CHAPTER 5

Solving Systems of Linear Equations

BIG Ideas

- Solve systems of linear equations by graphing.
- Solve systems of linear equations algebraically.
- Apply systems of linear equations.

Key Vocabulary

elimination (p. 266)
substitution (p. 260)
system of equations (p. 253)

Real-World Link

Travel Trends in the travel industry change with time. For example, the number of tourists traveling to South America, the Caribbean, and the Middle East is on the rise. These changes can be modeled by systems of linear equations.

FOLDABLES™ Study Organizer

Solving Systems of Linear Equations Make this Foldable to record information about solving systems of equations and inequalities. Begin with five sheets of grid paper.

1. **Fold** each sheet in half along the width.

2. **Unfold** and cut four rows from left side of each sheet, from the top to the crease.

3. **Stack** the sheets and staple to form a booklet.

4. **Label** each page with a lesson number and title.

Bob Coates/Mira

GET READY for Chapter 5

Diagnose Readiness You have two options for checking Prerequisite Skills.

Option 2

MathOnline Take the Online Readiness Quiz at algebra1.com.

Option 1

Take the Quick Check below. Refer to the Quick Review for help.

QUICKCheck

Graph each equation. (Lesson 3-3)

1. $y = 1$
2. $y = -2x$
3. $y = 4 - x$
4. $y = 2x + 3$
5. $y = 5 - 2x$
6. $y = \frac{1}{2}x + 2$
7. **HOUSES** The number on Craig's house is 7. The numbers of the houses on his block increase by 2. Graph the equation that models the house numbers on Craig's block.

Solve each equation or formula for the variable specified. (Lesson 2-8)

8. $4x + a = 6x$, for x
9. $8a + y = 16$, for a
10. $\frac{7bc - d}{10} = 12$, for b
11. $\frac{7m + n}{q} = 2m$, for q

Simplify each expression. If not possible, write *simplified*. (Lesson 1-6)

12. $(3x + y) - (2x + y)$
13. $(7x - 2y) - (7x + 4y)$
14. **MOWING** Jake and his brother charge x dollars to cut and y dollars to weed an average lawn. Simplify the expression that gives the total amount that their business earns in a weekend if Jake cuts and weeds 7 lawns and his brother cuts and weeds 10 lawns.

QUICKReview

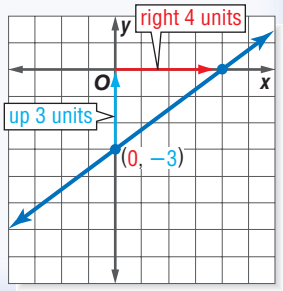
EXAMPLE 1

Graph $y = \frac{3}{4}x - 3$.

Step 1 The y-intercept is -3. So, graph $(0, -3)$.

Step 2 The slope is $\frac{3}{4}$. From $(0, -3)$, move up 3 units and right 4 units. Draw a dot.

Step 3 Draw the line.



EXAMPLE 2

Solve $\frac{2y}{3s} = \frac{2y}{13x}$ for x.

$\frac{2y}{3s} = \frac{2y}{13x}$ Original equation

$2y \cdot 3s = 2y \cdot 13x$ Find the cross products.

$6ys = 26xy$ Simplify.

$\frac{6ys}{26y} = x$ Divide each side by $26y$.

$\frac{3s}{13} = x$ Simplify.

EXAMPLE 3

Simplify $3(x - y) - (x - y)$. If not possible, write *simplified*.

$3(x - y) - (x - y)$ Original expression

$= 3x - 3y - x + y$ Distributive Property

$= 2x - 2y$ Combine like terms.

$= 2(x - y)$ Factor out a 2.

EXPLORE 5-1

Spreadsheet Lab

Systems of Equations

You can use a spreadsheet to investigate when two quantities will be equal. Enter each formula into the spreadsheet and look for the row in which both formulas have the same result.

EXAMPLE

Bill Winters is considering two job offers in telemarketing departments. The salary at the first job is $400 per week plus 10% commission on Mr. Winters' sales. At the second job, the salary is $375 per week plus 15% commission. For what amount of sales would the weekly salary be the same at either job?

Enter different amounts for Mr. Winters' weekly sales in column A. Then enter the formula for the salary at the first job in each cell in column B. In each cell of column C, enter the formula for the salary at the second job.

The spreadsheet shows that for sales of $500 the total weekly salary for each job is $450.

Job Salaries.xls

	A	B	C
1	Sales	Salary 1	Salary 2
2	0	400	375
3	100	410	390
4	200	420	405
5	300	430	420
6	400	440	435
7	500	450	450
8	600	460	465
9	700	470	480
10	800	480	495
11	900	490	510
12	1000	500	525

Sheet 1 Sheet 2 Sheet 3

Concepts in Motion
Interactive Lab
algebra1.com

Exercises

For Exercises 1–4, use the spreadsheet of weekly salaries above.

1. If x is the amount of Mr. Winters' weekly sales and y is his total weekly salary, write a linear equation for the salary at the first job.
2. Write a linear equation for the salary at the second job.
3. Which ordered pair is a solution for both of the equations you wrote for Exercises 1 and 2?

   a. (100, 410) **b.** (300, 420) **c.** (500, 450) **d.** (900, 510)
4. Use the graphing capability of the spreadsheet program to graph the salary data using a line graph. At what point do the two lines intersect? What is the significance of that point in the real-world situation?
5. How could you find the sales for which Mr. Winters' salary will be equal without using a spreadsheet?

5-1 Graphing Systems of Equations

Main Ideas

- Determine whether a system of linear equations has no, one, or infinitely many solutions.
- Solve systems of equations by graphing.

New Vocabulary

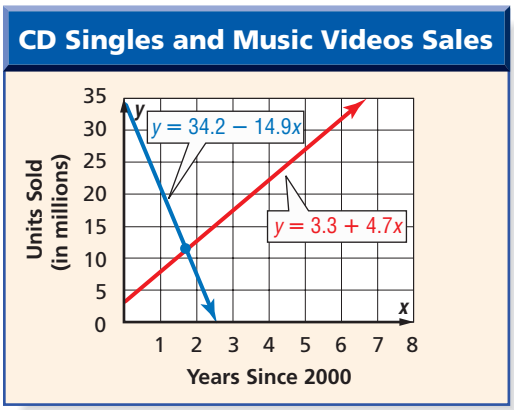
system of equations
consistent
inconsistent
independent
dependent

GET READY for the Lesson

If x is the number of years since 2000 and y is units sold in millions, the following equations represent the sales of CD singles and music videos.

CD singles: $y = 34.2 - 14.9x$
music videos: $y = 3.3 + 4.7x$

The point at which the graphs of the two equations intersect represents the time when the CD units sold equaled the music videos sold. The ordered pair of this point is a solution of both equations.

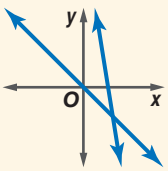
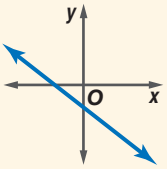
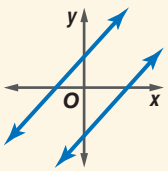


Source: The Recording Industry Association of America

Number of Solutions Two equations, such as $y = 34.2 - 14.9x$ and $y = 3.3 + 4.7x$, together are called a **system of equations**. A solution of a system is an ordered pair that satisfies both equations. A system of two linear equations can have no, one, or an infinite number of solutions.

- If the graphs intersect or coincide, the system of equations is **consistent**. That is, it has at least one ordered pair that satisfies both equations.
- If a consistent system has exactly one solution, it is **independent**. If it has infinite solutions, it is **dependent**.
- If the graphs are parallel, the system of equations is said to be **inconsistent**. There are *no* ordered pairs that satisfy both equations.

KEY CONCEPT *Graphing Systems of Equations*

Graph of a System			
Number of Solutions	exactly one solution	infinitely many	no solutions
Terminology	consistent and independent	consistent and dependent	inconsistent

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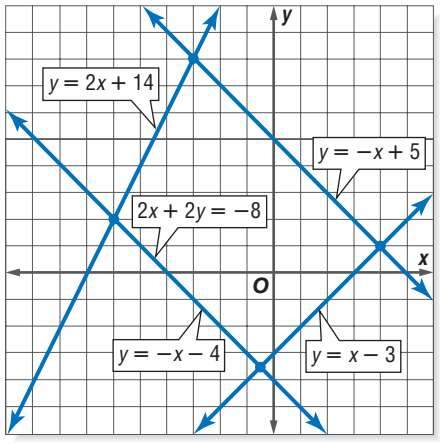
EXAMPLE Number of Solutions

1 **Use the graph at the right to determine whether each system has *no* solution, *one* solution, or *infinitely many* solutions.**



a. $y = -x + 5$
$y = x - 3$

Since the graphs are intersecting lines, there is one solution.

b. $y = -x + 5$
$2x + 2y = -8$

Since the graphs are parallel, there are no solutions.

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CHECK Your Progress

1A. $2x + 2y = -8$
$y = -x - 4$

1B. $y = 2x + 14$
$y = -x + 5$

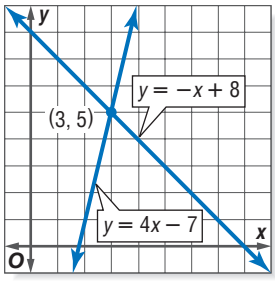
Solve By Graphing One method of solving systems of equations is to carefully graph the equations on the same coordinate plane.

EXAMPLE Solve a System of Equations

2 **Graph each system of equations. Then determine whether the system has *no* solution, *one* solution, or *infinitely many* solutions. If the system has one solution, name it.**

a. $y = -x + 8$
$y = 4x - 7$

The graphs appear to intersect at (3, 5). Check by replacing x with 3 and y with 5.



CHECK

$y = -x + 8$
$5 \stackrel{?}{=} -3 + 8$
$5 = 5$ ✓

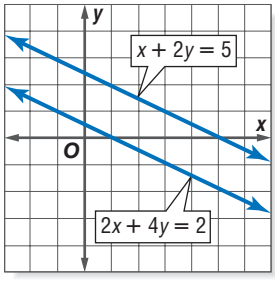
$y = 4x - 7$
$5 \stackrel{?}{=} 4(3) - 7$
$5 = 5$ ✓

The solution is (3, 5).

Study Tip
Look Back
To review **graphing linear equations**, see Lesson 3-3.

b. $x + 2y = 5$
$2x + 4y = 2$

The graphs are parallel lines. Since they do not intersect, there are no solutions to this system of equations. Notice that the lines have the same slope but different y-intercepts. *Recall that a system of equations that has no solution is said to be inconsistent.*



CHECK Your Progress

2A. $x - y = 2$
$3y + 2x = 9$

2B. $y = -2x - 3$
$2x + y = -3$

Online **Personal Tutor at** algebra1.com

Real-World Link

In 2004, 2.9 million girls participated in high school sports. This was an all-time high for female participation.

Source: National Federation of State High School Associations

Real-World EXAMPLE 3 Write and Solve a System of Equations

SPORTS The number of girls participating in high school soccer and track and field has steadily increased during the past few years. Use the information in the table to predict the year in which the number of girls participating in these two sports will be the same.

High School Sport	Number of Girls Participating in 2004 (thousands)	Average Rate of Increase (thousands per year)
soccer	309	8
track and field	418	3

Source: National Federation of State High School Associations

Words Number of girls participating | equals | rate of increase | times | number of years after 2004 | plus | number participating in 2004.

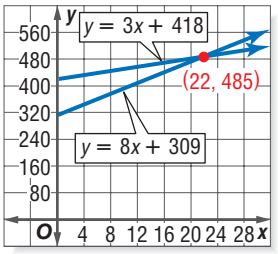
Variables Let y = number of girls competing. Let x = number of years after 2004.

Equations

soccer: $y = 8 \times x + 309$

track and field: $y = 3 \times x + 418$

Graph the equations $y = 8x + 309$ and $y = 3x + 418$. The graphs appear to intersect at (22, 485). Check by replacing x with 22 and y with 485 in each equation.



