent real-life situations.

Real-World Link

An average student between the ages of 12 and 17 spends 10.3 hours per week with friends outside of school and about 7.8 hours per week talking to friends using the telephone, E-mail, instant messaging, or text messaging.

Source: www.pewinternet.org

Real-World EXAMPLE

2 **ANALYZE TABLES** **The table shows the results of a recent survey in which students were asked about their use of text messaging.**

Percent of Students Who Use Text Messaging						
Age of Students	12	13	14	15	16	17
Percent	17	30	34	45	46	54

Source: www.pewinternet.org

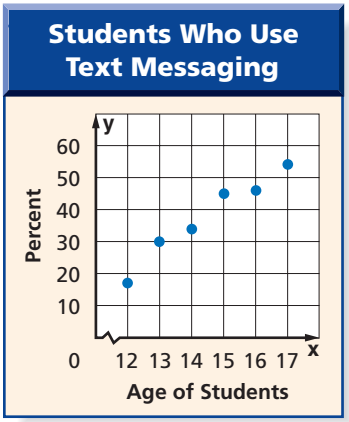
a. Determine the domain and range of the relation.

The domain is {12, 13, 14, 15, 16, 17} because age is the independent variable. It is unaffected by the percents.

The range is {17, 30, 34, 45, 46, 54} because the percent of teens who use text messaging depends on the age of the teens.

b. Graph the data.

- The values of the x-axis need to go from 12 to 17. It is not practical to begin the scale at 0. Begin at 12 and extend to 17 to include all of the data. The units can be 1 unit per grid square.
- The values on the y-axis need to go from 17 to 54. In this case, you can begin the scale at 0 and extend to 60. You can use units of 10.



c. What conclusions might you make from the graph of the data?

There is a steady increase in the percent of students who use text messaging as the students get older.

CHECK Your Progress

MONEY **Leticia earns $7 for walking 1 dog, $28 for walking 4 dogs, $42 for walking 6 dogs, and $49 for walking 7 dogs.**

2A. Determine the domain and range of the relation.

2B. Graph the data.

Online Personal Tutor at algebra1.com

Inverse Relations The **inverse** of any relation is obtained by switching the coordinates in each ordered pair. The domain of a relation becomes the range of the inverse and the range of a relation becomes the domain of the inverse.

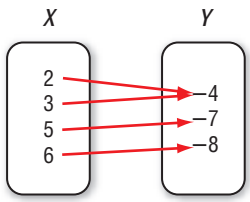
KEY CONCEPT *Inverse of a Relation*

Relation Q is the inverse of relation S if and only if for every ordered pair (a, b) in S, there is an ordered pair (b, a) in Q.

Relation	Inverse of Relation
$\{(0, 2), (-5, 4)\}$	$\{(2, 0), (4, -5)\}$

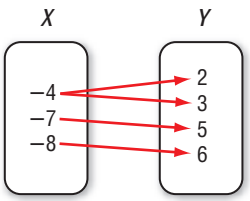
EXAMPLE Inverse Relation

3 **Express the relation shown in the mapping as a set of ordered pairs. Then write the inverse of the relation.**



Relation Notice that both 2 and 3 in the domain are paired with −4 in the range. $\{(2, -4), (3, -4), (5, -7), (6, -8)\}$

Inverse Exchange x and y in each ordered pair to write the inverse relation. $\{(-4, 2), (-4, 3), (-7, 5), (-8, 6)\}$



The mapping of the inverse is shown at the right. Compare this to the mapping of the relation.

Study Tip

Functions The relation in Example 3 is a function, but the inverse of the relation is not. You will learn more about functions in Lesson 3-2.

CHECK Your Progress

3. Express the relation shown in the table as a set of ordered pairs. Then write the inverse of the relation.

x	1	3	5	7
y	2	4	6	8

ALGEBRA LAB

Relations and Inverses

- Graph the relation $\{(3, 4), (-2, 5), (-4, -3), (5, -6), (-1, 0), (0, 2)\}$ on grid paper using a colored pencil. Connect the points in order.
- Use a different colored pencil to graph the inverse of the relation, connecting the points in order.
- Fold the grid paper so that the positive y-axis lies on top of the positive x-axis. Hold the paper up to a light to view the points.

ANALYZE

1. What do you notice about the location of the points?
2. Unfold the paper. Describe the transformation of each point and its inverse.
3. What do you think are the ordered pairs that represent the points on the fold line? Describe these in terms of x and y.
4. How could you graph the inverse of a function without writing ordered pairs first?

CHECK Your Understanding

Example 1 (pp. 143–144)

Express each relation as a table, a graph, and a mapping. Then determine the domain and range.

1. {(5, −2), (8, 3), (−7, 1)}

2. {(6, 4), (3, −3), (−1, 9), (5, −3)}

Example 2 (p. 144)

COOKING For Exercises 3 and 4, use the table. Recipes often have different cooking times for high altitudes because water boils at a lower temperature.

3. Determine the domain and range of the relation and then graph the data.

4. Use your graph to estimate the boiling point of water at an altitude of 7000 feet.

Altitude (feet)	Boiling Point of Water (°F)
0	212.0
1000	210.2
2000	208.4
3000	206.5
5000	201.9
10,000	193.7

Source: Stevens Institute of Technology

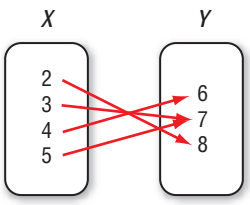
Example 3 (p. 145)

Express the relation shown in each table, mapping, or graph as a set of ordered pairs. Then write the inverse of the relation.

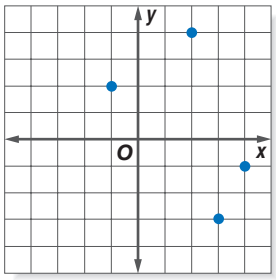
5.

x	y
3	−2
−6	7
4	3
−6	5

6.



7.



Exercises

HOMEWORK HELP

For Exercises	See Examples
8–13	1
14–21	2
22–27	3

Express each relation as a table, a graph, and a mapping. Then determine the domain and range.

8. {(0, 0), (6, −1), (5, 6), (4, 2)}

9. {(3, 8), (3, 7), (2, −9), (1, −9)}

10. {(4, −2), (3, 4), (1, −2), (6, 4)}

11. {(0, 2), (−5, 1), (0, 6), (−1, 9)}

12. {(3, 4), (4, 3), (2, 2), (5, −4), (−4, 5)}

13. {(7, 6), (3, 4), (4, 5), (−2, 6), (−3, 2)}

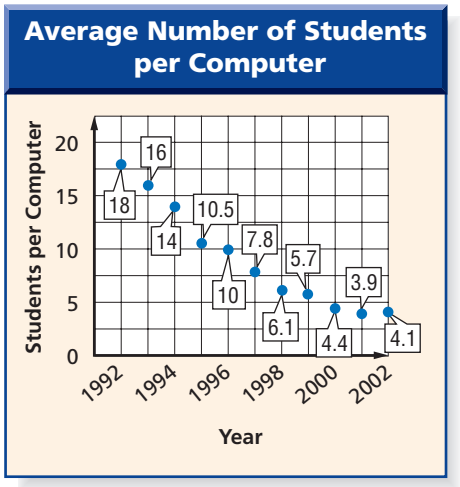
ANALYZE GRAPHS For Exercises 14–17, use the graph of the average number of students per computer in U.S. public schools.

14. Name three ordered pairs from the graph.

15. Determine the domain of the relation.

16. What are the least and greatest range values?

17. What conclusions can you make from the graph of the data?



Source: Quality Education Data

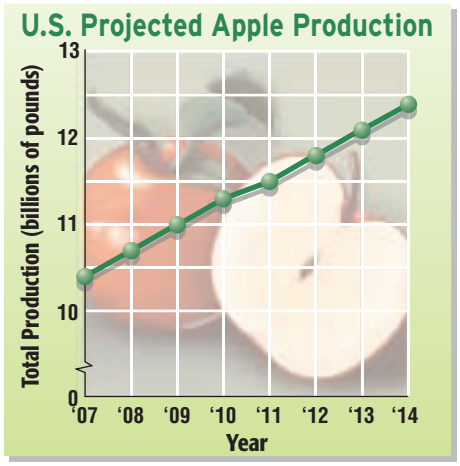
FOOD **For Exercises 18–21, use the graph that shows the projected annual production of apples from 2007–2014.**

18. Estimate the domain and range.

19. Which year is projected to have the lowest production? the highest?

20. Describe any patterns that you see.

21. What is a reasonable range value for a domain value of 2015? Explain what this ordered pair represents.



Source: National Food and Agricultural Policy Project Outlook

Express the relation shown in each table, mapping, or graph as a set of ordered pairs. Then write the inverse of the relation.

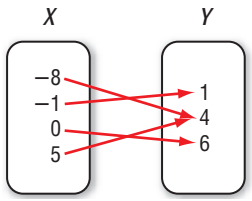
22.

x	y
0	3
−5	2
4	7
−8	2

23.

x	y
0	0
4	7
8	10.5
12	18
16	14.5

24.



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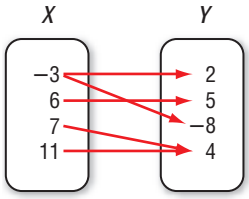

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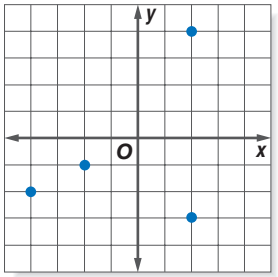

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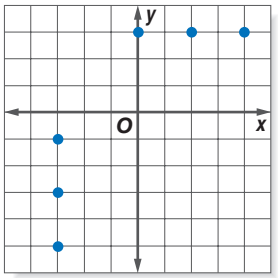


Real-World Link

Studies have shown that students who own fish score higher on both math and verbal SATs, with a combined score 200 points higher than non-pet owners.

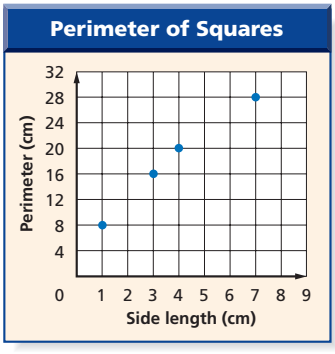
Source: diveintofish.com/media/med_facts.html

Express each relation as a set of ordered pairs and describe the domain and range. Then write the inverse of the relation.

28.

Buying Aquarium Fish	
Number of Fish	Total Cost ($)
1	2.50
2	5.50
5	10.00
8	18.75

29.



BIOLOGY **For Exercises 30–33, use the fact that a person typically has about 2 pounds of muscle for each 5 pounds of body weight.**

30. Make a table to show the relation between body and muscle weight for people weighing 100, 105, 110, 115, 120, 125, and 130 pounds.

31. State the domain and range and then graph the relation.

32. What are the domain and range of the inverse?

33. Graph the inverse relation.

EXTRA PRACTICE
See pages 723, 746.

Math Online
Self-Check Quiz at algebra1.com

H.O.T. Problems

34. CHALLENGE Find a counterexample to disprove the following.

The domain of relation F contains the same elements as the range of relation G. The range of relation F contains the same elements as the domain of relation G. Therefore, relation G must be the inverse of relation F.

35. OPEN ENDED Describe a real-life situation that can be represented using a relation and discuss how one of the quantities in the relation depends on the other. Then give an example of such a relation in three different ways.

36. *Writing in Math* Use the information about batting statistics on page 143 to explain how relations can be used to represent baseball statistics. Include a graph of the relation of the number of Ken Griffey, Jr.'s, home runs and his strikeouts. Describe the relationship between the quantities.

STANDARDIZED TEST PRACTICE

37. What is the domain of the function that contains the points at (0, -3), (−2, 4), (4, −3), and (−3, 1)?

A {−3, −2}

B {−3, 1, 4}

C {−3, −2, 0, 1}

D {−3, −2, 0, 4}

38. REVIEW Kara deposited $2000 into a savings account that pays 1.5% interest compounded annually. If she does not deposit any more money into her account, how much will she earn in interest at the end of one year?

F $30

G $35

H $300

J $350

Spiral Review

39. CHEMISTRY Jamaal has 20 milliliters of a 30% solution of nitric acid. How many milliliters of a 15% solution should he add to obtain a 25% solution of nitric acid? (Lesson 2-9)

Solve each equation or formula for the variable specified. (Lesson 2-8)

40. $3x + b = 2x + 5$ for x

41. $6w - 3h = b$ for h

42. HOURLY PAY Dominique earned $9.75 per hour before her employer increased her hourly rate to $10.15 per hour. What was the percent of increase in her salary? (Lesson 2-7)

GET READY for the Next Lesson

PREREQUISITE SKILL **Evaluate each expression.** (Lesson 1-2)

43. $12 \div 4 + 15 \cdot 3$

44. $12(19 - 15) - 3 \cdot 8$

45. $(25 - 4) \div (2^2 - 1)$

3-2 Representing Functions

Main Ideas

- Determine whether a relation is a function.
- Find functional values.

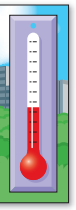
New Vocabulary

function
vertical line test
function notation
function value

GET READY for the Lesson

The table shows barometric pressures and temperatures recorded by the National Climatic Data Center over a three-day period.

Pressure (millibars)	1013	1006	997	995	995	1000	1006	1011	1016	1019
Temperature (C)	3	4	10	13	8	4	1	–2	–6	–9



Notice that when the pressure is 995 and 1006 millibars, there is more than one value for the temperature.

Study Tip

Look Back

To review **relations and functions,** see Lesson 1-9.

Identify Functions Recall that relations in which each element of the domain is paired with exactly one element of the range are called **functions**.

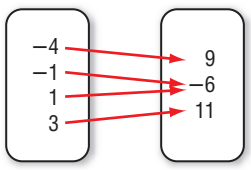
KEY CONCEPT *Function*

A function is a relation in which each element of the domain is paired with *exactly* one element of the range.

EXAMPLE 1 Identify Functions

Determine whether each relation is a function. Explain.

a.



For each element of the domain, there is only one corresponding element in the range. So, this mapping represents a function. It does not matter if two elements of the domain are paired with the same element in the range.

b.

x	y
−3	6
2	5
3	1
2	4

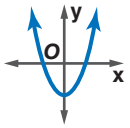
The element 2 in the domain is paired with both 5 and 4 in the range. So, if x is 2, there are two possible values for y. The relation in this table does not represent a function.

CHECK Your Progress

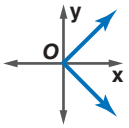
1. {(−2, 4), (1, 5), (3, 6), (5, 8), (7, 10)}

You can use the **vertical line test** to see if a graph represents a function. If no vertical line can be drawn that intersects the graph more than once, then the graph is a function. If a vertical line can be drawn so that it intersects the graph at two or more points, the graph is not a function.

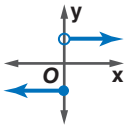
Function



Not a Function



Function



Concepts in Motion
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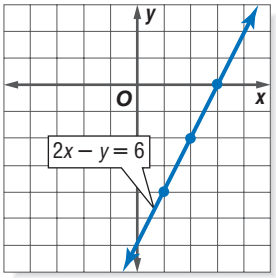
EXAMPLE 2 Equations as Functions

Determine whether $2x - y = 6$ represents a function.

Make a table and plot points to graph the equation.

x	1	2	3
y	−4	−2	0

Since the equation is in the form $Ax + By = C$, the graph of the equation will be a line. Place a pencil at the left of the graph to represent a vertical line. Slowly move the pencil to the right across the graph.



For each value of x, this vertical line passes through no more than one point on the graph. Thus, the graph represents a function.

CHECK Your Progress

2. Determine whether $x = -2$ is a function.

Review Vocabulary

Independent/ Dependent Variables
In a function, the value of the dependent variable depends on the value of the independent variable. (Lesson 1-9)

Function Values Equations that are functions can be written in a form called **function notation**. For example, consider $y = 3x - 8$.

equation	function notation
$y = 3x - 8$	$f(x) = 3x - 8$

In a function, x represents the independent quantity, or the elements of the domain and $f(x)$ represents the dependent quantity, or the elements of the range. For example, $f(5)$ is the element in the range that corresponds to the element 5 in the domain. We say that $f(5)$ is the **function value** of f for $x = 5$.

Reading Math

Function Notation The symbol $f(x)$ is read *f* of *x*. The symbol $f(5)$ is read *f of 5*.

EXAMPLE 3 Function Values

If $f(x) = 2x + 5$, find each value.

a. $f(-2)$

$f(-2) = 2(-2) + 5$ Replace x with -2.
$= -4 + 5$ Multiply.
$= 1$ Add.

b. $f(1) + 4$

$f(1) + 4 = [2(1) + 5] + 4$ Replace x with 1.
$= 7 + 4$ Simplify.
$= 11$ Add.

CHECK Your Progress

3A. $f(3)$ **3B.** $2 - f(0)$

Extra Examples at algebra1.com

Reading Math

Functions Other letters such as g and h can be used to represent functions. For example, $g(x)$ is read *g of x* and $h(t)$ is read *h of t*.

EXAMPLE 4 Nonlinear Function Values

PHYSICS **The function $h(t) = -16t^2 + 68t + 2$ represents the height $h(t)$ of a football in feet t seconds after it is kicked. Find each value.**

a. $h(4)$

$h(4) = -16(4)^2 + 68(4) +$ Replace t with 4.

$= -256 + 272 + 2$ Multiply.

$= 18$ Simplify.

b. $2[h(g)]$

$2[h(g)] = 2[-16(g)^2 + 68(g) + \ ]$ Evaluate $h(g)$ by replacing t with g.

$= 2(-16g^2 + 68g + \ )$ Simplify.

$= -32g^2 + 136g + 4$ Multiply the value of $h(g)$ by 2.

CHECK Your Progress **If $f(t) = 2t^3$, find each value.**

4A. $f(4)$ **4B.** $3[f(t)]$

Test-Taking Tip

Functions
When representing functions, determine the values of the domain and range that make sense for the given situation.

STANDARDIZED TEST EXAMPLE 5

The algebraic form of a function is $s = 9h$, where s is Barbara's weekly salary and h is the number of hours that she works in a week. Which of the following represents the same function?

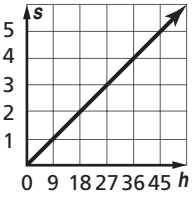
A For every week Barbara works, she earns \$9.

C $f(h) = 9$

B

h	s
0	0
2	18
4	36

D



Read the Test Item

The independent variable is h and the dependent variable is s.

Solve the Test Item

Choices A and C both represent a constant weekly salary of \$9. This is incorrect because the salary depends on the number of hours worked.

Choice D represents $s = \frac{1}{9}h$, which is incorrect. In choice B, the salary equals the number of hours times 9, which is correct. The answer is B.

CHECK Your Progress

5. Which statement represents the function that is described below?

For every minute that Beatriz walks, she walks 0.1 mile.

F $f(t) = 0.1t$ **G** $f(t) = t + 0.1$ **H** $f(t) = 0.1 - t$ **J** $f(t) = t - 0.1$

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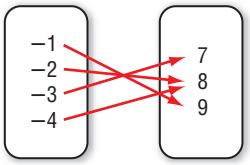
CHECK Your Understanding

Example 1 (p. 149)

Determine whether each relation is a function.

1.



2.

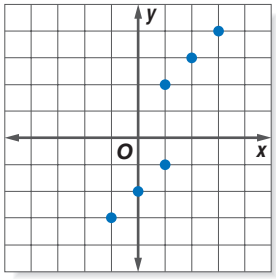
x	y
−3	0
2	1
2	4
6	5

3. $\{(-4, 1), (-1, 4), (3, -2), (-4, 5)\}$

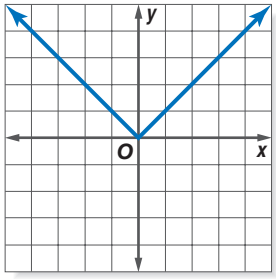
4. $y = x + 3$

Example 2 (p. 150)

5.



6.



Examples 3, 4 (pp. 150–151)

If $f(x) = 4x - 5$ and $g(x) = x^2 + 1$, find each value.

7. $f(2)$ **8.** $f(c)$ **9.** $f(x + 5)$

10. $g(-1)$ **11.** $g(t) - 4$ **12.** $g(3n)$

13. CELL PHONE PICTURES The cost of sending cell phone pictures is given by $y = 0.25x$, where x is the number of pictures that you send. Write the equation in function notation and then find $f(5)$ and $f(12)$. What do these values represent?

Example 5 (p. 151)

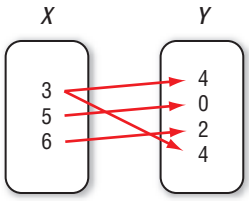
14. STANDARDIZED TEST PRACTICE Represent the function described in Exercise 13 in two different ways.

Exercises

HOMEWORK HELP	
For Exercises	See Examples
15–22	1
23–26	2
27–38	3, 4
39–44	5

Determine whether each relation is a function.

15.



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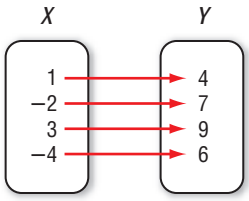


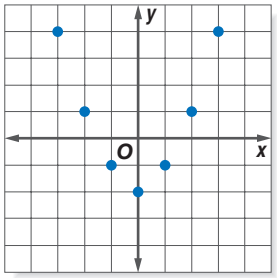
17.

x	y
2	7
4	9
5	5
8	−1

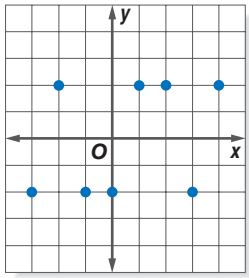
18.

x	y
−9	−5
−4	0
3	6
7	1
6	−5
3	2

19.



20.



Cross-Curricular Project

Math Online A graph of the winning Olympic swimming times will help you determine whether the winning time is a function of the year. Visit algebra1.com to continue work on your project.

Determine whether each relation is a function.

21. $\{(5, -7), (6, -7), (-8, -1), (0, -1)\}$

22. $\{(4, 5), (3, -2), (-2, 5), (4, 7)\}$

23. $y = -8$

24. $x = 15$

25. $y = 3x - 2$

26. $y = 3x + 2y$

If $f(x) = 3x + 7$ and $g(x) = x^2 - 2x$, find each value.

27. $f(3)$

28. $f(-2)$

29. $g(5)$

30. $g(0)$

31. $g(-3) + 1$

32. $f(8) - 5$

33. $g(2c)$

34. $g(4n)$

35. $f(k + 2)$

36. $f(a - 1)$

37. $3[f(r)]$

38. $2[g(t)]$

METEOROLOGY **For Exercises 39–42, use the following information.**
The temperature of the atmosphere decreases about 5°F for every 1000 feet increase in altitude. Thus, if the temperature at ground level is 77°F, the temperature at an altitude of h feet is found by using $t = 77 - 0.005h$.

39. Write the equation in function notation. Then find $f(100)$, $f(200)$, and $f(1000)$.

40. Suppose the temperature at ground level was less than 77°F. Describe how the range values in Exercise 39 would change. Explain.

41. Graph the function.

42. Use the graph of the function to estimate the temperature at 4000 feet.

Real-World Career
Meteorologist

Meteorologists study the physical characteristics of the atmosphere using instruments and tools such as weather balloons. This information is used to interpret and predict trends in the weather.

Math Online For more information, go to algebra1.com.

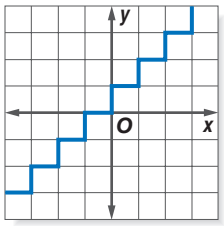
EDUCATION **For Exercises 43 and 44, use the following information.**
The average national math test scores $f(s)$ for 17-year-olds can be represented as a function of the national science scores s by $f(s) = 0.8s + 72$.

43. Graph this function.

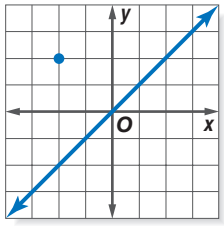
44. What is the science score that corresponds to a math score of 308?

Determine whether each relation is a function.

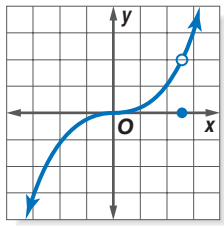
45.



46.

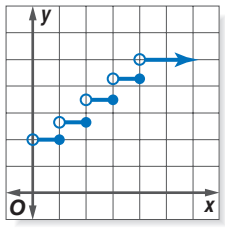


47.

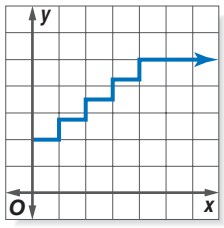


48. **PARKING** A parking garage charges \$2.00 for the first hour, \$2.75 for the second, \$3.50 for the third, \$4.25 for the fourth, and \$5.00 for any time over four hours. Choose the graph that best represents the information and determine whether the graph represents a function. Explain.