CHECK

$y = 8x + 309$ | $y = 3x + 418$

$485 = 8(22) + 309$ | $485 = 3(22) + 418$

$485 = 485$ ✓ | $485 \approx 484$ ✓

The solution means that approximately 22 years after 2004, or in 2026, the number of girls participating in high school soccer and track and field will be the same, about 485,000.

Concepts in Motion

Animation

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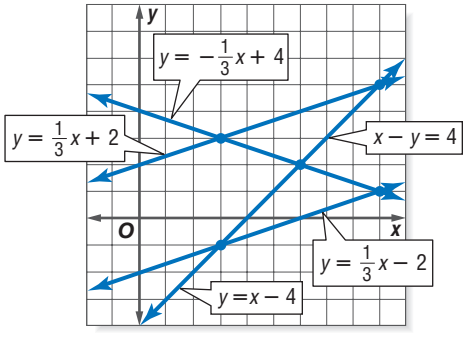
CHECK Your Progress

3. GARDENS A rectangular garden has a border around it consisting of 60 bricks. The width of the border has $\frac{2}{3}$ the number of bricks as the length. How many bricks are along one length of the garden?

CHECK Your Understanding

Example 1 (p. 254)

Use the graph to determine whether each system has *no* solution, *one* solution, or *infinitely many* solutions.



1. $y = x - 4$
$y = \frac{1}{3}x - 2$

2. $y = \frac{1}{3}x + 2$
$y = \frac{1}{3}x - 2$

3. $x - y = 4$
$y = x - 4$

4. $x - y = 4$
$y = -\frac{1}{3}x + 4$

Example 2 (p. 254) **Graph each system of equations. Then determine whether the system has *no* solution, *one* solution, or *infinitely many* solutions. If the system has one solution, name it.**

5. $y = 3x - 4$
$y = -3x - 4$

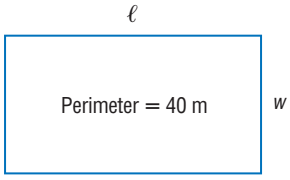
6. $x + y = 2$
$y = 4x + 7$

7. $x + y = 4$
$x + y = 1$

8. $2x + 4y = 2$
$3x + 6y = 3$

Example 3 (p. 255)

9. GEOMETRY The length of the rectangle is 1 meter less than twice its width. What are the dimensions of the rectangle?

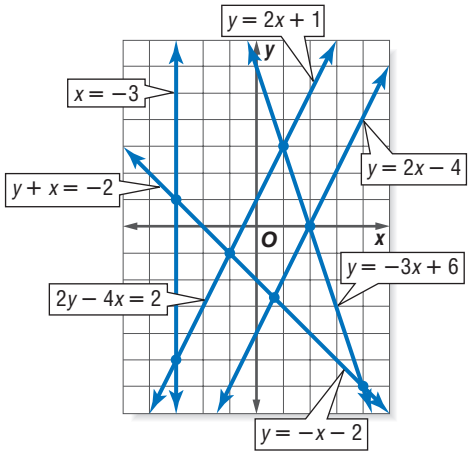


Exercises

HOMEWORK

For Exercises	See Examples
10–15	1
16–27	2
28–31	3

Use the graph to determine whether each system has *no* solution, *one* solution, or *infinitely many* solutions.



10. $x = -3$
$y = 2x + 1$

11. $y = -x - 2$
$y = 2x - 4$

12. $y = 2x + 1$
$2y - 4x = 2$

13. $y = 2x + 1$
$y = 2x - 4$

14. $y + x = -2$
$y = -x - 2$

15. $2y - 4x = 2$
$y = 2x - 4$

Graph each system of equations. Then determine whether the system has *no* solution, *one* solution, or *infinitely many* solutions. If the system has one solution, name it.

16. $y = -6$
$4x + y = 2$

17. $x = 2$
$3x - y = 8$

18. $y = \frac{1}{2}x$
$2x + y = 10$

19. $y = -x$
$y = 2x - 6$

20. $y = 2x + 6$
$y = -x - 3$

21. $x - 2y = 2$
$3x + y = 6$

22. $x + y = 2$
$2y - x = 10$

23. $3x + 2y = 12$
$3x + 2y = 6$

24. $2x + 3y = 4$
$-4x - 6y = -8$

25. $2x + y = -4$
$5x + 3y = -6$

26. $4x + 3y = 24$
$5x - 8y = -17$

27. $3x + y = 3$
$2y = -6x + 6$

SAVINGS **For Exercises 28 and 29, use the following information.**
Monica and Max Gordon each want to buy a scooter. Monica has already saved $25 and plans to save $5 per week until she can buy the scooter. Max has $16 and plans to save $8 per week.

28. In how many weeks will Monica and Max have saved the same amount of money?

29. How much will each person have saved at that time?

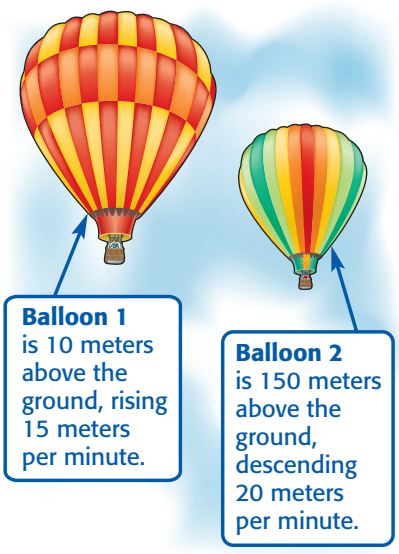
Cross-Curricular Project

Math Online You can graph a system of equations to predict when men's and women's Olympic times will be the same. Visit algebra1.com to continue work on your project.

BALLOONING **For Exercises 30 and 31, use the information in the graphic at the right.**

30. In how many minutes will the balloons be at the same height?

31. How high will the balloons be at that time? Is your answer reasonable? Explain.



Graph each system of equations. Then determine whether the system has *no* solution, *one* solution, or *infinitely many* solutions. If the system has one solution, name it.

32. $y = 0.6x - 5$
$2y = 1.2x$

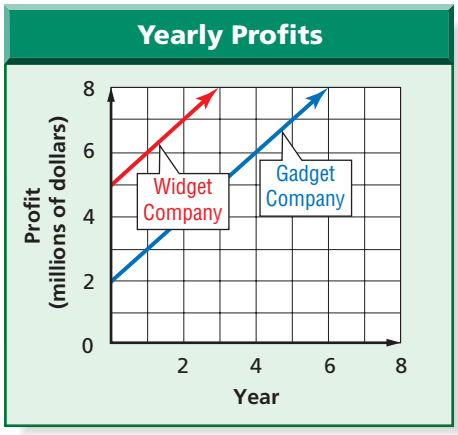
33. $6 - \frac{3}{8}y = x$
$\frac{2}{3}x + \frac{1}{4}y = 4$

ANALYZE GRAPHS **For Exercises 34–36, use the graph at the right.**

34. Which company had the greater profit during the ten years?

35. Which company had a greater rate of growth?

36. If the profit patterns continue, will the profits of the two companies ever be equal? Explain.



POPULATION **For Exercises 37–39, use the following information.**

The U.S. Census Bureau divides the country into four sections. They are the Northeast, the Midwest, the South, and the West. The populations and rates of growth for the Midwest and the West are shown in the table.

Section	2000 Population (millions)	Average Rate of Increase (millions per year)
Midwest	64.4	0.3
West	63.2	1.0

Source: U.S. Census Bureau

EXTRA PRACTICE
See pages 727, 748.
Math Online
Self-Check Quiz at algebra1.com

37. Write an equation to represent the population of the Midwest for the years since 2000.

38. Write an equation to represent the population of the West for the years since 2000.

39. Graph the population equations. Assume that the rate of growth of each of these areas remained the same. Estimate the solution and interpret what it means.

H.O.T. Problems

40. CHALLENGE The solution of the system of equations $Ax + y = 5$ and $Ax + By = 20$ is $(2, -3)$. What are the values of A and B? Justify your reasoning.

41. OPEN ENDED Write three equations such that they form a system of equations with $y = 5x - 3$. The systems should have *no*, *one*, and *infinitely many* solutions, respectively.

42. REASONING Determine whether a system of two linear equations with (0, 0) and (2, 2) as solutions *sometimes*, *always*, or *never* has other solutions. Explain.

43. *Writing in Math* Use the information on page 253 to explain how graphs can be used to compare the sales of two products. Include an estimate of the year in which the CD units sold equaled the music videos sold. Then determine the reasonableness of your solution in the context of the problem.

STANDARDIZED TEST PRACTICE

44. A buffet restaurant has one price for adults and another price for children. The Taylor family has two adults and three children, and their bill was \$40.50. The Wong family has three adults and one child. Their bill was \$38. Which system of equations could be used to determine the buffet price for an adult and the price for a child?

A $x + y = 40.50$
$x + y = 38$

B $2x + 3y = 40.50$
$x + 3y = 38$

C $2x + 3y = 40.50$
$3x + y = 38$

D $2x + 2y = 40.50$
$3x + y = 38$

45. REVIEW Francisco has 3 dollars more than $\frac{1}{4}$ the number of dollars that Kayla has. Which expression represents how much money Francisco has?

F $3\left(\frac{1}{4}k\right)$

G $3 - \frac{1}{4}k$

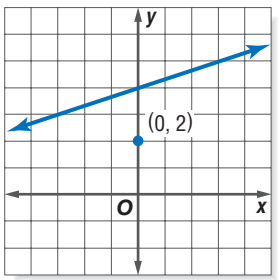
H $\frac{1}{4}k + 3$

J $\frac{1}{4} + 3k$

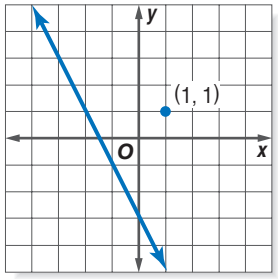
Spiral Review

Write the slope-intercept form of an equation for the line that passes through the given point and is parallel to the graph of each equation. (Lesson 4-7)

46.



47.



48. BIOLOGY The table shows the date of the month that 10 students were born, and their heights. Draw a scatter plot and determine what relationship exists, if any, in the data. Explain. (Lesson 4-6)

Date of Birth	12	28	24	15	3	11	20	5	3	9
Height (in.)	60	58	62	60	59	64	66	65	67	62

GET READY for the Next Lesson

PREREQUISITE SKILL **Solve each equation for the variable specified.** (Lesson 2-8)

49. $12x - y = 10x$, for y

50. $6a + b = 2a$, for a

51. $\frac{7m - n}{q} = 10$, for q

EXTEND 5-1

Graphing Calculator Lab

Systems of Equations

You can use a graphing calculator to solve a system of equations.

EXAMPLE

Solve the system of equations. State the decimal solution to the nearest hundredth.

$2.93x + y = 6.08$

$8.32x - y = 4.11$

Step 1 Solve each equation for y. Enter them into the calculator.

$2.93x + y = 6.08$	First equation
$2.93x + y - 2.93x = 6.08 - 2.93x$	Subtract $2.93x$ from each side.
$y = 6.08 - 2.93x$	Simplify.
$8.32x - y = 4.11$	Second equation
$8.32x - y - 8.32x = 4.11 - 8.32x$	Subtract $8.32x$ from each side.
$-y = 4.11 - 8.32x$	Simplify.
$(-1)(-y) = (-1)(4.11 - 8.32x)$	Multiply each side by -1.
$y = -4.11 + 8.32x$	Simplify.

Step 2 Enter these equations in the Y= list and graph.

KEYSTROKES: *Review on pages 162–163.*

Step 3 Use the CALC menu to find the point of intersection.

KEYSTROKES: 2nd [CALC] 5 ENTER ENTER ENTER

The solution is approximately (0.91, 3.43).

[−10, 10] scl: 1 by [−10, 10] scl: 1

Exercises

Use a graphing calculator to solve each system of equations. Write decimal solutions to the nearest hundredth.