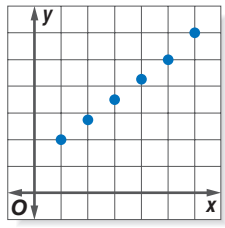
a.



b.



c.



EXTRA PRACTICE
See pages 723, 746.

Math Online Self-Check Quiz at algebra1.com.

H.O.T. Problems

REASONING **For Exercises 49 and 50, refer to the following information.** The ordered pairs (0, 1), (3, 2), (3, −5), and (5, 4) are on the graph of a relation between x and y.

49. Determine whether x is a function of y. Explain.

50. Determine whether y is a function of x. Explain.

51. CHALLENGE State whether the following is *sometimes*, *always*, or *never* true. Explain your reasoning. *The inverse of a function is also a function.*

52. OPEN ENDED Disprove the following statement by finding a counterexample. *All linear equations are functions.*

53. *Writing in Math* Use the information on page 149 to explain how functions are used in meteorology. Describe the relationship between pressure and temperature, and investigate whether the relation is a function.

STANDARDIZED TEST PRACTICE

54. Which relation is a function?

A {(−5, 6), (4, −3), (2, −1), (4, 2)}

B {(3, −1), (3, −5), (3, 4), (3, 6)}

C {(−2, 3), (0, 3), (−2, −1), (−1, 2)}

D {(−5, 6), (4, −3), (2, −1), (4, 2)}

55. REVIEW If $a = -4$ and $b = 8$, then $3a(b + 2) + a =$

F −124

G −98

H −26

J 18

Spiral Review

56. Express the relation shown in the table as a set of ordered pairs. Then write the inverse of the relation. (Lesson 3-1)

x	y
−4	9
2	5
−2	−2
11	12

57. AIRPLANES At 1:30 P.M., an airplane leaves Tucson for Baltimore, a distance of 2240 miles. The plane flies at 280 miles per hour. A second airplane leaves Tucson at 2:15 P.M. and is scheduled to land in Baltimore 15 minutes before the first airplane. At what rate must the second airplane travel to arrive on schedule? (Lesson 2-9)

56. RUNNING Lacey can run a 10K race (about 6.2 miles) in 45 minutes. If she runs a 26-mile marathon at the same pace, how long will it take her to finish? (Lesson 2-6)

GET READY for the Next Lesson

PREREQUISITE SKILL **Solve each equation.** (Lesson 2-4)

59. $r - 9 = 12$

60. $-4 = 5n + 6$

61. $3 - 8w = 35$

62. $\frac{g}{4} + 2 = 5$

3-3 Linear Functions

Main Ideas

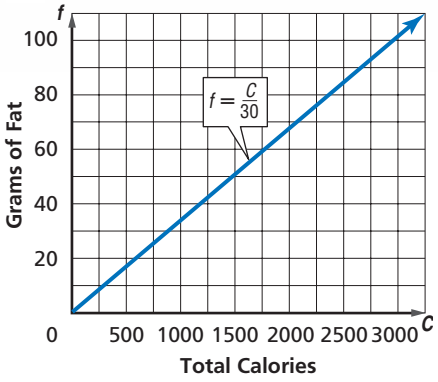
- Identify linear equations, intercepts, and zeros.
- Graph linear equations.

New Vocabulary

linear equation
standard form
x-intercept
y-intercept
zero

GET READY for the Lesson

It is recommended that no more than 30% of a person's daily caloric intake come from fat. Since each gram of fat contains 9 Calories, the most grams of fat f that you should have each day is given by $f = 0.3\left(\frac{C}{9}\right)$ or $f = \frac{C}{30}$. C is the total number of Calories C that you consume. The graph of this equation shows the maximum number of grams of fat you should consume based on the total number of Calories you consume.



Identify Linear Equations, Intercepts, and Zeros A **linear equation** is the equation of a line. Linear equations can often be written in the form $Ax + By = C$. This is called the **standard form** of a linear equation.

KEY CONCEPT *Standard Form of a Linear Equation*

The standard form of a linear equation is

$$Ax + By = C,$$

where $A \geq 0$, A and B are not both zero, and A, B, and C are integers with a greatest common factor of 1.

EXAMPLE **Identify Linear Equations**

1 **Determine whether each equation is a linear equation. If so, write the equation in standard form.**

a. $y = 5 - 2x$

Rewrite the equation so that both variables are on the same side of the equation.

$y = 5 - 2x$	Original equation
$y + 2x = 5 - 2x + 2x$	Add $2x$ to each side.
$2x + y = 5$	Simplify.

The equation is now in standard form where $A = 2$, $B = 1$, and $C = 5$. This is a linear equation.

b. $2xy - 5y = 6$

Since the term $2xy$ has two variables, the equation cannot be written in the form $Ax + By = C$. Therefore, this is not a linear equation.

CHECK Your Progress

1A. $\frac{1}{3}y = -1$ **1B.** $y = x^2 + 3$

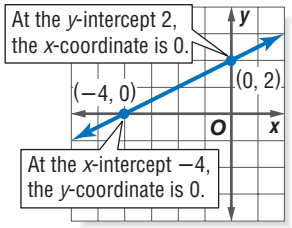
Study Tip

Linear Functions

The graph of a linear function has at most one x-intercept and one y-intercept, unless it is the function $f(x) = 0$, in which case every point of the graph is an x-intercept.

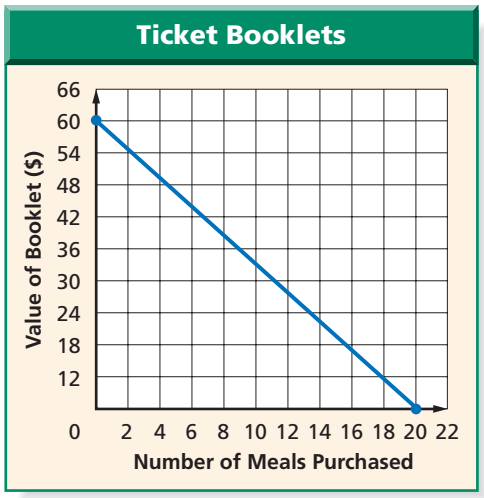
The x-coordinate of the point at which the graph of an equation crosses the x-axis is an **x-intercept**. The y-coordinate of the point at which the graph crosses the y-axis is called a **y-intercept**.

Values of x for which $f(x) = 0$ are called **zeros** of the function f. The zero of a function is its x-intercept.



Real-World EXAMPLE

2 **ANALYZE GRAPHS** **High school students in Palo Alto, California, can buy ticket booklets for lunch, as shown in the graph.**



a. Determine the x-intercept, y-intercept, and zero.

The x-intercept is 20 because it is the x-coordinate of the point where the line crosses the x-axis. The zero of the function is also 20.

The y-intercept is 60 because it is the y-coordinate of the point where the line crosses the y-axis.

Study Tip

Intercepts

Usually, the individual coordinates are called the x- and y-intercepts.

- The x-intercept 20 is located at (20, 0).
- The y-intercept 60 is located at (0, 60).

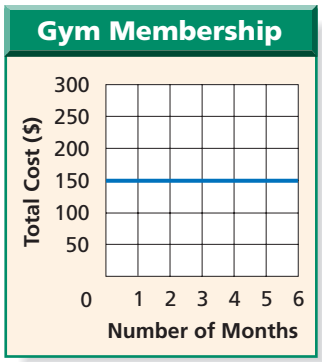
b. Describe what the intercepts mean.

The x-intercept 20 means that after 20 meals are purchased, the meal ticket booklet has a value of \$0.

The y-intercept 60 means before any meals are purchased, the booklet has a value of \$60.

CHECK Your Progress

HEALTH **Use the graph at the right that shows the cost of a gym membership.**



2A. Determine the x-intercept, y-intercept, and zero.

2B. Describe what the intercept(s) mean.

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Real-World EXAMPLE

Study Tip

Defining Variables

Time is the *independent* variable, and volume of water is the *dependent* variable.

3 **ANALYZE TABLES** **It is recommended that a swimming pool be drained at a maximum rate of 720 gallons per hour. The table shows the function relating the volume of water in a pool and the time in hours that the pool has been draining.**

Draining a Pool	
Time, x (h)	Volume, y (gal)
0	10,080
2	8640
6	5760
10	2880
12	1440
14	0

a. Determine the x-intercept, y-intercept, and zero of the graph of the function.

x-intercept $= 14$ — 14 is the value of x when $y = 0$.

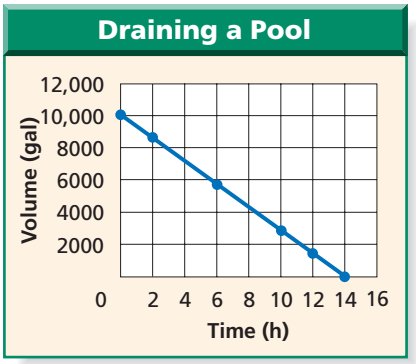
y-intercept $= 10{,}080$ — 10,080 is the value of y when $x = 0$.

zero $= 14$ — The zero of the function is the x-intercept of the graph.

b. Describe what the intercepts mean.

The x-intercept 14 means that after 14 hours, the water has a volume of 0 gallons, or the pool is completely drained.

The y-intercept 10,080 means that the pool contained 10,080 gallons of water at time 0, or before it started to drain. This is shown in the graph.



CHECK Your Progress

3. Use the table to determine the x-intercept, y-intercept, and zero of the graph of the function.

x	−3	−2	−1	0	1
y	2	0	−2	−4	−6

Concepts in Motion

Animation

algebra1.com

Graph Linear Equations

The graph of an equation represents all its solutions. So, every ordered pair that satisfies the equation represents a point on the line. An ordered pair that does not satisfy the equation represents a point *not* on the line.

EXAMPLE **Graph by Making a Table**

4 Graph $y = \frac{1}{2}x - 3$.

Select values from the domain and make a table. Then graph the ordered pairs. The domain is all real numbers, so there are infinitely many solutions. Draw a line through the points.

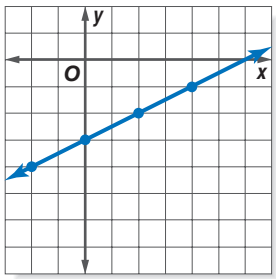
Study Tip

Rewriting Equations

When appropriate, first solve an equation for y in order to find values for y more easily.

$3x + y = -1 \rightarrow y = -1 - 3x$

x	$\frac{1}{2}x - 3$	y	(x, y)
−2	$\frac{1}{2}(-2) - 3$	−4	(−2, −4)
0	$\frac{1}{2}(0) - 3$	−3	(0, −3)
2	$\frac{1}{2}(2) - 3$	−2	(2, −2)
4	$\frac{1}{2}(4) - 3$	−1	(4, −1)



This line represents all of the solutions of $y = \frac{1}{2}x - 3$.

CHECK Your Progress **Graph each equation.**

4A. $3x + y = -1$ **4B.** $y = -2$

EXAMPLE Graph by Using Intercepts

5 **Graph $3x + 2y = 9$ using the x-intercept and y-intercept.**

To find the x-intercept, let $y = 0$.

$3x + 2y = 9$ Original equation

$3x + 2(0) = 9$ Replace y with 0.

$3x = 9$ Divide each side by 3.

$x = 3$

To find the y-intercept, let $x = 0$.

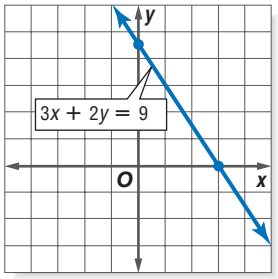
$3x + 2y = 9$ Original equation

$3(0) + 2y = 9$ Replace x with 0.

$2y = 9$ Divide each side by 2.

$y = 4.5$

The x-intercept is 3, so the graph intersects the x-axis at (3, 0). The y-intercept is 4.5, so the graph intersects the y-axis at (0, 4.5). Plot these points. Then draw the line through them.



Concepts in MOtion
Interactive Lab
algebra1.com

CHECK Your Progress

5. Graph $y = -x - 5$ using the x- and y-intercepts.

CHECK Your Understanding

Example 1 (pp. 155–156)

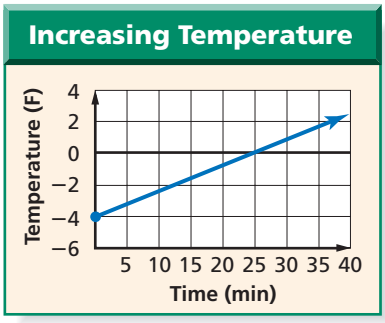
Determine whether each equation is a linear equation. If so, write the equation in standard form.