1. $y = 3x - 4$
$y = -0.5x + 6$

2. $y = 2x + 5$
$y = -0.2x - 4$

3. $x + y = 5.35$
$3x - y = 3.75$

4. $0.35x - y = 1.12$
$2.25x + y = -4.05$

5. $1.5x + y = 6.7$
$5.2x - y = 4.1$

6. $5.4x - y = 1.8$
$6.2x + y = -3.8$

7. $5x - 4y = 26$
$4x + 2y = 53.3$

8. $2x + 3y = 11$
$4x + y = -6$

9. $0.22x + 0.15y = 0.30$
$-0.33x + y = 6.22$

5-2 Substitution

Main Ideas

- Solve systems of equations algebraically by using substitution.
- Solve real-world problems involving systems of equations.

New Vocabulary

substitution

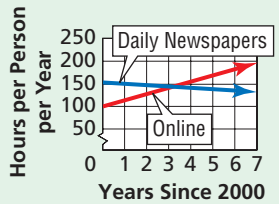
GET READY for the Lesson

Americans spend more time online than they spend reading daily newspapers. If x represents the number of years since 2000 and y represents the average number of hours per person per year, the following system represents the situation.

reading daily newspapers: $y = -2.8x + 150.4$
online: $y = 14.4x + 102.8$

The solution of the system represents the year that the number of hours spent on each activity will be the same. To solve this system, you could graph the equations and find the point of intersection. However, the exact coordinates of the point would be very difficult to determine from the graph. You could find a more accurate solution by using algebraic methods.

Media Usage



Substitution The exact solution of a system of equations can be found by using algebraic methods. One such method is called **substitution**.

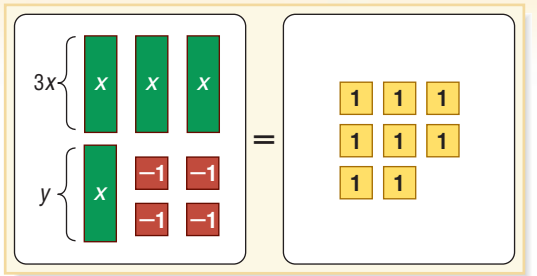
ALGEBRA LAB

Using Substitution

Use algebra tiles and an equation mat to solve the system of equations. $3x + y = 8$ and $y = x - 4$

MODEL AND ANALYZE

Since $y = x - 4$, use 1 positive x-tile and 4 negative 1-tiles to represent y. Use algebra tiles to represent $3x + y = 8$.



1. Use what you know about equation mats to solve for x. What is the value of x?
2. Use $y = x - 4$ to solve for y.
3. What is the solution of the system of equations?
4. **MAKE A CONJECTURE** Explain how to solve the following system of equations using algebra tiles. $4x + 3y = 10$ and $y = x + 1$
5. Why do you think this method is called substitution?

EXAMPLE 1 Solve Using Substitution

Use substitution to solve each system of equations.

a. $y = 3x$
$x + 2y = -21$

Since $y = 3x$, substitute $3x$ for y in the second equation.

$x + 2y = -21$	Second equation
$x + 2(3x) = -21$	$y = 3x$
$x + 6x = -21$	Simplify.
$7x = -21$	Combine like terms.
$x = -3$	Divide each side by 7 and simplify.

Use $y = 3x$ to find the value of y.

$y = 3x$	First equation
$y = 3(-3)$ or -9	$x = -3$

The solution is $(-3, -9)$. Check the solution by graphing.

Study Tip

Looking Back

To review **solving linear equations,** see Lesson 2-5.

b. $x + 5y = -3$
$3x - 2y = 8$

Solve the first equation for x since the coefficient of x is 1.

$x + 5y = -3$	First equation
$x + 5y - 5y = -3 - 5y$	Subtract $5y$ from each side.
$x = -3 - 5y$	Simplify.

Find the value of y by substituting $-3 - 5y$ for x in the second equation.

$3x - 2y = 8$	Second equation
$3(-3 - 5y) - 2y = 8$	$x = -3 - 5y$
$-9 - 15y - 2y = 8$	Distributive Property
$-9 - 17y = 8$	Combine like terms.
$-9 - 17y + 9 = 8 + 9$	Add 9 to each side.
$-17y = 17$	Simplify.
$\frac{-17y}{-17} = \frac{17}{-17}$	Divide each side by -17.
$y = -1$	Simplify.

Study Tip

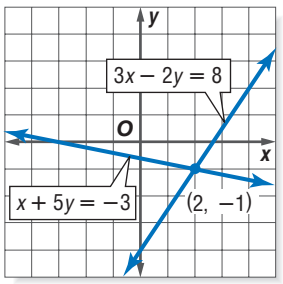
Substituting

When substituting to find the value of the second variable, choose the equation that is easier to solve.

Substitute -1 for y in either equation to find the value of x.

$x + 5y = -3$	First equation
$x + 5(-1) = -3$	$y = -1$
$x - 5 = -3$	Simplify.
$x = 2$	Add 5 to each side.

The solution is $(2, -1)$. The graph verifies the solution.



CHECK Your Progress

1A. $4x + 5y = 11$
$y = 3x - 13$

1B. $x - 3y = -9$
$5x - 2y = 7$

In general, if you solve a system of linear equations and the result is a true statement (an identity such as $-4 = -4$), the system has an infinite number of solutions. If the result is a false statement (such as $-4 = 5$), there is no solution.

EXAMPLE 2 Infinitely Many or No Solutions

Use substitution to solve the system of equations.

$$6x - 2y = -4$$
$$y = 3x + 2$$

Since $y = 3x + 2$, substitute $3x + 2$ for y in the first equation.

$$\begin{aligned} 6x - 2y &= -4 && \text{First equation} \\ 6x - 2(3x + 2) &= -4 && y = 3x + 2 \\ 6x - 6x - 4 &= -4 && \text{Distributive Property} \\ -4 &= -4 && \text{Simplify.} \end{aligned}$$

The statement $-4 = -4$ is true. So, there are infinitely many solutions.

Study Tip

Dependent Systems There are infinitely many solutions of the system in Example 2, because the slope-intercept form of both equations is $y = 3x + 2$. That is, the equations are equivalent, and they have the same graph.

CHECK Your Progress

2A. $2x - y = 8$
$y = 2x - 3$

2B. $4x - 3y = 1$
$6y - 8x = -2$

Real-World Problems Sometimes it is helpful to organize data tables, charts, graphs, or diagrams before solving a problem.

EXAMPLE 3 Write and Solve a System of Equations

METAL ALLOYS **A metal alloy is 25% copper. Another metal alloy is 50% copper. How much of each alloy should be used to make 1000 grams of a metal alloy that is 45% copper?**

Let a = the number of grams of the 25% copper alloy and b = the number of grams of the 50% copper alloy. Use a table to organize the information.

	25% Copper	50% Copper	45% Copper	
Total Grams	a	b	1000	$\rightarrow a + b = 1000$
Grams of Copper	$0.25a$	$0.50b$	$0.45(1000)$	$\rightarrow 0.25a + 0.5b = 0.45(1000)$

$$\begin{aligned} a + b &= 1000 && \text{First equation} \\ a + b - b &= 1000 - b && \text{Subtract } b \text{ from each side.} \\ a &= 1000 - b && \text{Simplify.} \end{aligned}$$

$$\begin{aligned} 0.25a + 0.50b &= 0.45(1000) && \text{Second equation} \\ 0.25(1000 - b) + 0.50b &= 0.45(1000) && a = 1000 - b \\ 250 - 0.25b + 0.50b &= 450 && \text{Distributive Property} \\ 250 + 0.25b &= 450 && \text{Combine like terms.} \\ 250 + 0.25b - 250 &= 450 - 250 && \text{Subtract 250 from each side.} \\ 0.25b &= 200 && \text{Simplify.} \\ \frac{0.25b}{0.25} &= \frac{200}{0.25} && \text{Divide each side by 0.25.} \\ b &= 800 && \text{Simplify.} \end{aligned}$$

Study Tip

Alternative Method Using a system of equations is an alternative method for solving the weighted average problems that you studied in Lesson 2-9.

$a + b = 1000$ First equation

$a + 800 = 1000$ $b = 800$

$a = 200$ Subtract 800 from each side and simplify.

200 grams of the 25% alloy and 800 grams of the 50% alloy should be used.

CHECK Your Progress

3. BASEBALL The New York Yankees and the Cincinnati Reds together have won a total of 31 World Series. The Yankees have won 5.2 times as many as the Reds. How many World Series did each team win?

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CHECK Your Understanding

Examples 1–2 (pp. 261–262)

Use substitution to solve each system of equations. If the system does *not* have exactly one solution, state whether it has *no* solution or *infinitely many* solutions.

1. $2x + 7y = 3$
$x = 1 - 4y$

2. $6x - 2y = -4$
$y = 3x + 2$

3. $y = \frac{3}{5}x$
$3x - 5y = 15$

4. $x + 3y = 12$
$x - y = 8$

5. $a + b = 1$
$5a + 3b = -1$

6. $\frac{1}{3}x - y = 2$
$x - 3y = 6$

Example 3 (p. 262)

7. TRANSPORTATION The Thrust SSC is the world's fastest land vehicle. Suppose the driver of a car with a top speed of 200 miles per hour requests a race against the SSC. The car gets a head start of one-half hour. If there is unlimited space to race, at what distance will the SSC pass the car?

Exercises

HOMEWORK HELP

For Exercises	See Examples
8–19	1–2
20, 21	3

Use substitution to solve each system of equations. If the system does *not* have exactly one solution, state whether it has *no* solution or *infinitely many* solutions.

8. $y = 5x$
$2x + 3y = 34$

9. $x = 4y$
$2x + 3y = 44$

10. $x = 4y + 5$
$x = 3y - 2$

11. $y = 2x + 3$
$y = 4x - 1$

12. $4c = 3d + 3$
$c = d - 1$

13. $x = \frac{1}{2}y + 3$
$2x - y = 6$

14. $8x + 2y = 13$
$4x + y = 11$

15. $2x - y = -4$
$-3x + y = -9$

16. $3x - 5y = 11$
$x - 3y = 1$

17. $2x + 3y = 1$
$-x + \frac{1}{3}y = 5$

18. $c - 5d = 2$
$2c + d = 4$

19. $5r - s = 5$
$-4r + 5s = 17$

20. GEOMETRY Angles X and Y are supplementary, and the measure of angle X is 24 degrees greater than the measure of angle Y. Find the angle measures.

Review Vocabulary

Supplementary Angles two angles with measures that have the sum of 180 degrees

21. CHEMISTRY MX Labs needs to make 500 gallons of a 34% acid solution. The only solutions available are a 25% acid solution and a 50% acid solution. How many gallons of each solution should be mixed to make the 34% solution?

Use substitution to solve each system of equations. If the system does *not* have exactly one solution, state whether it has *no* solutions or *infinitely many* solutions.

22. $x - 3y = 0$
$3x + y = 7$

23. $-0.3x + y = 0.5$
$0.5x - 0.3y = 1.9$

24. $0.5x - 2y = 17$
$2x + y = 104$

25. $y = \frac{1}{2}x + 3$
$y = 2x - 1$

JOBS **For Exercises 26 and 27, use the following information.**

Shantel Jones has a job offer in which she will receive $600 per month plus a commission of 2% of the total price of the cars she sells. At her current job, she receives $1000 per month plus a commission of 1.5% of her total sales.

26. What is the total price of the cars that Ms. Jones must sell each month to make the same income from either dealership?

27. How much must Ms. Jones sell to make the new job a better deal?

Real-World Career

Chemist

Many chemists study the properties and composition of substances. This research can be applied to the development of products including cosmetics, pharmaceuticals, and cleaning products.

Math Online

For more information, go to algebra1.com.

28. LANDSCAPING A blue spruce grows an average of 6 inches per year. A hemlock grows an average of 4 inches per year. If a blue spruce is 4 feet tall and a hemlock is 6 feet tall, write a system of equations to represent their growth. Find and interpret the solution in the context of the situation.

29. ANALYZE TABLES The table shows the approximate number of tourists in two areas during a recent year and the average rates of change in tourism. If the trends continue, in how many years would you expect the number of tourists to the regions to be equal?

Destination	Number of Tourists	Average Rates of Change in Tourists (millions per year)
South America and the Caribbean	40.3 million	increase of 0.8
Middle East	17.0 million	increase of 1.8

EXTRA PRACTICE
See pages 727, 748.
Math Online
Self-Check Quiz at algebra1.com

30. RESEARCH Use the Internet or other resource to find the pricing plans for various cell phones. Determine the number of minutes you would need to use the phone for two plans to cost the same amount of money.

H.O.T. Problems

31. FIND THE ERROR In the system $a + b = 7$ and $1.29a + 0.49b = 6.63$, a represents the pounds of apples bought and b represents pounds of bananas. Josh and Lydia are finding and interpreting the solution. Who is correct? Explain.

Josh

$1.29a + 0.49b = 6.63$
$1.29a + 0.49(7 - a) = 6.63$
$1.29a + 3.43 - 0.49a = 6.63$
$0.8a = 3.2$
$a = 4$

$a + b = 7$, so $b = 3$. The solution $(4, 3)$ means that 4 apples and 3 bananas were bought.

Lydia

$1.29a + 0.49b = 6.63$
$1.29(7 - b) + 0.49b = 6.63$
$9.03 - 1.29b + 0.49b = 6.63$
$-0.8b = -2.4$
$b = 3$

The solution $b = 3$ means that 3 apples and 3 bananas were bought.

32. CHALLENGE Solve the system of equations below. Write the solution as an ordered triple of the form (x, y, z). Describe the steps that you used.

$2x + 3y - z = 17$ $\quad y = -3z - 7$ $\quad 2x = z + 2$

33. OPEN ENDED Create a system of equations that has one solution. Illustrate how the system could represent a real-world situation and describe the significance of the solution in the context of the situation.

34. ***Writing in Math*** Use the data about the time Americans spend online and reading newspapers on page 260 to explain how substitution can be used to analyze problems. Then solve the system and interpret its meaning in the situation.

STANDARDIZED TEST PRACTICE

35. The debate team plans to make and sell trail mix for a fundraiser.

Item	Cost Per Pound
sunflower seeds	\$4.00
raisins	\$1.50

The number of pounds of raisins in the mix is to be 3 times the pounds of sunflower seeds. If they can spend \$34, which system can be used to find r, the pounds of raisins, and s, pounds of sunflower seeds, they should buy?

A $3s = r$
$4s + 1.5r = 34$

B $3r = s$
$4s + 1.5r = 34$

C $3s = r$
$4r + 1.5s = 34$

D $3r = s$
$4r + 1.5s = 34$

36. What is the solution to this system of equations?

$$\begin{cases} x + 4y = 1 \\ 2x - 3y = -9 \end{cases}$$

F $(2, -8)$

G $(-3, 1)$

H no solution

J infinitely many solutions

37. REVIEW What is the value of x if $4x - 3 = -2x$?

A -2

B $-\frac{1}{2}$

C $\frac{1}{2}$

D 2

Spiral Review

Graph each system of equations. Then determine whether the system has *no* solution, *one* solution, or *infinitely many* solutions. If the system has one solution, name it. (Lesson 5-1)

38. $x + y = 3$
$x + y = 4$

39. $x + 2y = 1$
$2x + y = 5$

40. $2x + y = 3$
$4x + 2y = 6$

41. Draw a scatter plot that shows a positive correlation. (Lesson 4-7)

42. RECYCLING When a pair of blue jeans is made, the leftover denim scraps can be recycled. One pound of denim is left after making every fifth pair of jeans. How many pounds of denim would be left from 250 pairs of jeans? (Lesson 2-6)

GET READY for the Next Lesson

PREREQUISITE SKILL Simplify each expression. (Lesson 1-5)

43. $6a - 9a$

44. $8t + 4t$

45. $-7g - 8g$

46. $7d - (2d + b)$

47. $(2x + 5y) + (x - 2y)$

48. $(3m + 2n) - (5m + 7n)$

5-3 Elimination Using Addition and Subtraction

Main Ideas

- Solve systems of equations algebraically by using elimination with addition.
- Solve systems of equations algebraically by using elimination with subtraction.

New Vocabulary

elimination

GET READY for the Lesson

The winter solstice marks the shortest day and longest night of the year in the Northern Hemisphere. On that day in Seward, Alaska, the difference between the number of hours of darkness n and the number of hours of daylight d is 12. The following system of equations represents the situation.

$$n + d = 24$$
$$n - d = 12$$

Notice that if you add these equations, the variable d is eliminated.

$$\begin{array}{rr} n + d = & 24 \\ (+)\ n - d = & 12 \\ \hline 2n = & 36 \end{array}$$

Elimination Using Addition Sometimes adding two equations together will eliminate one variable. Using this step to solve a system of equations is called **elimination**.

EXAMPLE Elimination Using Addition

1 **Use elimination to solve the system of equations.**

$$3x - 5y = -16$$
$$2x + 5y = 31$$

Since the coefficients of the y-terms, -5 and 5, are additive inverses, you can eliminate these terms by adding the equations.

$$\begin{array}{rrl} 3x - 5y = & -16 & \text{Write the equations in column form and add.} \\ (+)\ 2x + 5y = & 31 & \\ \hline 5x = & 15 & \text{The } y \text{ variable is eliminated.} \\ \frac{5x}{5} = & \frac{15}{5} & \text{Divide each side by 5.} \\ x = & 3 & \text{Simplify.} \end{array}$$

Now substitute 3 for x in either equation to find the value of y.

$3x - 5y = -16$ First equation

$3(3) - 5y = -16$ Replace x with 3.

$9 - 5y = -16$ Simplify.

$9 - 5y - 9 = -16 - 9$ Subtract 9 from each side.

$-5y = -25$ Simplify.

$\frac{-5y}{-5} = \frac{-25}{-5}$ Divide each side by -5.

$y = 5$ Simplify.

The solution is (3, 5).

CHECK Your Progress

Use elimination to solve each system of equations.

1A. $-4x + 3y = -3$
$4x - 5y = 5$

1B. $4y + 3x = 22$
$3x - 4y = 14$

Study Tip

Eliminating Variables

When solving systems by elimination, be sure to add like terms.

EXAMPLE 2 Write and Solve a System of Equations

Twice one number added to another number is 18. Four times the first number minus the other number is 12. Find the numbers.

Let x represent the first number and y represent the second number.

Twice one number	added to	another number	is	18.
$2x$	$+$	y	$=$	18

Four times the first number	minus	the other number	is	12.
$4x$	$-$	y	$=$	12

Use elimination to solve the system.

$2x + y = 18$ Write the equations in column form and add.
$(+)\ 4x - y = 12$
$6x = 30$ The variable y is eliminated.

$\frac{6x}{6} = \frac{30}{6}$ Divide each side by 6.

$x = 5$ Simplify.

Now substitute 5 for x in either equation to find the value of y.

$4x - y = 12$ Second equation

$4(5) - y = 12$ Replace x with 5.

$20 - y = 12$ Simplify.

$-y = -8$ Simplify.

$\frac{-y}{-1} = \frac{-8}{-1}$ Divide each side by -1.

$y = 8$

The numbers are 5 and 8.

CHECK Your Progress

2. The sum of two numbers is -10. Negative three times the first number minus the second number equals 2. Find the numbers.

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Elimination Using Subtraction Sometimes subtracting one equation from another will eliminate one variable.

EXAMPLE Elimination Using Subtraction

3 **Use elimination to solve the system of equations.**

$5s + 2t = 6$

$9s + 2t = 22$

Since the coefficients of the t-terms, 2 and 2, are the same, you can eliminate the t-terms by subtracting the equations.

$$\begin{array}{rl} 5s + 2t = 6 & \text{Write the equations in column form and subtract.} \\ (-)\ 9s + 2t = 22 & \\ \hline -4s = -16 & \text{The variable } t \text{ is eliminated.} \\ \frac{-4s}{-4} = \frac{-16}{-4} & \text{Divide each side by } -4. \\ s = 4 & \text{Simplify.} \end{array}$$

Now substitute 4 for s in either equation to find the value of t.

$$\begin{array}{rl} 5s + 2t = 6 & \text{First equation} \\ 5(4) + 2t = 6 & s = 4 \\ 20 + 2t = 6 & \text{Simplify.} \\ 2t = -14 & \text{Subtract 20 from each side and simplify.} \\ \frac{2t}{2} = -\frac{14}{2} & \text{Divide each side by 2.} \\ t = -7 & \text{The solution is } (4, -7). \end{array}$$

CHECK Your Progress

Use elimination to solve each system of equations.

3A. $8b + 3c = 11$
$8b + 7c = 7$

3B. $12n - m = -14$
$6n - m = -8$

CHECK Your Understanding

Examples 1, 3 (pp. 266–268) **Use elimination to solve each system of equations.**

1. $2a - 3b = -11$
$a + 3b = 8$

2. $4x + y = -9$
$4x + 2y = -10$

3. $6x + 2y = -10$
$2x + 2y = -10$

4. $-4m + 2n = 6$
$-4m + n = 8$

Example 2 (p. 267)

5. The sum of two numbers is 24. Five times the first number minus the second number is 12. What are the two numbers?

6. FOOTBALL In 2003, Rich Gannon, the Oakland Raiders quarterback, earned about \$4 million more than Charles Woodson, the Raiders cornerback. Together they cost the Raiders approximately \$9 million. How much did each make?

Exercises

HOMEWORK HELP	
For Exercises	See Examples
7–18	1, 3
19–24	2

Use elimination to solve each system of equations.

7. $x + y = -3$
$x - y = 1$

8. $s - t = 4$
$s + t = 2$

9. $3m - 2n = 13$
$m + 2n = 7$

10. $-4x + 2y = 8$
$4x - 3y = -10$

11. $3a + b = 5$
$2a + b = 10$

12. $2m - 5n = -6$
$2m - 7n = -14$

13. $3r - 5s = -35$
$2r - 5s = -30$

14. $13a + 5b = -11$
$13a + 11b = 7$

15. $3x - 5y = 16$
$-3x + 2y = -10$

16. $6s + 5t = 1$
$6s - 5t = 11$

17. $4x - 3y = 12$
$4x + 3y = 24$

18. $a - 2b = 5$
$3a - 2b = 9$

19. The sum of two numbers is 48, and their difference is 24. What are the numbers?

20. Find the two numbers whose sum is 51 and whose difference is 13.

21. Three times one number added to another number is 18. Twice the first number minus the other number is 12. Find the numbers.

22. One number added to twice another number is 13. Four times the first number added to twice the other number is −2. What are the numbers?

23. PARKS A youth group traveling in two vans visited Mammoth Cave in Kentucky. The number of people in each van and the total cost of a tour of the cave are shown in the table. Find the adult price and the student price of the tour.

Van	Number of Adults	Number of Students	Total Cost
A	2	5	$77
B	2	7	$95

Real-World Link
On average, 2 million people visit Mammoth Cave National Park each year. On a busy summer day, about 5000 to 7000 people come to the park.

Source: Mammoth Cave National Park

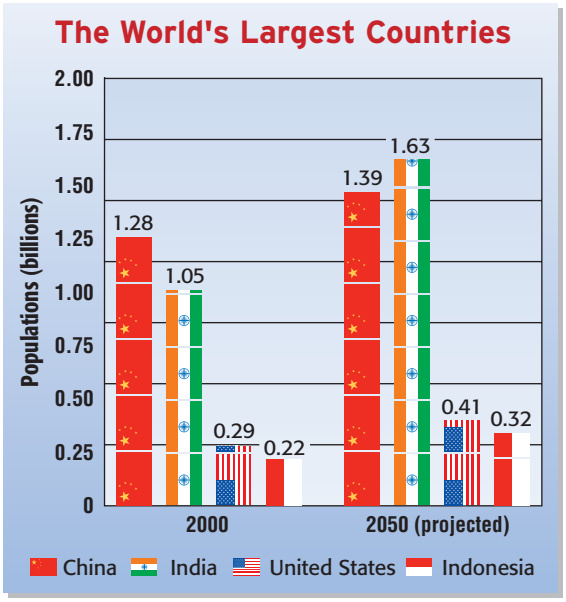
24. ARCHITECTURE The total height of an office building b and the granite statue that stands on top of it g is 326.6 feet. The difference in heights between the building and the statue is 295.4 feet. How tall is the statue?