1. $x + y^2 = 25$ **2.** $3y + 2 = 0$ **3.** $\frac{3}{5}x - \frac{2}{5}y = 5$

Examples 2, 3 (pp. 156–157)

Determine the x-intercept and y-intercept of each linear function and describe what the intercepts mean.

4.



5.

Position of Scuba Diver	
Time, x (s)	Depth, y (m)
0	−24
3	−18
6	−12
9	−6
12	0

6. What are the zeros of the functions represented in Exercises 4 and 5?

Example 4 (p. 157)

Graph each equation by making a table.

7. $x - y = 0$ **8.** $x = 3$

Example 5 (p. 158)

Graph each equation by using the x- and y-intercepts.

9. $y = -3 - x$ **10.** $x + 4y = 10$

11. RODEOS Tickets for a rodeo cost \$5 for children and \$10 for adults. The equation $5x + 10y = 60$ represents the number of children x and adults y who can attend the rodeo for \$60. Use the x- and y-intercepts to graph the equation. What do these values mean?

Exercises

HOMEWORK HELP

For Exercises	See Examples
12–17	1
18–23	2, 3
24–38	4, 5

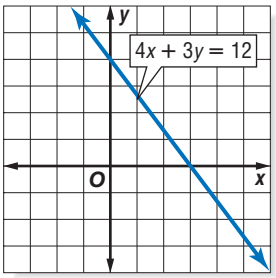
Determine whether each equation is a linear equation. If so, write the equation in standard form.

12. $3x = 5y$ **13.** $6 - y = 2x$ **14.** $6xy + 3x = 4$

15. $y + 5 = 0$ **16.** $7y = 2x + 5x$ **17.** $y = 4x^2 - 1$

Determine the *x*-intercept, *y*-intercept, and zero of each linear function.

18.

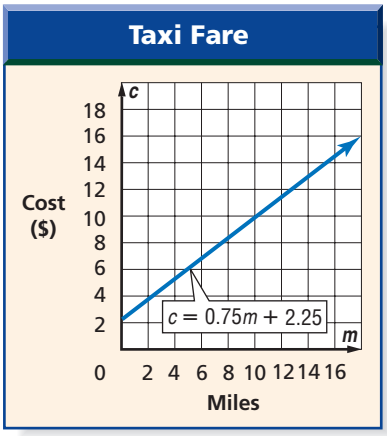


19.

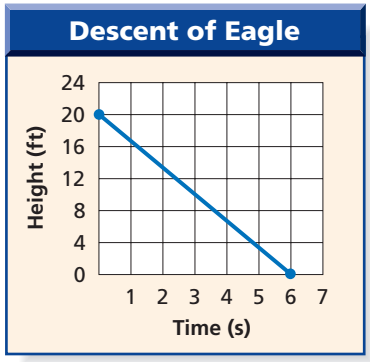
x	y
−3	−1
−2	0
−1	1
0	2
1	3

Determine the *x*-intercept and *y*-intercept of each linear function and describe what the intercepts mean.

20.



21.



22.

Swimming to Burn Calories	
Time, *x* (min)	Calories Burned, *y*
0	0
10	106
15	159
20	212
30	318

23.

Eva's Distance from Home	
Time, *x* (min)	Distance, *y* (mi)
0	4
2	3
4	2
6	1
8	0

Graph each equation.

24. $y = -1$ **25.** $y = 2x$ **26.** $y = x - 6$

27. $y = 5 - x$ **28.** $y = 4 - 3x$ **29.** $x = 3y$

30. $x = 4y - 6$ **31.** $x - y = -3$ **32.** $4x + 6y = 8$

METEOROLOGY **For Exercises 33–35, use the following information.**
The distance d in miles that the sound of thunder travels in t seconds is given by the equation $d = 0.21t$.

33. Make a table of values.

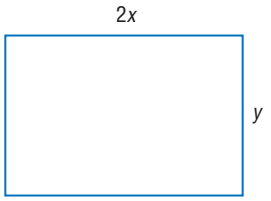
34. Graph the equation.

35. Use the graph to estimate how long it will take you to hear thunder from a storm that is 3 miles away.

EXTRA PRACTICE
See pages 723, 746.

Math Online
Self-Check Quiz at algebra1.com

GEOMETRY **For Exercises 36–38, refer to the figure at right.**
The perimeter P of a rectangle is given by $2\ell + 2w = P$, where ℓ is the length of the rectangle and w is the width.



36. If the perimeter of the rectangle is 30 inches, write an equation for the perimeter in standard form.

37. What are the x- and y-intercepts of the graph? Do they make sense in this problem? Explain.

38. Graph the equation.

Real-World Link
How heavy is air? The atmospheric pressure is a measure of the weight of air. At sea level, air pressure is 14.7 pounds per square inch.
Source: www.brittanica.com

Determine whether each equation is a linear equation. If so, write the equation in standard form.

39. $x + \frac{1}{y} = 7$

40. $\frac{x}{2} = 10 + \frac{2y}{3}$

41. $7n - 8m = 4 - 2m$

42. $3a + b - 2 = b$

43. $2r - 3rs + 5s = 1$

44. $\frac{3m}{4} = \frac{2n}{3} - 5$

Graph each equation.

45. $1.5x + y = 4$

46. $75 = 2.5x + 5y$

47. $\frac{4x}{3} = \frac{3y}{4} + 1$

48. $y + \frac{1}{3} = \frac{1}{4}x - 3$

49. $\frac{1}{2}x + y = 4$

50. $1 = x - \frac{2}{3}y$

51. Find the x- and y-intercepts of the graph of $4x - 7y = 14$.

52. Graph $5x + 3y = 15$. Where does the line intersect the x-axis? Where does the line intersect the y-axis? What is the slope?

OCEANOGRAPHY **For Exercises 53 and 54, use the information at left and below.**
Under water, pressure increases 4.3 pounds per square inch (psi) for every 10 feet you descend. This can be expressed by the equation $p = 0.43d + 14.7$, where p is the pressure in pounds per square inch and d is the depth in feet.

53. Graph the equation and find the y-intercept.

54. Divers cannot work at depths below about 400 feet. Given this information, determine a reasonable domain and range for this situation.

H.O.T. Problems

55. REASONING Verify that the point at $(-4, 2)$ lies on the line with the equation $y = \frac{1}{2}x + 4$.

OPEN ENDED **Describe a linear equation in the form $Ax + By = C$ for each condition.**

56. $A = 0$

57. $B = 0$

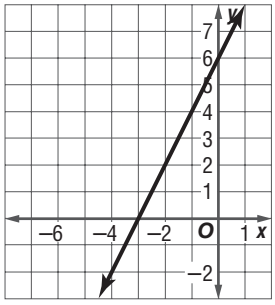
58. $C = 0$

59. CHALLENGE Demonstrate how you can determine whether a point at (x, y) is *above*, *below*, or *on* the line given by $2x - y = 8$ without graphing it. Give an example of each.

60. ***Writing in Math*** Use the information about nutrition on page 155 to explain how linear equations can be used in nutrition. Explain how you could use the Nutrition Information labels on packages to limit your fat intake.

STANDARDIZED TEST PRACTICE

61. What are the x- and y-intercept points of the function graphed?



A $(-3, 0)$ and $(0, 6)$

B $(-3, 0)$ and $(6, 0)$

C $(0, -3)$ and $(0, 6)$

D $(0, -3)$ and $(6, 0)$

62. Which is the best estimate for the x-intercept of the graph of the linear function represented in the table?

x	y
0	5
1	3
2	1
3	−1
4	−3

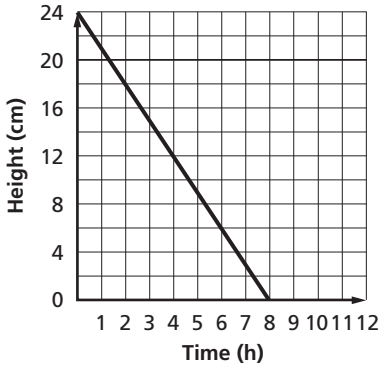
F between 0 and 1

G between 1 and 2

H between 2 and 3

J between 3 and 4

63. REVIEW A candle is 24 centimeters high and burns 3 centimeters per hour, as shown in the graph.



If the height of the candle is 8 centimeters, approximately how long has the candle been burning?

A 0 hours

B 24 minutes

C 64 minutes

D $5\frac{1}{2}$ hours

Spiral Review

If $f(x) = 3x - 2$ and $g(x) = x^2 - 5$, find each value. (Lesson 3-2)

64. $f(4)$ **65.** $g(-3)$ **66.** $2[f(6)]$

67. NUTRITION The cost of buying energy bars for a camping trip is given by $y = 2.25x$. Write the equation in function notation and then find $f(4)$ and $f(7)$. What do these values represent? (Lesson 3-2)

Solve each equation. Then check your solution. (Lesson 2-5)

68. $3n - 12 = 5n - 20$ **69.** $6(x + 3) = 3x$ **70.** $2(x - 2) = 3x - (4x - 5)$

71. BALLOONS Brandon slowly fills a deflated balloon with air. Without tying the balloon, he lets it go. Draw a graph to represent this situation. (Lesson 1-9)

GET READY for the Next Lesson

PREREQUISITE SKILL **Find each difference.**

72. $12 - 16$ **73.** $-5 - (-8)$ **74.** $16 - (-4)$ **75.** $-9 - 6$

EXTEND 3-3

Graphing Calculator Lab

Graphing Linear Functions

The power of a graphing calculator is the ability to graph different types of equations accurately and quickly. Often linear equations are graphed in the standard viewing window. The **standard viewing window** is [−10, 10] by [−10, 10] with a scale of 1 on each axis. To quickly choose the standard viewing window on a TI-83/84 Plus, press ZOOM 6.

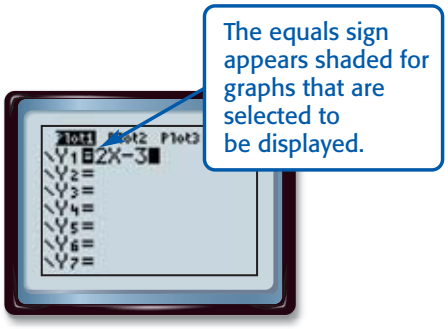
ACTIVITY 1

Graph $2x - y = 3$ on a TI-83/84 Plus graphing calculator.

Step 1 Enter the equation in the Y= list.

- The Y= list shows the equation or equations that you will graph.
- Equations must be entered with the y isolated on one side of the equation. Solve the equation for y, then enter it into the calculator.

$$\begin{aligned} 2x - y &= 3 && \text{Original equation} \\ 2x - y - 2x &= 3 - 2x && \text{Subtract } 2x \text{ from each side.} \\ -y &= -2x + 3 && \text{Simplify.} \\ y &= 2x - 3 && \text{Multiply each side by } -1. \end{aligned}$$

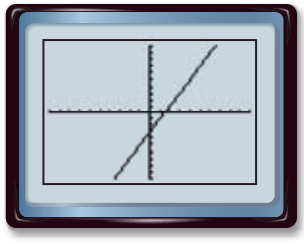


The equals sign appears shaded for graphs that are selected to be displayed.

KEYSTROKES: Y= 2 X,T,θ,n − 3

Step 2 Graph the equation in the standard viewing window.

KEYSTROKES: ZOOM 6



[−10, 10] scl: 1 by [−10, 10] scl: 1

Sometimes a complete graph is not displayed using the standard viewing window. A **complete graph** includes all of the important characteristics of the graph on the screen. These include the origin and the x- and y-intercepts. Notice that the graph of $2x - y = 3$ is a complete graph because all of these points are visible.

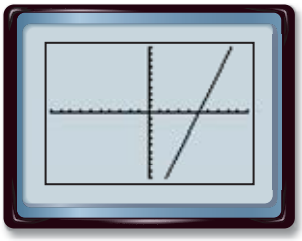
When a complete graph is not displayed using the standard viewing window, you will need to change the viewing window to accommodate these important features. You can use what you have learned about intercepts to help you choose an appropriate viewing window.

Math nline **Other Calculator Keystrokes at** algebra1.com

ACTIVITY 2

Graph $y = 3x - 15$ on a graphing calculator.

Step 1 Enter the equation in the Y= list and graph in the standard viewing window.



[−10, 10] scl: 1 by [−10, 10] scl: 1

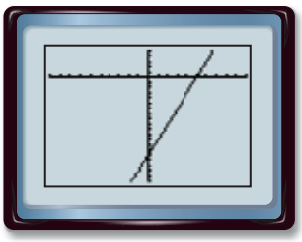
Clear the previous equation from the Y= list. Then enter the new equation and graph.

KEYSTROKES: [Y=] [CLEAR] 3 [X,T,θ,n] [−] 15 [ZOOM] 6

Step 2 Modify the viewing window and graph again.

The origin and the x-intercept are displayed in the standard viewing window. But notice that the y-intercept is outside of the viewing window. Find the y-intercept.

$y = 3x - 15$ Original equation
$= 3(0) - 15$ Replace x with 0.
$= -15$ Simplify.



[−10, 10] scl: 1 by [−20, 5] scl: 1

Since the y-intercept is −15, choose a viewing window that includes a number less than −15. The window [−10, 10] by [−20, 5] with a scale of 1 on each axis is a good choice.

KEYSTROKES: [WINDOW] −10 [ENTER] 10 [ENTER] 1 [ENTER] −20 [ENTER] 5 [ENTER] 1 [GRAPH]

EXERCISES

Graph each linear equation in the standard viewing window. Determine whether the graph is complete. If the graph is not complete, choose a viewing window that will show a complete graph and graph the equation again.

1. $y = x + 2$ **2.** $y = 4x + 5$ **3.** $y = 6 - 5x$ **4.** $2x + y = 6$

5. $x + y = -2$ **6.** $x - 4y = 8$ **7.** $y = 5x + 9$ **8.** $y = 10x - 6$

9. $y = 3x - 18$ **10.** $3x - y = 12$ **11.** $4x + 2y = 21$ **12.** $3x + 5y = -45$

For Exercises 13–15, consider the linear equation $y = 2x + b$.

13. Choose several different positive and negative values for b. Graph each equation in the standard viewing window.

14. For which values of b is the complete graph in the standard viewing window?

15. How is the value of b related to the y-intercept of the graph of $y = 2x + b$?

CHAPTER 3

Mid-Chapter Quiz

Lessons 3-1 through 3-3

State the domain, range, and inverse of each relation. (Lesson 3-1)

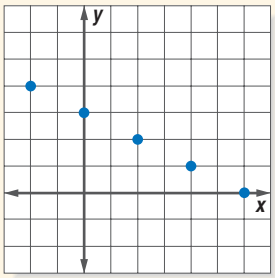
1. {(1, 3), (4, 6), (2, 3), (1, 5)}

2. {(−5, 8), (−1, 0), (−1, 4), (2, 7), (6, 3)}

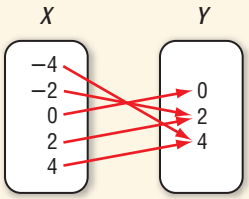
3.

x	y
11	5
15	3
−8	22
11	31

4.



5. MULTIPLE CHOICE What are the domain and range of the relation? (Lesson 3-1)



A D = {0, 2, 4}; R = {−4, −2, 0, 2, 4}

B D = {−4, −2, 0, 2, 4}; R = {0, 2, 4}

C D = {0, 2, 4}; R = {−4, −2, 0}

D D = {−4, −2, 0, 2, 4}; R = {−4, −2, 0, 2, 4}

CHEERLEADING For Exercises 6–8, use the following information.
The cost of a cheerleading camp is shown in the table. (Lesson 3-1)

Number of Cheerleaders	Total Cost ($)
1	70
2	140
3	210
4	280

6. Determine the domain and range of the relation.

7. Graph the data.

8. Describe the independent and dependent quantities in this situation.

If $f(x) = 3x + 5$, find each value. (Lesson 3-2)

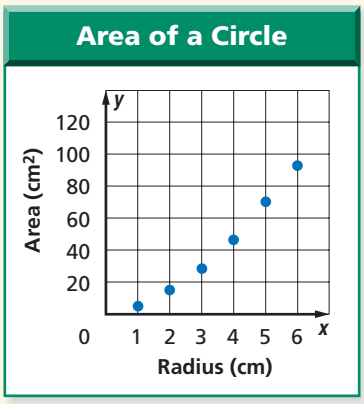
9. $f(-4)$ **10.** $f(2a)$ **11.** $f(x + 2)$

Determine whether each relation is a function. (Lesson 3-2)

12. {(3, 4), (5, 3), (−1, 4), (6, 2)}

13. {(−1, 4), (−2, 5), (7, 2), (3, 9), (−2, 1)}

14. MULTIPLE CHOICE Which is a true statement about the relation? (Lesson 3-2)



F As the radius increases, the area decreases.

G The relation is a linear function.

H The area is a function of the radius.

J The relation is not a function.

Graph each equation. (Lesson 3-3)

15. $y = x - 2$ **16.** $3x + 2y = 6$

17. MULTIPLE CHOICE If $(a, -7)$ is a solution to the equation $8a + 3b = 3$, what is a? (Lesson 3-3)

A 2 **B** 3 **C** 3.5 **D** −6.5

ENTERTAINMENT For Exercises 18–20, use the following information.
The equation $200x + 80y = 600$ represents the number of premium tickets x and the number of discount tickets y to a car race that can be bought with $600. (Lesson 3-3)

18. Graph the function.

19. Describe a domain and range that makes sense for this situation. Explain.

20. Describe what the x-and y-intercepts represent in the context of this situation.

3-4 Arithmetic Sequences

Main Ideas

- Recognize arithmetic sequences.
- Extend and write formulas for arithmetic sequences

New Vocabulary

sequence
terms
arithmetic sequence
common difference

GET READY for the Lesson

A probe to measure air quality is attached to a hot-air balloon. The probe has an altitude of 6.3 feet after the first second, 14.5 feet after the next second, 22.7 feet after the third second, and so on. You can make a table and look for a pattern in the data.

Time (s)	1	2	3	4	5	6	7	8
Altitude (ft)	6.3	14.5	22.7	30.9	39.1	47.3	55.5	63.7

+ 8.2 + 8.2 + 8.2 + 8.2 + 8.2 + 8.2 + 8.2

Recognize Arithmetic Sequences A **sequence** is a set of numbers, called **terms**, in a specific order. If the difference between successive terms is constant, then it is called an **arithmetic sequence**. The difference between the terms is called the **common difference**.

KEY CONCEPT *Arithmetic Sequence*

An arithmetic sequence is a numerical pattern that increases or decreases at a constant rate or value called the common difference.

Study Tip

Ellipsis

The three dots after the last number in a sequence are called an *ellipsis*. The ellipsis indicates that there are more terms in the sequence that are not listed.

EXAMPLE 1 **Identify Arithmetic Sequences**

Determine whether each sequence is arithmetic. Explain.

a. 1, 2, 4, 8, …

1 → 2 (+1) → 4 (+2) → 8 (+4)

This is not an arithmetic sequence because the difference between terms is not constant.

b. $\frac{1}{2}, \frac{1}{4}, 0, -\frac{1}{4}, \ldots$

$\frac{1}{2} \to \frac{1}{4}$ $\left(-\frac{1}{4}\right)$ → 0 $\left(-\frac{1}{4}\right)$ → $-\frac{1}{4}$ $\left(-\frac{1}{4}\right)$

This is an arithmetic sequence because the difference between terms is constant.

CHECK Your Progress

1A. −26, −22, −18, −14, …

1B. 1, 4, 9, 25, …

Write Arithmetic Sequences You can use the common difference of an arithmetic sequence to find the next term in the sequence.

KEY CONCEPT *Writing Arithmetic Sequences*

Words Each term of an arithmetic sequence after the first term can be found by adding the common difference to the preceding term.

Symbols An arithmetic sequence can be found as follows:

$$a_1, a_1 + d, a_2 + d, a_3 + d \ldots,$$

where d is the common difference, a_1 is the first term, a_2 is the second term, and so on.

Real-World EXAMPLE

2 **MONEY** **The arithmetic sequence 74, 67, 60, 53, ... represents the amount of money that Tiffany owes her mother at the end of each week. Find the next three terms.**

Find the common difference by subtracting successive terms.

74 → 67 → 60 → 53 → ? → ? → ?

−7 −7 −7 −7 −7 −7

The common difference is −7.

Add −7 to the last term of the sequence to get the next term in the sequence. Continue adding −7 until the next three terms are found.

53 → 46 → 39 → 32 The next three terms are 46, 39, 32.

−7 −7 −7

CHECK Your Progress

2. Find the next four terms of the arithmetic sequence 9.5, 11.0, 12.5, 14.0, …

Each term in an arithmetic sequence can be expressed in terms of the first term a_1 and the common difference d.

Study Tip

Formulas The formula for the nth term of an arithmetic sequence is called a *recursive formula*. This means that each succeeding term is formulated from one or more of the previous terms.

Term	Symbol	In Terms of a_1 and d	Numbers
first term	a_1	a_1	8
second term	a_2	$a_1 + d$	$8 + 1(3) = 11$
third term	a_3	$a_1 + 2d$	$8 + 2(3) = 14$
fourth term	a_4	$a_1 + 3d$	$8 + 3(3) = 17$
⋮	⋮	⋮	⋮
nth term	a_n	$a_1 + (n - 1)d$	$8 + (n - 1)(3)$

This leads to the formula that can be used to find any term in an arithmetic sequence.

Extra Examples at algebra1.com

KEY CONCEPT *nth Term of an Arithmetic Sequence*

The nth term a_n of an arithmetic sequence with first term a_1 and common difference d is given by

$$a_n = a_1 + (n - 1)d,$$

where n is a positive integer.

Real-World EXAMPLE

3 **SHIPPING** **The arithmetic sequence 12, 23, 34, 45, ... represents the total number of ounces that a box weighs after each additional book is added.**

a. Write an equation for the nth term of the sequence.

In this sequence, the first term, a_1, is 12. Find the common difference.

12 → 23 → 34 → 45

+ 11 + 11 + 11

The common difference is 11.

Use the formula for the nth term to write an equation.

$a_n = a_1 + (n - 1)d$ Formula for nth term

$= 12 + (n - 1)11$ $a_1 = 12$, $d = 11$

$= 12 + 11n - 11$ Distributive Property

$= 11n + 1$ Simplify.

b. Find the 10th term in the sequence.

$a_n = 11n + 1$ Equation for the nth term

$a_{10} = 11(10) + 1$ Replace n with 10.

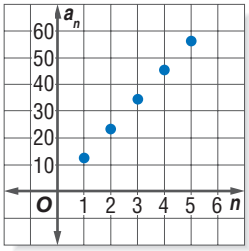
$a_{10} = 111$ Simplify.

Study Tip

Graph of a Sequence
Notice that the points on the graph fall on a line. The graph of an arithmetic sequence is linear.

c. Graph the first five terms of the sequence.

n	$11n + 1$	a_n	(n, a_n)
1	11(1) + 1	12	(1, 12)
2	11(2) + 1	23	(2, 23)
3	11(3) + 1	34	(3, 34)
4	11(4) + 1	45	(4, 45)
5	11(5) + 1	56	(5, 56)



CHECK Your Progress

Consider the arithmetic sequence 3, −10, −23, −36, ….

3A. Write an equation for the nth term of the sequence.

3B. Find the 15th term in the sequence.

3C. Graph the first five terms of the sequence.

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CHECK Your Understanding

Example 1 (p. 165)

Determine whether each sequence is an arithmetic sequence. If it is, state the common difference.

1. 24, 16, 8, 0, …

2. $3\frac{1}{4}, 6\frac{1}{2}, 13, 26, \ldots$

Example 2 (p. 166)

Find the next three terms of each arithmetic sequence.

3. 7, 14, 21, 28, …

4. −34, −29, −24, −19, …

Example 3 (p. 167)

Find the *n*th term of each arithmetic sequence described.

5. $a_1 = 3, d = 4, n = 8$

6. $a_1 = 10, d = -5, n = 21$

7. 23, 25, 27, 29, … for $n = 12$

8. −27, −19, −11, −3, … for $n = 17$

9. FITNESS Latisha is beginning an exercise program that calls for 20 minutes of walking each day for the first week. Each week thereafter, she has to increase her walking by 7 minutes a day. Which week of her exercise program will be the first one in which she will walk over an hour a day?

Write an equation for the *n*th term of each arithmetic sequence. Then graph the first five terms of the sequence.

10. 6, 12, 18, 24, …

11. 12.1, 17.2, 22.3, 27.4, …

Exercises

HOMEWORK HELP	
For Exercises	See Examples
12–15	1
16–19	2
20–33	3

Determine whether each sequence is an arithmetic sequence. If it is, state the common difference.

12. 7, 6, 5, 4, …

13. 10, 12, 15, 18, …

14. −15, −11, −7, −3, …

15. −0.3, 0.2, 0.7, 1.2, …

Find the next three terms of each arithmetic sequence.