ANALYZE GRAPHS For Exercises 25–27, use the information in the graph.

25. Let x represent the number of years since 2000 and y represent population in billions. Write an equation to represent the population of China.

26. Write an equation to represent the population of India.

27. Use elimination to find the solution to the system of equations. Interpret the solution.



Source: Population Reference Bureau

ONLINE CATALOGS For Exercises 28 and 29, use the table that shows the number of online catalogs and print catalogs in 2004 and the growth rates of each type.

Catalogs	Number in 2004	Growth Rate (number per year)
Online	7440	1293
Print	3805	−1364

Source: MediaPost Publications

EXTRA PRACTICE
See pages 727, 748.
Math Online
Self-Check Quiz at algebra1.com

H.O.T. Problems

28. Let x represent the number of years since 2004 and y represent the number of catalogs. Write a system of equations to represent this situation.

29. Solve the system of equations. Analyze the solution in terms of the situation. Determine the reasonableness of the solution.

30. OPEN ENDED Create a system of equations that can be solved by adding to eliminate a variable. Formulate a general rule for creating such systems.

31. CHALLENGE The graphs of $Ax + By = 15$ and $Ax - By = 9$ intersect at (2, 1). Find A and B and describe the steps that you used to find the values.

32. *Writing in Math* Use the information on page 266 to explain how to use elimination to solve a system of equations. Include a step-by-step solution of the Seward daylight problem.

STANDARDIZED TEST PRACTICE

33. What is the solution to this system of equations?

$2x - 3y = -9$
$-x + 3y = 6$

A (3, 3) **C** (−3, 3)
B (−3, 1) **D** (1, −3)

34. REVIEW Rhiannon is paid $52 for working 4 hours. At this rate, how many hours of work will it take her to earn $845?

F 13 hours **H** 3380 hours
G 65 hours **J** 10,985 hours

Spiral Review

Use substitution to solve each system of equations. If the system does *not* have exactly one solution, state whether it has *no* solution or *infinitely many* solutions. (Lesson 5-2)

35. $y = 5x$
$x + 2y = 22$

36. $x = 2y + 3$
$3x + 4y = -1$

37. $2y - x = -5$
$4y - 3x = -1$

Graph each system of equations. Then determine whether the system has *no* solution, *one* solution, or *infinitely many* solutions. If the system has one solution, name it. (Lesson 5-1)

38. $x - y = 3$
$3x + y = 1$

39. $2x - 3y = 7$
$3y = 7 + 2x$

40. $4x + y = 12$
$x = 3 - \frac{1}{4}y$

41. PHYSICAL SCIENCE If x cubic centimeters of water is frozen, the ice that is formed has a volume of $\left(x + \frac{1}{11}x\right)$ cubic centimeters. Simplify the expression for the volume of the ice. (Lesson 1-5)

GET READY for the Next Lesson

PREREQUISITE SKILL **Rewrite each expression without parentheses.** (Lesson 1-5)

42. $2(3x + 4y)$ **43.** $6(2a - 5b)$ **44.** $-3(-2m + 3n)$ **45.** $-5(4t - 2s)$

CHAPTER 5

Mid-Chapter Quiz

Lessons 5-1 through 5-3

Graph each system of equations. Then determine whether the system has *no* solution, *one* solution, or *infinitely many* solutions. If the system has one solution, name it. (Lesson 5-1)

1. $y = -x - 1$
$y = x + 5$

2. $x + y = 3$
$x - y = 1$

3. $3x - 2y = -6$
$3x - 2y = 6$

4. $3x + 2y = 4$
$6x + 4y = 8$

WORLD RECORDS **For Exercises 5 and 6, use the following information.**

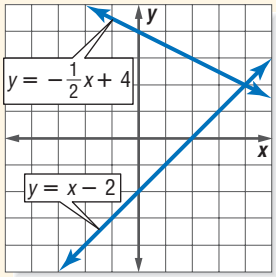
A swimmer broke a world record by crossing the Atlantic Ocean on a raft. He traveled about 44 miles each day by swimming s hours and floating on a raft f hours. The rates that he traveled are shown in the table. (Lesson 5-1)

Activity	Rate
swimming	3 mph
floating	1 mph

5. Write a system of equations to represent this situation.

6. Graph the system. Describe what the solution means in the context of the problem.

7. **MULTIPLE CHOICE** The graphs of the linear equations $y = -\frac{1}{2}x + 4$ and $y = x - 2$ are shown below. If $-\frac{1}{2}x + 4 = x - 2$, what is the value of x? (Lesson 5-1)



A 2

B 4

C 6

D 8

Use substitution to solve each system of equations. If the system does *not* have exactly one solution, state whether it has *no* solution or *infinitely many* solutions. (Lesson 5-2)

8. $y = 6x$
$4x + y = 10$

9. $c + d = 0$
$3c + d = -8$

10. $x - 2y = 5$
$3x - 5y = 8$

11. $x + y = 2$
$y = 2 - x$

12. **MULTIPLE CHOICE** Sydney has \$115 and she is earning \$50 per week at her summer job. Felipe has \$130 and is earning \$50 per week at his summer job. Which is a true statement about the system of equations that represents this situation? (Lesson 5-2)

F The system has 1 solution, which represents their hourly rates.

G The system has 1 solution, which represents the number of weeks in which they will have earned the same amount of money.

H The system has infinitely many solutions.

J The system has no solution.

Use elimination to solve each system of equations. (Lesson 5-3)

13. $a + b = 9$
$a - b = 7$

14. $3x + y = 1$
$-6x + y = 10$

15. $5x + 4y = 2$
$3x - 4y = 14$

16. $2s - 3t = 13$
$2s + 2t = -2$

WATER PARKS **For Exercises 17 and 18, use the following information.**

The cost of two groups going to a water park is shown in the table. (Lesson 5-3)

Group	Total Cost (\$)
2 adults, 1 child	68.97
2 adults, 4 children	125.94

17. Define variables and write a system of equations that you can use to find the admission cost of an adult and a child.

18. Solve the system of equations and explain what the solution means.

5-4 Elimination Using Multiplication

Main Ideas

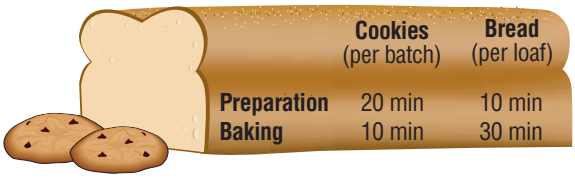
- Solve systems of equations algebraically by using elimination with multiplication.
- Solve real-world problems involving systems of equations.

GET READY for the Lesson

The Finneytown Bakery is making peanut butter cookies and loaves of quick bread. The preparation and baking times for each are shown.

For these two items, the management has allotted 800 minutes of employee time and 900 minutes of oven time. The following system of equations can be used to determine how many of each to bake.

$20c + 10b = 800$
$10c + 30b = 900$



	Cookies (per batch)	Bread (per loaf)
Preparation	20 min	10 min
Baking	10 min	30 min

Elimination Using Multiplication Neither variable in the system above can be eliminated by simply adding or subtracting the equations. However, you can use the Multiplication Property of Equality so that adding or subtracting eliminates one of the variables.

EXAMPLE **Multiply One Equation to Eliminate**

1 **Use elimination to solve the system of equations.** $3x + 4y = 6$, $5x + 2y = -4$

Multiply the second equation by -2 so the coefficients of the y-terms are additive inverses. Then add the equations.

$3x + 4y = 6$
$5x + 2y = -4$ Multiply by -2.

$$\begin{array}{rl} 3x + 4y = 6 & \\ (+)\ -10x - 4y = 8 & \\ \hline -7x = 14 & \text{Add the equations.} \\ x = -2 & \text{Divide each side by } -7. \end{array}$$

Now substitute -2 for x in either equation to find the value of y.

$$\begin{array}{rl} 3x + 4y = 6 & \text{First equation} \\ 3(-2) + 4y = 6 & x = -2 \\ -6 + 4y = 6 & \text{Simplify.} \\ 4y = 12 & \text{Add 6 to each side and simplify.} \\ \frac{4y}{4} = \frac{12}{4} & \text{Divide each side by 4 and simplify.} \\ y = 3 & \text{The solution is } (-2, 3). \end{array}$$

CHECK Your Progress

Use elimination to solve each system of equations.

1A. $6x - 2y = 10$, $3x - 7y = -19$

1B. $9p + q = 13$, $3p + 2q = -4$

Sometimes you have to multiply each equation by a different number in order to solve the system.

EXAMPLE 2 Multiply Both Equations to Eliminate

Use elimination to solve the system of equations. $3x + 4y = -25$, $2x - 3y = 6$

Method 1 Eliminate x.

$3x + 4y = -25$
$2x - 3y = 6$

Multiply by 2.
Multiply by -3.

$$\begin{array}{rl} 6x + 8y = -50 & \\ (+)\ -6x + 9y = -18 & \\ \hline 17y = -68 & \text{Add the equations.} \\ \frac{17y}{17} = \frac{-68}{17} & \text{Divide each side by 17.} \\ y = -4 & \text{Simplify.} \end{array}$$

Now substitute -4 for y in either equation to find the value of x.

$$\begin{array}{rl} 2x - 3y = 6 & \text{Second equation} \\ 2x - 3(-4) = 6 & y = -4 \\ 2x + 12 = 6 & \text{Simplify.} \\ 2x + 12 - 12 = 6 - 12 & \text{Subtract 12 from each side.} \\ 2x = -6 & \text{Simplify.} \\ \frac{2x}{2} = \frac{-6}{2} & \text{Divide each side by 2 and simplify.} \\ x = -3 & \text{The solution is } (-3, -4). \end{array}$$

Method 2 Eliminate y.

$3x + 4y = -25$
$2x - 3y = 6$

Multiply by 3.
Multiply by 4.

$$\begin{array}{rl} 9x + 12y = -75 & \\ (+)\ 8x - 12y = 24 & \\ \hline 17x = -51 & \text{Add the equations.} \\ \frac{17x}{17} = \frac{-51}{17} & \text{Divide each side by 17.} \\ x = -3 & \text{Simplify.} \end{array}$$

Now substitute -3 for x in either equation to find the value of y.

$$\begin{array}{rl} 2x - 3y = 6 & \text{Second equation} \\ 2(-3) - 3y = 6 & x = -3 \\ -6 - 3y = 6 & \text{Simplify.} \\ -6 - 3y + 6 = 6 + 6 & \text{Add 6 to each side.} \\ -3y = 12 & \text{Simplify.} \\ \frac{-3y}{-3} = \frac{12}{-3} & \text{Divide each side by } -3. \\ y = -4 & \text{Simplify.} \end{array}$$

The solution is $(-3, -4)$, which matches the result obtained with Method 1.

Study Tip

Using Multiplication

There are many other combinations of multipliers that could be used to solve the system in Example 2. For instance, the first equation could be multiplied by -2 and the second by 3.

CHECK Your Progress

Use elimination to solve each system of equations.

2A. $5x - 3y = 6$, $2x - 5y = 10$

2B. $6a + 2b = 2$, $4a + 3b = 8$

STANDARDIZED TEST EXAMPLE Writing Systems of Equations

3 **Anita has a total of 28 e-mail addresses of family and friends stored on her PDA. Twice the number of family addresses minus the number of friends' addresses is 2. Which system of equations can be used to find how many e-mail addresses of family a and friends b that Anita has stored?**

A $a + b = 28$, $2a - b = 2$ **B** $a + b = 28$, $2a + b = 2$ **C** $a - b = 28$, $2a - b = 2$ **D** $a - b = 28$, $a - 2b = 2$

Read the Test Item

You are asked to find a system of equations to represent this situation using a, the number of family addresses, and b, the number of friends' addresses.