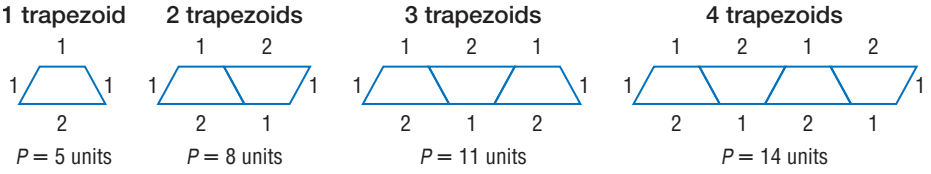
16. 4, 7, 10, 13, …

17. 18, 24, 30, 36, …

18. −66, −70, −74, −78, …

19. –31, −22, −13, −4, …

GEOMETRY For Exercises 20 and 21, use the diagram below that shows the perimeter of the pattern consisting of trapezoids.



20. Write a formula that can be used to find the perimeter of a pattern containing n trapezoids.

21. What is the perimeter of the pattern containing 12 trapezoids?

Find the *n*th term of each arithmetic sequence described.

22. $a_1 = 8, d = 3, n = 16$

23. $a_1 = 52, d = 12, n = 102$

24. $a_1 = \frac{5}{8}, d = \frac{1}{8}, n = 22$

25. –9, –7, –5, –3, … for $n = 18$

26. −7, −3, 1, 5, … for $n = 35$

27. 0.5, 1, 1.5, 2, … for $n = 50$

Real-World Link

The open-air theaters of ancient Greece held about 20,000 people. They became the models for amphitheaters, Roman coliseums, and modern sports arenas.

Source: encarta.msn.com

THEATER **For Exercises 28 and 29, use the following information.**
The Coral Gables Actors' Playhouse has 7 rows of seats in the orchestra section. The number of seats in each row forms an arithmetic sequence, as shown in the table. On opening night, 368 tickets were sold for the orchestra section.

Row	Number of Seats
7	76
6	68
5	60

28. Write a formula to find the number of seats in any given row of the orchestra section of the theater.

29. How many seats are in the first row? Was this section oversold?

Write an equation for the *n*th term of the arithmetic sequence. Then graph the first five terms in the sequence.

30. $-3, -6, -9, -12, \ldots$

31. $8, 9, 10, 11, \ldots$

32. $2, 8, 14, 20, \ldots$

33. $-18, -16, -14, -12, \ldots$

Find the next three terms of each arithmetic sequence.

34. $2\frac{1}{3}, 2\frac{2}{3}, 3, 3\frac{1}{3}, \ldots$

35. $\frac{7}{12}, 1\frac{1}{3}, 2\frac{1}{12}, 2\frac{5}{6}$

36. 200 is the __?__th term of 24, 35, 46, 57....

37. –34 is the __?__th term of 30, 22, 14, 6, …

38. Find the value of y that makes $y + 4, 6, y, \ldots$ an arithmetic sequence.

39. Find the value of y that makes $y + 8, 4y + 6, 3y, \ldots$ an arithmetic sequence.

ANALYZE TABLES **For Exercises 40–43, use the following information.**
Taylor and Brooklyn are recording how far a ball rolls down a ramp during each second. The table shows the data they have collected.

Time (s)	1	2	3	4	5	6
Distance traveled (cm)	9	13	17	21	25	29

40. Do the distances traveled by the ball form an arithmetic sequence? Justify your answer.

41. Write an equation for the sequence. How far will the ball have traveled after 35 seconds?

42. Graph the sequence.

43. Suppose that for each second, the ball rolls twice the distance shown in the table. Is the graph representing this sequence linear? If so, describe how its rate of change is different from the rate of change shown in your original graph.

EXTRA PRACTICE
See pages 724, 746.
MathOnline
Self-Check Quiz at algebra1.com

GAMES **For Exercises 44 and 45, use the following information.**
Contestants on a game show win money by answering 10 questions. The value of each question increases by $1500.

44. If the first question is worth $2500, find the value of the 10th question.

45. If the contestant answers all 10 questions correctly, how much money will he or she win?

H.O.T. Problems

46. **OPEN ENDED** Create an arithmetic sequence with a common difference of -10.

47. **CHALLENGE** Is $2x + 5, 4x + 5, 6x + 5, 8x + 5, \ldots$ an arithmetic sequence? Explain your reasoning.

48. FIND THE ERROR Marisela and Richard are finding the common difference for the arithmetic sequence -44, -32, -20, -8. Who is correct? Explain.

Marisela	Richard
$-32 - (-44) = 12$	$-44 - (-32) = -12$
$-20 - (-32) = 12$	$-32 - (-20) = -12$
$-8 - (-20) = 12$	$-20 - (-8) = -12$

49. *Writing in Math* Refer to the data about measuring air quality on page 165. Write a formula for the arithmetic sequence that represents the altitude of the probe after each second, and an explanation of how you could use this information to predict the altitude of the probe after 15 seconds.

STANDARDIZED TEST PRACTICE

50. REVIEW Luis deposits $25 each week into a savings account from his part-time job. If he has $350 in savings now, how much will he have in 12 weeks?

A $600 **C** $650

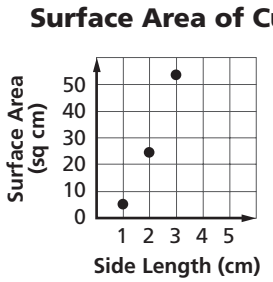
B $625 **D** $675

51. What is the slope of a line that contains the point at $(1, -5)$ and has the same y-intercept as $2x - y = 9$?

F -9 **H** 2

G -7 **J** 4

52. REVIEW Which is a true statement about the relation graphed?



A As the side length of a cube increases, the surface area decreases.

B Surface area is the independent quantity.

C The surface area of a cube is a function of the side length.

D The relation is not a function.

Spiral Review

Determine whether each equation is a linear equation. If so, write the equation in standard form. (Lesson 3-3)

53. $x^2 + 3x - y = 8$ **54.** $y - 8 = 10 - x$ **55.** $2y = y + 2x - 3$

56. TAX The amount of sales tax in California is given by $y = 0.0725x$, where x is the cost of an item that you buy. Write the equation in function notation and then find $f(40)$. What does this value represent? (Lesson 3-2)

57. Translate the sentence *The sum of twice r and three times s is identical to thirteen* into an algebraic equation. (Lesson 2-1)

GET READY for the Next Lesson

PREREQUISITE SKILL **Graph each point on the same coordinate plane.** (Lesson 1-9)

58. $J(3, 0)$ **59.** $L(-3, -4)$ **60.** $M(3, 5)$ **61.** $N(5, -1)$

READING MATH

Inductive and Deductive Reasoning

Throughout your life, you have used reasoning skills, possibly without even knowing it. As a child, you used inductive reasoning to conclude that your hand would hurt if you touched the stove while it was hot. Now, you use inductive reasoning when you decide, after many trials, that one of the worst ways to prepare for an exam is by studying only an hour before you take it. **Inductive reasoning** is used to derive a general rule after observing many individual events.

Inductive reasoning involves:

- observing many examples
- looking for a pattern
- making a conjecture
- checking the conjecture
- discovering a likely conclusion

With **deductive reasoning**, you use a general rule to help you decide about a specific event. You come to a conclusion by accepting facts. There is no conjecturing involved. Read the two statements below.

1) If a person wants to play varsity sports, he or she must have a C average in academic classes.
2) Jolene is playing on the varsity tennis team.

If these two statements are accepted as facts, then the obvious conclusion is that Jolene has at least a C average in her academic classes. This is an example of deductive reasoning.

Reading to Learn

1. Explain the difference between inductive and deductive reasoning. Then give an example of each.

2. When Sherlock Holmes reaches a conclusion about a murderer's height because he knows the relationship between a man's height and the distance between his footprints, what kind of reasoning is he using? Explain.

3. When you examine a sequence of numbers and decide that it is an arithmetic sequence, what kind of reasoning are you using? Explain.

4. Once you have found the common difference for an arithmetic sequence, what kind of reasoning do you use to find the 100th term in the sequence?

5. a. Copy and complete the following table.

3^1	3^2	3^3	3^4	3^5	3^6	3^7	3^8	3^9
3	9	27						

b. Write the sequence of numbers representing the numbers in the ones place.

c. Find the number in the ones place for the value of 3100. Explain your reasoning. State the type of reasoning that you used.

6. A sequence contains all numbers less than 50 that are divisible by 5. You conclude that 35 is in the sequence. Is this an example of inductive or deductive reasoning? Explain.

3-5 Proportional and Nonproportional Relationships

Main Ideas

- Write an equation for a proportional relationship.
- Write an equation for a nonproportional relationship.

New Vocabulary

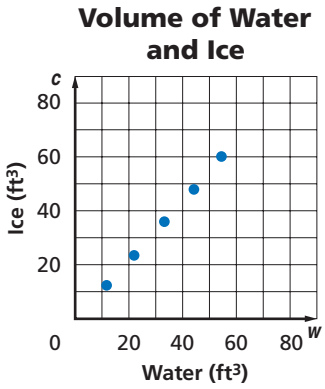
inductive reasoning

GET READY for the Lesson

Water is one of the few substances that expands when it freezes. The table shows volumes of water and the corresponding volumes of ice.

Volume of Water (ft^3)	11	22	33	44	55
Volume of Ice (ft^3)	12	24	36	48	60

The relation in the table can be represented by a graph. Let w represent the volume of water, and let c represent the volume of ice. When the ordered pairs are graphed, they form a linear pattern. This pattern can be described by an equation.



Proportional Relationships Using a pattern to find a general rule utilizes **inductive reasoning**. If the relationship between the domain and range of a relation is linear, the relationship can be described by a linear equation. If the equation is of the form $y = kx$, then the relationship is proportional. In a proportional relationship, the graph will pass through (0, 0).

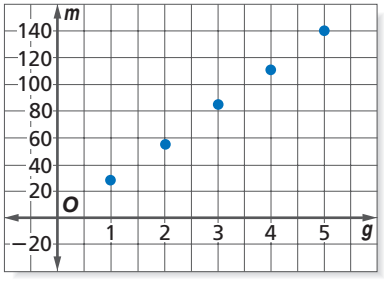
Real-World EXAMPLE Proportional Relationships

1 **FUEL ECONOMY** **The table below shows the average amount of gas Rogelio's car uses, depending on how many miles he drives.**

Gallons of gasoline	1	2	3	4	5
Miles driven	28	56	84	112	140

a. Graph the data. What conclusion can you make about the relationship between the number of gallons used and the number of miles driven?

The graph shows a linear relationship between the number of gallons used g and the number of miles driven m.



Real-World Link
Hybrid cars have a small, fuel-efficient gas engine combined with an electric motor. The electric motor is powered by batteries that recharge automatically while the car is being driven.

Source: artheasy.com/live_hybrid_cars.htm

b. Write an equation to describe this relationship.

Look at the relationship between the values in the domain and range to find a pattern that can be described by an equation.

	+1	+1	+1	+1	
Gallons of gasoline	1	2	3	4	5
Miles driven	28	56	84	112	140
	+28	+28	+28	+28	

The difference of the values for g is 1, and the difference of the values for m is 28. This suggests that $m = 28g$. Check to see if this equation is correct by substituting values of g into the equation.

CHECK If $g = 1$, then $m = 28(1)$ or 28. ✓
If $g = 2$, then $m = 28(2)$ or 56. ✓
If $g = 3$, then $m = 28(3)$ or 84. ✓

The equation checks. Since this relation is a function, we can write the equation as $f(g) = 28g$, where $f(g)$ represents the number of miles driven.

CHECK Your Progress

FUND-RAISING **The table shows the cost of buying Spanish Club T-shirts.**

Number of T-shirts	1	2	3	4
Total Cost ($)	7.50	15.00	22.50	30.00

1A. Graph the data and describe the relationship between the number of T-shirts bought and the amount spent.

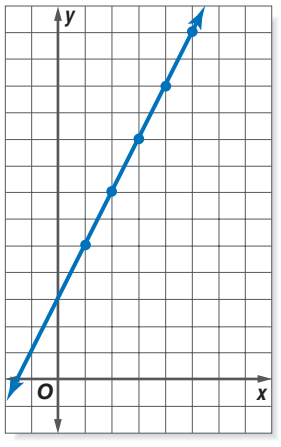
1B. Write an equation to describe this relationship.

Nonproportional Relationships

Nonproportional Relationships Some linear relationships are nonproportional. In the equation of a nonproportional situation, a constant must be added or subtracted from the variable expression.

EXAMPLE Nonproportional Relationships

2 **Write an equation in function notation for the relation graphed at the right.**



Make a table of ordered pairs.

	+1	+1	+1	+1	
x	1	2	3	4	5
y	5	7	9	11	13
	+2	+2	+2	+2	

The difference of the x values is 1, and the difference of the y values is 2. The difference in y values is twice the difference of x values. This suggests that $y = 2x$.

(continued on the next page)

Study Tip

Nonproportional Relationships
The x- and y-values in Example 2 do not have a proportional relationship because $\frac{x}{y}$ is not always the same; for example, $\frac{1}{5} \neq \frac{2}{7}$.

CHECK Suppose the equation is $y = 2x$. If $x = 1$, then $y = 2(1)$ or 2. But the y-value for $x = 1$ is 5. This is a difference of 3. Try some other values in the domain to see if the same difference occurs.

x	1	2	3	4	5
$2x$	2	4	6	8	10
y	5	7	9	11	13

y is always 3 more than $2x$.

This pattern suggests that 3 should be added to one side of the equation in order to correctly describe the relation. Check $y = 2x + 3$.

If $x = 2$, then $y = 2(2) + 3$ or 7.
If $x = 3$, then $y = 2(3) + 3$ or 9.

Thus, $y = 2x + 3$ correctly describes this relation. Since this relation is a function, the equation in function notation is $f(x) = 2x + 3$.

CHECK Your Progress

2. Write an equation in function notation for the relation shown in the table.

x	1	2	3	4
y	3	2	1	0

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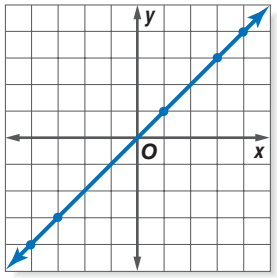
CHECK Your Understanding

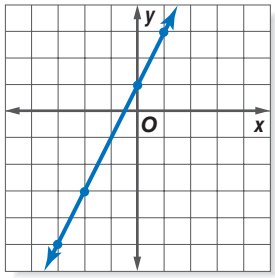
Example 1 (pp. 172–173)

Write an equation in function notation for each relation.

1.



2.



GEOMETRY For Exercises 3 and 4, use the table below that shows the perimeter of a square with sides of a given length.

Side length (in)	1	2	3	4	5
Perimeter (in)	4	8	12	16	20

3. Graph the data. What can you conclude about the relationship between side length and perimeter?

4. Write an equation to describe the relationship.

Example 2 (pp. 173–174)

ANALYZE TABLES For Exercises 5–7, use the table below that shows the underground temperature of rocks at various depths below Earth's surface.

Depth (km)	1	2	3	4	5	6
Temperature (°C)	55	90	125	160	195	230

5. Graph the data.

6. Write an equation in function notation for the relation.

7. Find the temperature of a rock that is 10 kilometers below the surface.

Exercises

HOMEWORK HELP	
For Exercises	See Examples
14, 15	1
16–22	2

Find the next three terms in each sequence.

8. 0, 2, 6, 12, 20, …

9. 9, 7, 10, 8, 11, 9, 12, …

10. 1, 4, 9, 16, …

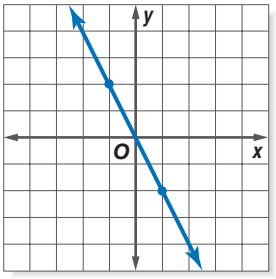
11. 0, 2, 5, 9, 14, 20, …

12. $a + 1, a + 2, a + 3, \ldots$

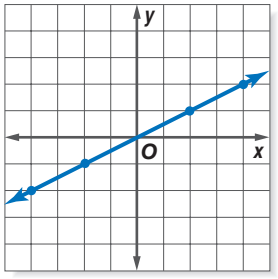
13. $x + 1, 2x + 1, 3x + 1, \ldots$

Write an equation in function notation for each relation.

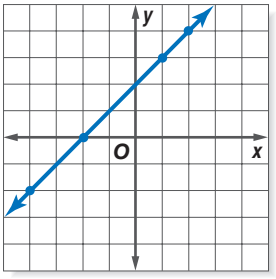
14.



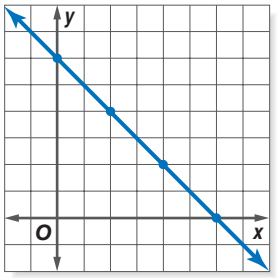
15.



16.



17.



18. TRAVEL On an island cruise in Hawaii, each passenger is given a lei. A crew member hands out 3 red, 3 blue, and 3 green leis in that order. If this pattern is repeated, what color lei will the 50th person receive?

Real-World Link

Fibonacci numbers occur in many areas of nature, including pine cones, shell spirals, flower petals, branching plants, and many fruits and vegetables.

Source: mathworld.wolfram.com

NUMBER THEORY **For Exercises 19 and 20, use the following information.**
In 1201, Leonardo Fibonacci introduced his now famous pattern of numbers called the Fibonacci sequence.

1, 1, 2, 3, 5, 8, 13, …

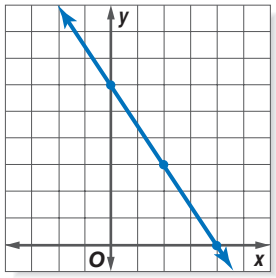
Notice the pattern in this sequence. After the second number, each number in the sequence is the sum of the two numbers that precede it. That is, $2 = 1 + 1$, $3 = 2 + 1$, $5 = 3 + 2$, and so on.

19. Write the first 12 terms of the Fibonacci sequence.

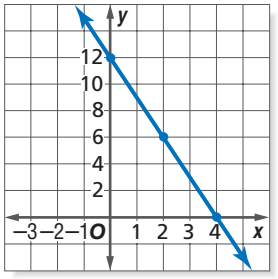
20. Notice that every third term is divisible by 2. What do you notice about every fourth term? every fifth term?

Write an equation in function notation for each relation.

21.



22.



EXTRA PRACTICE
See pages 724, 746.
Math Online
Self-Check Quiz at algebra1.com

FITNESS **For Exercises 23 and 24, use the table below that shows the maximum heart rate to maintain during aerobic activities such as biking.**

Age (yr)	20	30	40	50	60	70
Pulse rate (beats/min)	175	166	157	148	139	130

Source: Ontario Association of Sport and Exercise Sciences

23. Write an equation in function notation for the relation.

24. What would be the maximum heart rate to maintain in aerobic training for a 10-year-old? an 80-year-old?

H.O.T. Problems

25. CHALLENGE Describe how inductive reasoning can be used to write an equation from a pattern.

26. OPEN ENDED Create a number sequence in which the first term is 4. Explain the pattern that you used.

27. *Writing in Math* Use the information about science on page 172 to explain how writing equations from patterns is important in science. Explain the relationship between the volumes of water and ice.

STANDARDIZED TEST PRACTICE

28. The table below shows the cost C of renting a pontoon boat for h hours.

Hours	1	2	3
Cost	7.25	14.50	21.75

Which equation best represents the data?

A $C = 7.25h$

B $C = h + 7.25$

C $C = 21.75 - 7.25h$

D $C = 7.25h + 21.75$

29. REVIEW Donald can ride 8 miles on his bicycle in 30 minutes. At this rate, how long would it take him to ride 30 miles?

F 8 hours

G 6 hours 32 minutes

H 2 hours

J 1 hour 53 minutes

Spiral Review

Find the next three terms of each arithmetic sequence. (Lesson 3-4)

30. 9, 5, 1, −3, …

31. −25, −19, −13, −7, …

32. 22, 34, 46, 58, …

Graph each equation. (Lesson 3-3)

33. $y = x + 3$

34. $y = 2x - 4$

35. $2x + 5y = 10$

36. IN THE MEDIA The following statement appeared on a news Web site shortly after a giant lobster named Bubba was found near Nantucket, Massachusetts. Approximately how much did Bubba weigh? (Lesson 2-3)

"At Tuesday's price of $14.98 a pound, Bubba would retail for about $350."

Source: cnn.com

CHAPTER 3

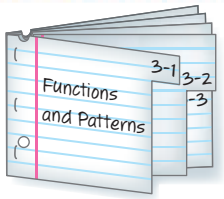
Study Guide and Review

Download Vocabulary Review from algebra1.com

FOLDABLES Study Organizer

GET READY to Study

Be sure the following Key Concepts are noted in your Foldable.



Key Concepts

Representing Relations and Functions

(Lessons 3-1 and 3-2)

- Relation Q is the inverse of relation S if, and only if, for every ordered pair (a, b) in S, there is an ordered pair (b, a) in Q.
- A function is a relation in which each element of the domain is paired with *exactly* one element of the range.

Linear Functions (Lesson 3-3)

- The standard form of a linear equation is $Ax + By = C$, where $A \geq 0$, A and B are not both zero, and A, B, and C are integers with the greatest common factor of 1.

Arithmetic Sequences (Lesson 3-4)

- An arithmetic sequence is a numerical pattern that increases or decreases at a constant rate or value called the common difference.
- The nth term a_n of an arithmetic sequence with first term a_1 and common difference d is given by $a_n = a_1 + (n - 1)d$, where n is a positive integer.

Number Patterns (Lesson 3-5)

- When you make a conclusion based on a pattern of examples, you are using inductive reasoning.

Key Vocabulary

arithmetic sequence (p. 165)	mapping (p. 143)
common difference (p. 165)	sequence (p. 165)
function (p. 149)	standard form (p. 155)
function notation (p. 150)	terms (p. 165)
function value (p. 150)	vertical line test (p. 150)
inductive reasoning (p. 172)	x-intercept (p. 156)
inverse (p. 144)	y-intercept (p. 156)
linear equation (p. 155)	zero (p. 156)

Vocabulary Check

State whether each sentence is *true* or *false*. If *false*, replace the underlined word or number to make a true sentence.

1. The mapping of a relation is obtained by switching the coordinates of each ordered pair.
2. The functional value of $g(x)$ for $x = 8$ is $g(8)$.
3. To determine if a graph represents a function, you can use the vertical line test.
4. A relation is a set of ordered pairs.
5. In a function, $f(x)$ represents the elements of the domain.
6. The x-coordinate of the point at which a graph of an equation crosses the x-axis is an x-intercept.
7. A linear equation is the equation of a line.
8. The difference between the terms of an arithmetic sequence is called the inverse.
9. The regular form of a linear equation is $Ax + By = C$.
10. Values of x for which $f(x) = 0$ are called zeros of the function f.

Lesson-by-Lesson Review

3–1 Representing Relations (pp. 143–148)

Express each relation as a table, a graph, and a mapping. Then determine the domain and range.

11. {(−2, 6), (3, −2), (3, 0), (4, 6)}

12. {(2, 5), (−3, 1), (4, −2), (2, 3)}

RIDES **For Exercises 13 and 14, use the table. It shows the angles of descent and the vertical drops for five roller coasters.**

Angle of Descent (°)	Vertical Drop (ft)
45	72
52	137
55	118
60	195
80	300

13. Determine the domain and range.

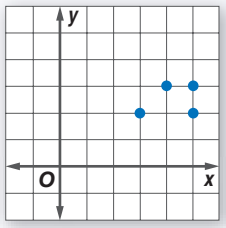
14. Graph the data. What conclusions might you make from the graph?

Example 1 **Express the relation {(3, 2), (5, 3), (4, 3), (5, 2)} as a table, a graph, and a mapping. Then determine the domain and range.**

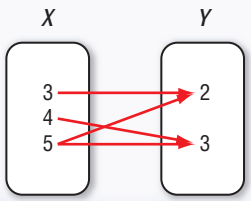
Table

x	y
3	2
5	3
4	3
5	2

Graph



Mapping



The domain is {3, 4, 5}. The range is {2, 3}.

3–2 Representing Functions (pp. 149–154)

Determine whether each relation is a function.

15. {(5, 3), (1, 4), (−6, 5), (1, 6), (−2, 7)}

16. {(2, 3), (−3, −4), (−1, 3), (6, 7)}

If $f(x) = x^2 - x + 1$, find each value.

17. $f(-1)$ **18.** $f(5) - 3$ **19.** $f(a)$

20. DOLPHINS The amount of food that an adult bottlenose dolphin eats per day can be approximated by $y = 0.05x$, where x is the dolphin's body weight in pounds. Write the equation in function notation and then find $f(460)$. What does this value represent?

Example 2 **Determine whether the relation shown is a function. Explain.**

x	y
0	−4
1	−1
2	2
6	3

Since each element of the domain is paired with exactly one element of the range, the relation is a function.

Example 3 **If $g(x) = 2x - 1$, find $g(-6)$.**

$g(-6) = 2(-6) - 1$ Replace x with -6.

$= -12 - 1$ Multiply.

$= -13$ Subtract.