Solve the Test Item

Represent the situation algebraically by writing two equations.

The total of family's and friends' e-mail addresses is 28.

$a + b = 28$

One equation is $a + b = 28$.

Twice the number of family addresses minus the number of friends' addresses is 2.

$2a - b = 2$

The second equation is $2a - b = 2$.

The system of equations that represents this situation is $a + b = 28$ and $2a - b = 2$. The answer is A.

Test-Taking Tip

Checking the Complete Answer
Since choices A and B both have $a + b = 28$ as one of the equations, be careful to choose the answer with the correct second equation.

CHECK Your Progress

3. The cost of 4 notebooks and 5 pens is \$20. The cost of 6 notebooks and 2 pens is \$19. Which system of equations can be used to find the cost of a notebook n and the cost of a pen p?

F $4n + 5p = 20$, $2n + 6p = 19$ **G** $5n + 4p = 20$, $6n + 2p = 19$ **H** $4n + 5p = 20$, $6n + 2p = 19$ **J** $5n + 4p = 20$, $2n + 6p = 19$

Online **Personal Tutor at** algebra1.com

Real-World Link
About 203 million tons of freight are transported on the Ohio River each year, making it the second most used commercial river in the United States.

Source: *World Book Encyclopedia*

Real-World Problems When formulating systems of linear equations to solve real-world problems, be sure to determine the reasonableness of the solutions. Recall that only positive values will make sense as solutions for most real-world problems.

EXAMPLE Write and Solve a System of Equations

4 **TRANSPORTATION** **A coal barge on the Ohio River travels 24 miles upstream in 3 hours. The return trip takes the barge only 2 hours. Find the rate of the barge in still water.**

ESTIMATE The average speed upstream is $\frac{24}{3}$, or 8 mph. The average speed downstream is $\frac{24}{2}$ or 12 mph. So, the rate in still water should be between 8 and 12 miles per hour.

WORDS You know the distance traveled each way and the times spent traveling.

VARIABLES Let b = the rate of the barge in still water and c = the rate of the current. Use the formula rate × time = distance, or $rt = d$.

EQUATIONS

	r	t	d	$rt = d$	
Downstream	$b + c$	2	24	$(b + c)2 = 24$	$\rightarrow 2b + 2c = 24$
Upstream	$b - c$	3	24	$(b - c)3 = 24$	$\rightarrow 3b - 3c = 24$

Use elimination with multiplication to solve this system. Since the problem asks for b, eliminate c.

$2b + 2c = 24$ Multiply by 3.
$3b - 3c = 24$ Multiply by 2.

$$\begin{array}{rr} 6b + 6c = & 72 \\ (+)\ 6b - 6c = & 48 \\ \hline 12b = & 120 \end{array}$$ Add the equations.

$\frac{12b}{12} = \frac{120}{12}$ Divide each side by 12.

$b = 10$ Simplify.

The rate of the barge in still water is 10 miles per hour. This solution is close to the estimate since it is between 8 and 12 miles per hour. So, the solution is reasonable.

CHECK Your Progress

4. CANOEING A canoe travels 4 miles upstream in 1 hour. The return trip takes the canoe 1.5 hours. Find the rate of the boat in still water.

CHECK Your Understanding

Use elimination to solve each system of equations.

Examples 1 (p. 272)

1. $2x - y = 6$
$3x + 4y = -2$

2. $2x + 7y = 1$
$x + 5y = 2$

Example 2 (p. 273)

3. $4x + 7y = 6$
$6x + 5y = 20$

4. $9a - 2b = -8$
$-7a + 3b = 12$

Example 3 (p. 274)

5. MULTIPLE CHOICE At a restaurant, the cost for 2 burritos and 1 tortilla salad is \$20.57. The cost for 3 burritos and 3 tortilla salads is \$36.24. Which pair of equations can be used to determine b, the cost of a burrito, and t, the cost of a tortilla salad?

A $b + t = 20.57$
$3b + 3t = 36.24$

B $2b + t = 20.57$
$3b + 3t = 36.24$

C $2b + t = 20.57$
$b + t = 36.24$

D $b + 2t = 20.57$
$b + t = 36.24$

Example 4 (pp. 274–275)

6. BUSINESS The owners of the River View Restaurant have hired enough servers to handle 17 tables of customers, and the fire marshal has approved the restaurant for a limit of 56 customers. How many two-seat tables and how many four-seat tables should the owners purchase?

Exercises

HOMEWORK	
For Exercises	See Examples
7–14	1, 2
15, 16	4
17, 18	3

Use elimination to solve each system of equations.

7. $x + y = 3$
$2x - 3y = 16$

8. $-5x + 3y = 6$
$x - y = 4$

9. $2x + y = 5$
$3x - 2y = 4$

10. $4x - 3y = 12$
$x + 2y = 14$

11. $8x - 3y = -11$
$2x - 5y = 27$

12. $5x - 2y = -15$
$3x + 3y = 12$

13. $4x - 7y = 10$
$3x + 2y = -7$

14. $2x - 3y = 2$
$5x + 4y = 28$

15. $12x - 3y = -3$
$6x + y = 1$

16. $-4x + 2y = 0$
$10x + 3y = 8$

17. NUMBER THEORY Seven times a number plus three times another number equals negative one. The sum of the two numbers is negative three. What are the numbers?

18. BASKETBALL In basketball, a free throw is 1 point and a field goal is either 2 or 3 points. In a recent season, Tim Duncan of the San Antonio Spurs scored a total of 1342 points. The total number of 2-point field goals and 3-point field goals was 517, and he made 305 of the 455 free throws that he attempted. Find the number of 2-point field goals and 3-point field goals Duncan made that season.

Use elimination to solve each system of equations.

19. $1.8x - 0.3y = 14.4$
$x - 0.6y = 2.8$

20. $0.4x + 0.5y = 2.5$
$1.2x - 3.5y = 2.5$

21. $3x - \frac{1}{2}y = 10$
$5x + \frac{1}{4}y = 8$

22. $2x + \frac{2}{3}y = 4$
$x - \frac{1}{2}y = 7$

23. NUMBER THEORY The sum of the digits of a two-digit number is 14. If the digits are reversed, the new number is 18 less than the original number. Find the original number.

ANALYZE TABLES **For Exercises 24 and 25, use the information below.**
At an entertainment center, two groups of people bought batting tokens and miniature golf games, as shown in the table.

Group	Number of Batting Tokens	Number of Miniature Golf Games	Total Cost
A	16	3	$30
B	22	5	$43

24. Define variables and formulate a system of linear equations from this situation.

25. Solve the system of equations and explain what the solution represents in terms of the situation.

26. TRANSPORTATION Traveling against the wind, a plane flies 1080 miles from Omaha, Nebraska, to San Diego, California, in the time shown at right. On the return trip, the plane is traveling with a wind that is twice as fast. Find the rate of the plane in still air.

Trip	Time
traveling against the wind	2 h 30 min
traveling with the wind	2 h

27. GEOMETRY The graphs of $x + 2y = 6$ and $2x + y = 9$ contain two of the sides of a triangle. A vertex of the triangle is at the intersection of the graphs. What are the coordinates of the vertex?

EXTRA PRACTICE
See pages 728, 748.
MathOnline
Self-Check Quiz at algebra1.com

28. TESTS Mrs. Henderson discovered that she had accidentally reversed the digits of a test score and shorted a student 36 points. Mrs. Henderson told the student that the sum of the digits was 14 and agreed to give the student his correct score plus extra credit if he could determine his actual score without looking at his test. What was his actual score on the test?

H.O.T. Problems

29. REASONING Explain why multiplication is sometimes needed to solve a system of equations by elimination.

30. FIND THE ERROR David and Yoomee are solving a system of equations. Who is correct? Explain your reasoning.

David

$$\begin{aligned} 2r + 7s &= 11 \quad \Rightarrow \\ r - 9s &= -7 \quad \Rightarrow \\ \hline 2r + 7s &= 11 \\ (-)\ 2r - 18s &= -14 \\ \hline 25s &= 25 \\ s &= 1 \\ 2r + 7s &= 11 \\ 2r + 7(1) &= 11 \\ 2r + 7 &= 11 \\ 2r &= 4 \\ \frac{2r}{2} &= \frac{4}{2} \\ r &= 2 \end{aligned}$$

The solution is (2, 1).

Yoomee

$$\begin{aligned} 2r + 7s &= 11 \\ (-)\ r - 9s &= -7 \\ \hline r \qquad &= 18 \\ 2r + 7s &= 11 \\ 2(18) + 7s &= 11 \\ 36 + 7s &= 11 \\ 7s &= -25 \\ \frac{7s}{7} &= -\frac{25}{7} \\ s &= -3.6 \end{aligned}$$

The solution is (18, -3.6).

31. OPEN ENDED Formulate a system of equations that could be solved by multiplying one equation by 5 and then adding the two equations together to eliminate one of the variables.

32. CHALLENGE The solution of the system $4x + 5y = 2$ and $6x - 2y = b$ is $(3, a)$. Find the values of a and b. Discuss the steps that you used.

33. *Writing in Math* Use the information about the bakery on page 272 to explain how a manager can use a system of equations to plan employee time. Include a demonstration of how to solve the system of equations given in the problem and an explanation of how a restaurant manager would schedule oven and employee time.

STANDARDIZED TEST PRACTICE

34. If $5x + 3y = 12$ and $4x - 5y = 17$, what is the value of y?

A -1 **C** $(-1, 3)$

B 3 **D** $(3, -1)$

35. What is the solution to the system of equations?

$x + 2y = -1$

$2x + 4y = -2$

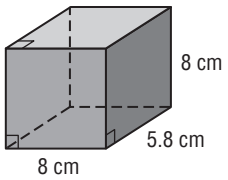
F $(-1, -1)$

G $(2, 1)$

H no solution

J infinitely many solutions

36. REVIEW What is the surface area of the rectangular solid shown below?



A 249.6 cm^2

B 278.4 cm^2

C 313.6 cm^2

D 371.2 cm^2

Spiral Review

Use elimination to solve each system of equations. (Lesson 5-3)

37. $x + y = 8$
$x - y = 4$

38. $2r + s = 5$
$r - s = 1$

39. $x + y = 18$
$x + 2y = 25$

Use substitution to solve each system of equations. If the system does *not* have exactly one solution, state whether it has *no* solution or *infinitely many* solutions. (Lesson 5-2)

40. $2x + 3y = 3$
$x = -3y$

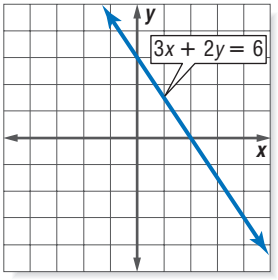
41. $x + y = 0$
$3x + y = -8$

42. $x - 2y = 7$
$-3x + 6y = -21$

43. PAINTING A ladder reaches a height of 16 feet on a wall. If the bottom of the ladder is placed 4 feet away from the wall, what is the slope of the ladder as a positive number? Explain the meaning of the slope. (Lesson 4-1)

Determine the x-intercept, y-intercept, and zero of each linear function. (Lesson 3-3)

44.