Mixed Problem Solving
For mixed problem-solving practice, see page 746.

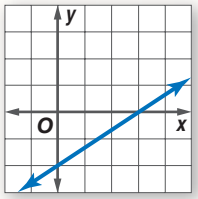
3–3 Linear Functions (pp. 155–161)

Determine the x-intercept, y-intercept, and zero of each linear function.

21.

x	y
−8	0
−4	3
0	6
4	9

22.



Graph each equation.

23. $y = -x + 2$

24. $x + 5y = 4$

25 $2x - 3y = 6$

26. $5x + 2y = 10$

27. SOUND The distance d in kilometers that sound waves travel through water is given by $d = 1.6t$, where t is the time in seconds. Graph the equation. Estimate how far sound can travel through water in 7 seconds.

Example 4 **Graph $3x - y = 4$ by using the x- and y-intercepts.**

Find the x-intercept.

$3x - y = 4$

$3x - 0 = 4$ Let $y = 0$.

$3x = 4$

$x = \frac{4}{3}$

Find the y-intercept.

$3x - y = 4$

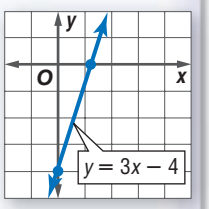
$3(0) - y = 4$ Let $x = 0$.

$-y = 4$

$y = -4$

x-intercept: $\frac{4}{3}$, y-intercept: -4

The graph intersects the x-axis at $\left(\frac{4}{3}, 0\right)$ and the y-axis at $(0, -4)$. Plot these points. Then draw the line through them.



3–4 Arithmetic Sequences (pp. 165–170)

Find the next three terms of each arithmetic sequence.

28. 6, 11, 16, 21, …

29. 1.4, 1.2, 1.0, 0.8, …

30. −3, −11, −19, −27, …

Find the nth term of each arithmetic sequence described.

31. $a_1 = 6, d = 5, n = 11$

32. 28, 25, 22, 19, … for $n = 8$

33. MONEY The table represents Tiffany's income. Write an equation for this sequence and use the equation to find her income if she works 20 hours.

Hours Worked	1	2	3	4
Income ($)	20.50	29	37.50	46

Example 5 **Find the next three terms of the arithmetic sequence 10, 23, 36, 49, ….**

Find the common difference.

10 23 36 49

+ 13 + 13 + 13

So, $d = 13$.

Add 13 to the last term of the sequence. Continue adding 13 until the next three terms are found.

49 62 75 88

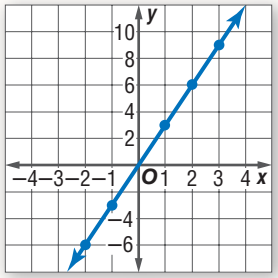
+ 13 + 13 + 13

The next three terms are 62, 75, and 88.

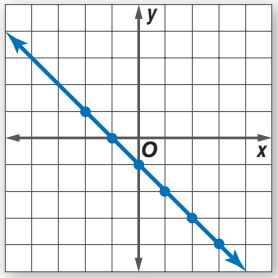
3–5 Proportional and Nonproportional Relationships (pp. 172–176)

Write an equation in function notation for each relation.

34.



35.



ANALYZE TABLES For Exercises 36–38, use the table below that shows the cost of picking your own strawberries at a local farm.

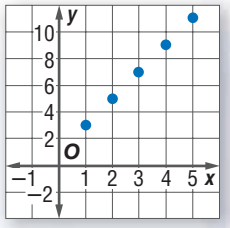
Number of Pounds	1	2	3	4
Total Cost ($)	1.25	2.50	3.75	5.00

36. Graph the data.

37. Write an equation in function notation to describe this relationship.

38. How much would 6 pounds of strawberries cost if you picked them yourself?

Example 6 **Write an equation in function notation for the relation graphed at the right.**



Make a table of ordered pairs for several points on the graph.

x	1	2	3	4	5
y	3	5	7	9	11

The difference in y-values is twice the difference of x-values. This suggests that $y = 2x$. However, $3 \neq 2(1)$. Compare the values of y to the values of $2x$.

x	1	2	3	4	5
2x	2	4	6	8	10
y	3	5	7	9	11

y is always 1 more than $2x$.

The difference between y and $2x$ is always 1. So the equation is $y = 2x + 1$. Since this relation is also a function, it can be written as $f(x) = 2x + 1$.

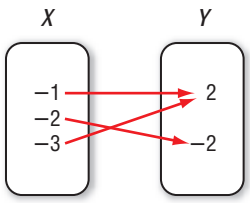
CHAPTER 3 Practice Test

Express the relation shown in each table, mapping, or graph as a set of ordered pairs. Then write the inverse of the relation.

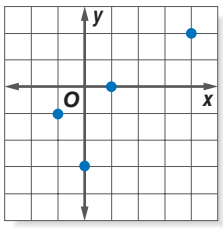
1.

x	$f(x)$
0	−1
2	4
4	5
6	10

2.



3.



Determine whether each relation is a function.

4. {(2, 4), (3, 2), (4, 6), 5, 4)}

5. {(3, 1), (2, 5), (4, 0), (3, −2)}

6. $8y = 7 + 3x$

If $f(x) = -2x + 5$ and $g(x) = x^2 - 4x + 1$, find each value.

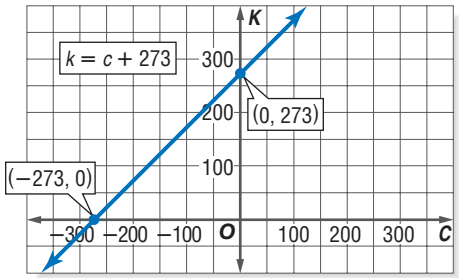
7. $g(-2)$

8. $f\left(\frac{1}{2}\right)$

9. $g(3a) + 1$

10. $f(x + 2)$

TEMPERATURE The equation to convert Celsius temperature C to Kelvin temperature K is shown in the graph.



11. State the independent and dependent variables. Explain.

12. Determine the x-intercept and y-intercept and describe what the intercepts mean.

13. MULTIPLE CHOICE If $f(x) = 3x - 2$, find $f(8) - f(-5)$.

A 7 **C** 37

B 9 **D** 39

Graph each equation.

14. $y = x + 2$ **15.** $y = 4x$

16. $x + 2y = -1$ **17.** $-3x = 5 - y$

Find the next three terms in each sequence.

18. 5, −10, 15, −20, 25, …

19. 5, 5, 6, 8, 11, 15, …

BIOLOGY For Exercises 20 and 21, use the following information.
The amount of blood in the body can be predicted by the equation $y = 0.07w$, where y is the number of pints of blood and w is the weight of a person in pounds.

20. Graph the equation.

21. Predict the weight of a person whose body holds 12 pints of blood.

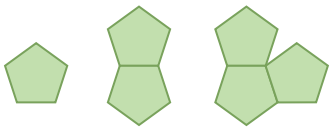
Determine whether each sequence is an arithmetic sequence. If it is, state the common difference.

22. −40, −32, −24, −16, …

23. 0.75, 1.5, 3, 6, 12, …

24. 5, 17, 29, 41, …

25. MULTIPLE CHOICE In each figure, only one side of each regular pentagon touches. Each side of each pentagon is 1 centimeter. If the pattern continues, what is the perimeter of a figure that has 6 pentagons?



F 15 cm **H** 20 cm

G 25 cm **J** 30 cm

CHAPTER 3

Standardized Test Practice

Cumulative, Chapters 1–3

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. The function $c(x) = 0.50(x - 100) + 20$ represents the charge for renting a car at Scott's Rental Cars when a car is driven x miles. Which statement best represents the formula for this charge?

A The charge consists of a set fee of \$.50 and 20 dollars for each mile over 100 miles.

B The charge consists of a set fee of \$100 and \$0.50 for each mile over 20 miles.

C The charge consists of a set fee of \$20 and \$0.50 per mile for each mile.

D The charge consists of a set fee of \$20 and \$0.50 per mile over 100 miles.

2. Which problem is best represented by the number sentence $20 + 2(20 - x) = 54$?

F Kayla babysat for 2 weeks. For the first week she babysat for 20 hours. For the second week she babysat for less than 20 hours. She babysat for a total of 54 hours in those two weeks. How many less hours did she babysit for in the second week?

G Jocelyn ran 20 miles the first week of a training program. The second and third week she ran less than 20 miles. In the second and third week she ran the same number of miles. In the three weeks she ran a total of 54 miles. How many miles less did she run each of the second and third weeks?

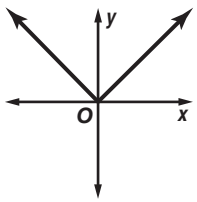
H Steven earned \$20 at his job and washed 2 cars in less than 20 minutes. He earned a total of \$54. How much did he earn per washed car?

J Ely earned \$20 walking dogs and sold 2 magazine subscriptions for \$20 each. Now he has \$54. How much did he earn?

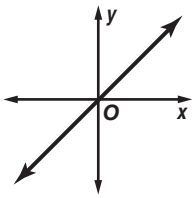
3. GRIDDABLE What is the value of the expression $(4 \cdot 1)^2 - \frac{(2 + 6)}{(4 \cdot 2)}$?

4. Which is the best representation of the function $y = |x|$?

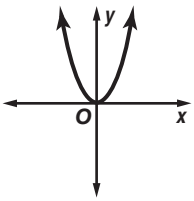
A



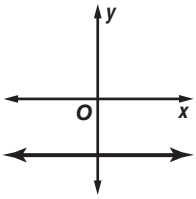
C



B

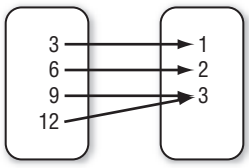


D

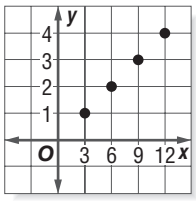


5. Which is NOT a correct representation of the function $f(x) = \{(3, 1), (6, 2), (9, 3), (12, 4)\}$?

F



G



H $y = \frac{1}{3}x$ and the domain is $\{3, 6, 9, 12\}$

J y is a natural number less than or equal to 4 and x is three times y

TEST-TAKING TIP

Question 5 Always read every answer choice, particularly in questions that ask, "Which is NOT a correct representation of the function?"

Standardized Test Practice at algebra1.com

6. Michael wants to write an expression that will always produce an odd integer. Which of the following will always produce an odd integer, n?

A $n + 1$

B $2n + 2$

C $2n + 1$

D $3n + 1$

7. GRIDDABLE Marcus and Peter are swimming laps together. Marcus gains 4 laps on Peter in 2 hours. How many laps will he gain in 45 minutes?

8. Thomas recorded data on a game at the carnival which awards points for throwing a dart at a dart board. If the dart lands on a yellow space you get x points and if the dart lands on a red space you receive y points. Amy threw 12 darts that landed in the yellow space and 9 darts that landed in the red space. Which expression gives the average point per dart throw?

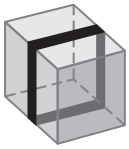
F $21\left(\frac{12}{x} + \frac{9}{y}\right)$

G $\frac{9x + 12y}{21}$

H $\frac{12x + 9y}{21}$

J $\frac{x + y}{21}$

9. Carmen wrapped a ribbon around the girth of a cube-shaped present. She used 48 inches of ribbon to fit exactly around the present.



What is the volume of the present?

A 12 in^3

B 48 in^3

C 144 in^3

D 1728 in^3

10. The area of a trapezoid A is $\frac{1}{2}$ times the sum of the bases a and b times the height. Which equation best represents this relationship?

F $A = \frac{1}{2}a + bh$

G $A = \frac{1}{2}(a + bh)$

H $A = \frac{1}{2}(a + b)h$

J $A = \frac{1}{2}(a + b + h)$

11. The odometer on Jenna's car is broken. It advances 1.1 miles for every mile Jenna drives. If the odometer showed that she drove 290.4 miles since she last filled the gas tank, how many miles did she actually drive?

A 264 miles

B 289.3 miles

C 291.5 miles

D 319.4 miles

Pre-AP

Record your answers on a sheet of paper. Show your work.

12. A car company lists the stopping distances of a car at different speeds.

Speed (ft/s)	Minimum Stopping Distance (ft)
10	2
20	8
40	31
60	70
100	194

a. Does the table of values represent a function? Explain.

b. Is this a proportional relationship? Explain.

NEED EXTRA HELP?												
If You Missed Question...	1	2	3	4	5	6	7	8	9	10	11	12
Go to Lesson or Page...	2-4	2-4	1-2	3-3	3-1	2-1	2-6	2-1	708	2-1	2-6	3-2