45.

x	y
−8	0
−4	1
0	2
4	3

GET READY for the Next Lesson

PREREQUISITE SKILL **Solve each equation.** (Lesson 2-4)

46. $3(x + 5) - x = 1$

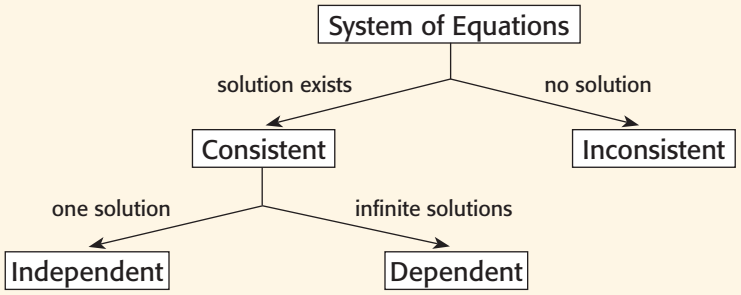
47. $14 = 5(n - 1) + 9$

48. $y - 2(y + 8) = 6$

Making Concept Maps

After completing a chapter, it is wise to review each lesson's main topics and vocabulary. In Lesson 5-1, the new vocabulary words were *system of equations*, *consistent*, *inconsistent*, *independent*, and *dependent*. They are all related in that they explain how many and what kind of solutions a system of equations has.

A graphic organizer called a *concept map* is a convenient way to show these relationships. A concept map is shown below for the vocabulary words for Lesson 5-1. The main ideas are placed in boxes. Any information that describes how to move from one box to the next is placed along the arrows.

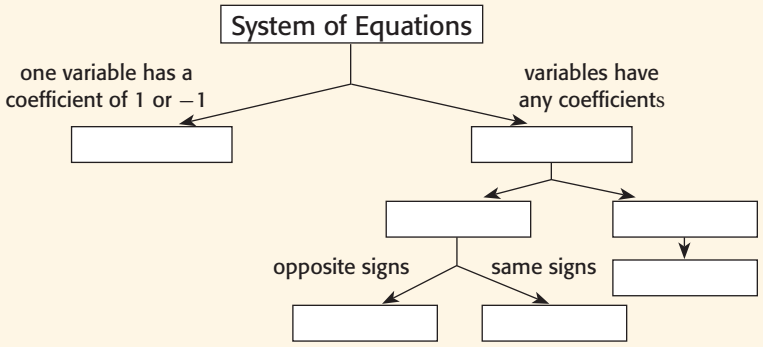


Concept maps are used to organize information. They clearly show how ideas are related to one another. They also show the flow of mental processes needed to solve problems.

Reading to Learn

Review Lessons 5-2, 5-3, and 5-4.

1. Write a couple of sentences describing the information in the concept map above.
2. How do you decide whether to use substitution or elimination? Give an example of a system that you would solve using each method.
3. How do you decide whether to multiply an equation by a factor?
4. How do you decide whether to add or subtract two equations?
5. Copy and complete the concept map at the right for solving systems of equations by using either substitution or elimination.



5-5 Applying Systems of Linear Equations

Main Ideas

- Determine the best method for solving systems of equations.
- Apply systems of linear equations.

GET READY for the Lesson

Northern California is home to several caves and caverns. Mercer Caverns is in Calaveras County near Murphys. Walter J. Mercer discovered the cavern in 1885. He descended to the base of the first chamber. Tours are currently offered at the caverns that descend approximately 4 times the depth that Mr. Mercer descended initially.

Tour	Depth (ft)
Mr. Mercer's tour	x
Current tour	y

The total depth of both tours is approximately 160 feet. How could you use a system of equations to determine the depth of each tour?

Determine the Best Method You have learned five methods for solving systems of linear equations. The table summarizes the methods and the systems for which each method works best.

KEY CONCEPT *Solving Systems of Equations*

Method	The Best Time to Use
Graphing	to estimate the solution, since graphing usually does not give an exact solution
Substitution	if one of the variables in either equation has a coefficient of 1 or -1
Elimination Using Addition	if one of the variables has opposite coefficients in the two equations
Elimination Using Subtraction	if one of the variables has the same coefficient in the two equations
Elimination Using Multiplication	if none of the coefficents are 1 or -1 and neither of the variables can be eliminated by simply adding or subtracting the equations

For an exact solution, an algebraic method is best. It is also usually a quicker method for solving linear equations than graphing. Graphing, with or without technology, is a good way to estimate solutions.

Real-World EXAMPLE Determine the Best Method

1 **SHOPPING** **At a sale, Sarah bought 4 T-shirts and 3 pairs of jeans for $181. Jenna bought 1 T-shirt and 2 pairs of jeans for $94. The T-shirts were all the same price and the jeans were all the same price, so the following system of equations represents this situation. Determine the best method to solve the system of equations. Then solve the system.**

$4x + 3y = 181$

$x + 2y = 94$

- Since neither the coefficients of x nor the coefficients of y are the same or additive inverses, you cannot add or subtract to eliminate.
- Since the coefficient of x in the second equation is 1, you can use the substitution method. You could also use elimination using multiplication.

Study Tip

Alternative Method

This system could also be solved easily by multiplying the second equation by 4 and then subtracting the equations.

The following solution uses substitution. Which method would you prefer?

$x + 2y = 94$	Second equation
$x + 2y - 2y = 94 - 2y$	Subtract $2y$ from each side.
$x = 94 - 2y$	Simplify.
$4x + 3y = 181$	First equation
$4(94 - 2y) + 3y = 181$	$x = 94 - 2y$
$376 - 8y + 3y = 181$	Distributive Property
$376 - 5y = 181$	Combine like terms.
$376 - 5y - 376 = 181 - 376$	Subtract 376 from each side.
$-5y = -195$	Simplify.
$y = 39$	Divide each side by -5 and simplify.
$x + 2y = 94$	Second equation
$x + 2(39) = 94$	$y = 39$
$x + 78 = 94$	Simplify.
$x + 78 - 78 = 94 - 78$	Subtract 78 from each side.
$x = 16$	Simplify.

The solution is (16, 39). This means that the cost of a T-shirt was $16 and the cost of a pair of jeans was $39.

CHECK Your Progress

Determine the best method to solve each system of equations. Then solve the system.

1A. $5x + 7y = 2$
$-2x + 7y = 9$

1B. $3x - 4y = -10$
$5x + 8y = -2$

Apply Systems of Linear Equations When applying systems of linear equations to problem situations, it is important to analyze each solution in the context of the situation.

Real-World Link

Digital photography can trace its roots back almost 40 years, when NASA needed a technology for spacecraft to send images back to Earth.

Source: technology.com

Real-World EXAMPLE

2 **PHOTOGRAPHY** **Since 2000, the number of film cameras sold has decreased at an average rate of 2.5 million per year. At the same time, the number of digital cameras sold has increased at an average rate of 2.6 million per year. Use the table to estimate the year in which the sales of digital cameras equaled the sales of film cameras.**

Cameras Sold in 2000	
Type of Camera	Number Sold (millions)
film	20.0
digital	4.7

Source: Mediamark Research, Inc.

Explore You know the number of each type of camera sold in 2000 and the rates of change in numbers sold.

Plan Write an equation to represent the number of cameras sold for each type of camera. Then solve.

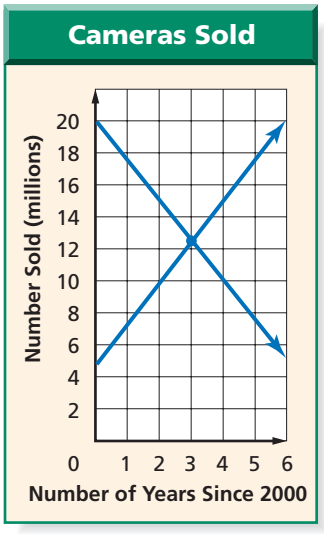
Solve Let x = the number of years after 2000 and let y = the total number of cameras sold.

	number sold		number sold in 2000		rate of change times number of years after 2000
film cameras	y	$=$	20.0	$+$	$-2.5x$
digital cameras	y	$=$	4.7	$+$	$2.6x$

You can use elimination by subtraction to solve this system.

$$\begin{array}{rl} y = 20.0 + -2.5x & \text{Write the equations in column form and subtract.} \\ (-)\ y = \ 4.7 + \ \ 2.6x & \\ \hline 0 = 15.3 - \ \ 5.1x & \text{The variable } y \text{ is eliminated.} \\ 5.1x = 15.3 & \text{Add } 5.1x \text{ to each side and simplify.} \\ x = 3 & \text{Divide each side by 5.1 and simplify.} \end{array}$$

This means that 3 years after 2000, or in 2003, the sales of digital cameras equaled the sales of film cameras.



Check Does this solution make sense in the context of the problem? After 1 year, the number of film cameras would be about 20 − 3 or 17. The number of digital cameras would be about 4.7 + 3 or 7.7. Continue estimating.

Check by sketching a graph of the equations. The graphs appear to intersect at (3, 12.5), which verifies the solution of $x = 3$.

In 2003, approximately 12.5 million film and digital cameras were sold.

Study Tip

Elimination

Equations do not have to be written in standard form to use elimination by addition or subtraction.

CHECK Your Progress

2. VOLUNTEERING Jared has volunteered 50 hours and plans to continue volunteering 3 hours each week. Clementine just started volunteering 5 hours each week. Find the number of weeks in which Jared and Clementine will have both volunteered the same number of hours.

Online Personal Tutor at algebra1.com

CHECK Your Understanding

Example 1 (p. 281)

Determine the best method to solve each system of equations. Then solve the system.

1. $4x + 3y = 19$
$3x - 4y = 8$

2. $3x - 7y = 6$
$2x + 7y = 4$

3. $y = 4x + 11$
$3x - 2y = -7$

4. $5x - 2y = 12$
$3x - 2y = -2$

Example 2 (p. 282)

5. FUND-RAISING For a Future Teachers of America fund-raiser, Denzell sold subs and pizzas as shown in the table. He sold 11 more subs than pizzas and earned a total of $233. Write and solve a system of equations to represent this situation. Then describe what the solution means.

Item	Selling Price
pizza	$5.00
sub	$3.00

Exercises

HOMEWORK HELP	
For Exercises	See Examples
6–11	1
12–13	2

Determine the best method to solve each system of equations. Then solve the system.

6. $9x - 8y = 42$
$4x + 8y = -16$

7. $y = 3x$
$3x + 4y = 30$

8. $x = 4y + 8$
$2x - 8y = -3$

9. $x - y = 2$
$5x + 3y = 18$

10. $y = 2x + 9$
$2x - y = -9$

11. $6x - y = 9$
$6x - y = 11$

12. ENTERTAINMENT Miranda has a total of 40 DVDs of movies and television shows. The number of movies is 4 less than 3 times the number of television shows. Write and solve a system of equations to find how many movies and television shows that she has on DVD.

13. YEARBOOKS The *break-even point* is the point at which income equals expenses. McGuffey High School is paying $13,200 for the writing and research of their yearbook, plus a printing fee of $25 per book. If they sell the books for $40 each, how many will they have to sell to break even? Explain.

Determine the best method to solve each system of equations. Then solve the system.

14. $2.3x - 1.9y = -2.5$
$x - 0.4y = 3.6$

15. $1.6x - 0.7y = -11$
$3.2x + 2.1y = -15$

16. $\frac{1}{2}x - \frac{1}{4}y = 6$
$\frac{5}{8}x - \frac{1}{4}y = 8$

For Exercises 17 and 18, use the table and the information at the right.

Material	Pounds Recycled	
	Mara	Ling
aluminum cans	9	9
newspaper	26	114

Mara and Ling each recycled aluminum cans and newspaper, as shown in the table. Mara earned $3.77 and Ling earned $4.65.

17. Define variables and formulate a system of linear equations from this situation.

18. What was the price per pound of aluminum? Determine the reasonableness of your solution.

EXTRA PRACTICE
See pages 728, 748.
Math Online
Self-Check Quiz at algebra1.com

H.O.T. Problems

19. OPEN ENDED Formulate a system of equations that represents a situation in your school. Describe the method that you would use to solve the system. Then solve the system and explain what the solution means.

20. Which One Doesn't Belong? Identify the system of equations that is not the same as the other three. Explain your reasoning.

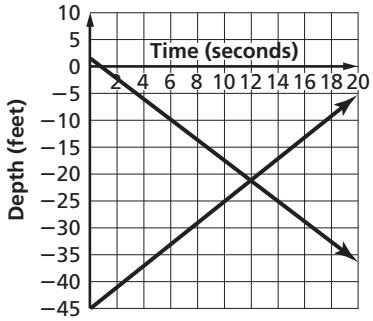
$x - y = 3$ $x + \frac{1}{2}y = 1$	$-x + y = 0$ $5x = 2y$	$y = x - 4$ $y = \frac{2}{x}$	$y = x + 1$ $y = 3x$

21. *Writing in Math* Suppose that in a system of equations, x represents the time spent riding a bike, y represents the distance traveled, and you determine the solution to be $(1, -7)$. Use this problem to discuss the importance of analyzing solutions in the context of real-world problems.

STANDARDIZED TEST PRACTICE

22. Marcus descends at a rate of 2 feet per second from the surface of the ocean. Toshiko is 45 feet below sea level and she is rising to the surface at a rate of 3 feet per second. Which graph represents when the two divers will be at the same depth?

A



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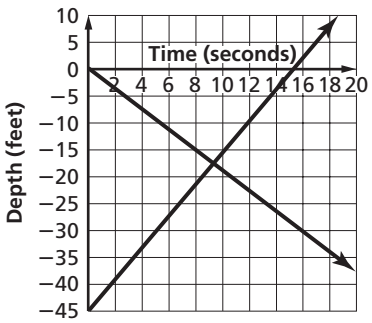

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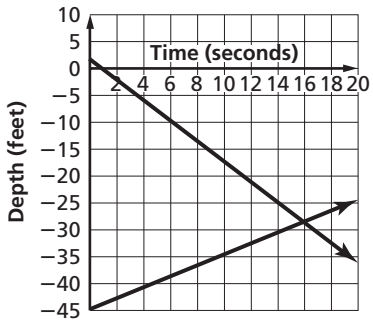

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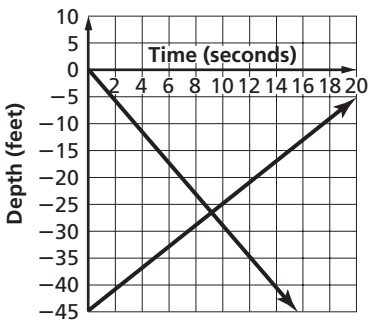


Spiral Review

Use elimination to solve each system of equations. (Lesson 5-4)

23. $x + y = -3$
$3x + 2y = -6$

24. $4x - 5y = 22$
$3x - 10y = 4$

25. $2x - 7y = -3$
$5x + 2y = -27$

26. BUSINESS In 2003, the United States produced about 2 million more motor vehicles than Japan. Together, the two countries produced about 22 million motor vehicles. How many vehicles were produced in each country? (Lesson 5-3)

CHAPTER 5

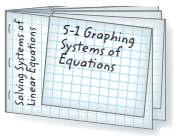
Study Guide and Review

Download Vocabulary Review from algebra1.com

FOLDABLES Study Organizer

GET READY to Study

Be sure the following Key Concepts are noted in your Foldable.



Key Concepts

Graphing Systems of Equations (Lesson 5-1)

- A system of intersecting lines has exactly one solution and is consistent and independent.
- A system whose graphs coincide has infinitely many solutions and is consistent and dependent.
- A system of parallel lines has no solution and is inconsistent.
- Graphing is best used to estimate the solution of a system of equations, since graphing usually does not give an exact solution.

Solving Systems of Equations Using Algebra (Lessons 5-2, 5-3, 5-4, and 5-5)

- Substitution is best used if one of the variables in either equation has a coefficient of 1 or −1.
- Elimination using addition is best used if one of the variables has opposite coefficients in the two equations.
- Elimination using subtraction is best used if one of the variables has the same coefficient in the two equations.
- Elimination using multiplication is best used if none of the coefficients are 1 or −1 and neither of the variables can be eliminated by simply adding or subtracting the equations.

Key Vocabulary

consistent (p. 254)
dependent (p. 254)
elimination (p. 266)
inconsistent (p. 254)
independent (p. 253)
substitution (p. 260)
system of equations (p. 253)

Vocabulary Check

State whether each sentence is *true* or *false*. If *false*, replace the underlined word or phrase to make a true sentence.

1. Two or more equations together are called a system of equations.
2. The best method for solving the system $3x - y = 9$ and $6x + y = 12$ is to use elimination using subtraction.
3. The system $2x + y = 5$ and $4x + 2y = 10$ is dependent.
4. If the graphs of the equations in a system have the same slope and different y-intercepts, the graph of the system is a pair of intersecting lines.
5. If a system has infinitely many solutions, it is inconsistent and independent.
6. The best method for solving the system $x = 4y$ and $2x + 3y = 6$ is to use substitution.
7. The system $y = 3x - 1$ and $y = 3x + 4$ is consistent.
8. Adding or subtracting two equations to solve a system of equations is known as substitution.
9. A system of equations whose solution is $(3, -5)$ is said to be independent.
10. If the graphs of the equations in a system have the same slope and y-intercept(s), the system has exactly one solution.

CHAPTER 5

Study Guide and Review

Lesson-by-Lesson Review

5–1 Graphing Systems of Equations (pp. 253–258)

Graph each system of equations. Then determine whether the system has *no* solution, *one* solution, or *infinitely many* solutions. If the system has one solution, name it.

11. $x - y = 3$
$x + y = 5$

12. $9x + 2 = 3y$
$y - 3x = 8$

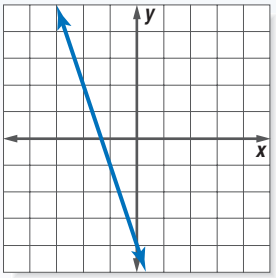
13. $2x - 3y = 4$
$6y = 4x - 8$

14. $3x - y = 8$
$3x = 4 - y$

15. RACE In a race, Pablo is 3 miles behind Marc. Pablo increases his speed to 5 miles per hour, while Marc continues to run at 4 miles per hour. At this rate, how many miles will Pablo have to run to catch up to Marc? How long will this take?

Example 1 **Graph the system of equations. Then determine whether the system has *no* solution, *one* solution, or *infinitely many* solutions. If the system has one solution, name it.**

$3x + y = -4$
$6x + 2y = -8$



When the lines are graphed, they coincide. There are infinitely many solutions.

5–2 Substitution (pp. 260–265)

Use substitution to solve each system of equations. If the system does *not* have exactly one solution, state whether it has *no* solution or *infinitely many* solutions.

16. $2m + n = 1$
$m - n = 8$

17. $x = 3 - 2y$
$2x + 4y = 6$

18. $3x - y = 1$
$-12x + 4y = 3$

19. $6m - 2n = 24$
$n = 12 - 3m$

20. PHONES The table shows the long-distance plans of Companies A and B. For how many minutes is the cost the same for the two long-distance companies?

Company	Flat Fee	Rate
A	$0	$0.06/minute
B	$5.80	$0.04/minute

Example 2 **Use substitution to solve the system of equations.**

$y = x - 1$
$4x - y = 19$

Since $y = x - 1$, substitute $x - 1$ for y in the second equation.

$4x - y = 19$ Second equation
$4x - (x - 1) = 19$ $y = x - 1$
$4x - x + 1 = 19$ Distributive Property
$3x + 1 = 19$ Combine like terms.
$3x = 18$ Subtract 1 from each side.
$x = 6$ Divide each side by 3.

You can find the value of y by replacing x with 6 in the first equation.

$y = x - 1$ First equation
$= 6 - 1$ $x = 6$
$= 5$ The solution is (6, 5).

Mixed Problem Solving
For mixed problem-solving practice, see page 748.

5–3 Elimination Using Addition and Subtraction (pp. 266–270)

Use elimination to solve each system of equations.

21. $x + 2y = 6$
$x - 3y = -4$

22. $2m - n = 5$
$2m + n = 3$

23. $3x - y = 11$
$x + y = 5$

24. $3x + 1 = -7y$
$6x + 7y = -16$

25. AIRPORTS The Detroit Wayne County Airport and the Denver International Airport appeared in the top 20 rankings of busiest airports by number of passengers. The sum of their rankings was 29, and the difference was 9. If Denver was busier than Detroit, what were their rankings?

Example 3 **Use elimination to solve the system of equations.**

$2m - n = 4$
$m + n = 2$

Eliminate the n-terms by adding the equations.

$2m - n = 4$ Write the equations in column form and add.
$(+)\ m + n = 2$
$3m = 6$ Notice the variable n is eliminated.
$m = 2$ Divide each side by 3.

Substitute 2 for m in either equation to find n.

$m + n = 2$ Second equation
$2 + n = 2$ Replace m with 2.
$n = 0$ Subtract 2 from each side.

The solution is (2, 0).

5–4 Elimination Using Multiplication (pp. 272–278)

Use elimination to solve each system of equations.

26. $x - 5y = 0$
$2x - 3y = 7$

27. $x - 2y = 5$
$3x - 5y = 8$

28. $2x + 3y = 8$
$x - y = 2$

29. $-5x + 8y = 21$
$10x + 3y = 15$

30. ENTERTAINMENT The cost for tickets to see a play are shown in the table. A group of 11 adults and students bought tickets for the play. If the total cost was \$156, how many of each type of ticket did they buy?

Ticket	Cost
adult	\$15
student	\$12

Example 4 **Use elimination to solve the system of equations.**

$x + 2y = 7$
$3x + y = 1$

Multiply the second equation by -2 so the coefficients of the y-terms are additive inverses. Then add the equations.

$x + 2y = 7$
$3x + y = 1$ Multiply by -2.

$x + 2y = 7$
$(+)\ -6x - 2y = -2$
$-5x = 5$
$\frac{-5x}{-5} = \frac{5}{-5}$
$x = -1$

$x + 2y = 7$ First equation
$-1 + 2y = 7$ $x = -1$
$2y = 8$ Add 1 to each side.
$\frac{2y}{2} = \frac{8}{2}$ Divide each side by 2.
$y = 4$ Simplify.

The solution is $(-1, 4)$.

5–5 Applying Systems of Linear Equations (pp. 280–284)

Determine the best method to solve each system of equations. Then solve the system.

31. $y = 2x$
$x + 2y = 8$

32. $9x + 8y = 5$
$18x + 15y = 6$

33. $3x + 5y = 2x$
$x + 3y = y$

34. $2x + y = 3x - 15$
$x + 5 = 4y + 2x$

35. SAVINGS Raul invests $1500 into two savings accounts, one earning 4% annual interest and the other earning 6% annual interest. At the end of one year, Raul has earned $72 in interest. How much did he invest at each rate?

Example 5 **Determine the best method to solve the system. Then solve the system.**

$4x - 3y = 7$
$3x + 1 = y$

Since the coefficient of y in the second equation is 1, you can use the substitution method.

$4x - 3y = 7$	First equation
$4x - 3(3x + 1) = 7$	$y = 3x + 1$
$4x - 9x - 3 = 7$	Distributive Property
$-5x - 3 = 7$	Combine like terms.
$-5x - 3 + 3 = 7 + 3$	Add 3 to each side.
$-5x = 10$	Simplify.
$\frac{-5x}{-5} = \frac{10}{-5}$	Divide each side by -5.
$x = -2$	Simplify.
$3x + 1 = y$	Second equation
$3(-2) + 1 = y$	$x = -2$
$-5 = y$	Simplify.

The solution is (–2, –5).

CHAPTER 5

Practice Test

Graph each system of equations. Then determine whether the system has *no* solution, *one* solution, or *infinitely many* solutions. If the system has one solution, name it.

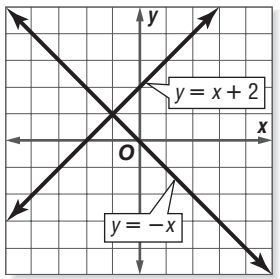
1. $y = x + 2$
$y = 2x + 7$

2. $x + 2y = 11$
$x = 14 - 2y$

3. $3x + y = 5$
$2y - 10 = -6x$

4. MULTIPLE CHOICE Which graph represents a system of equations with no solution?

A



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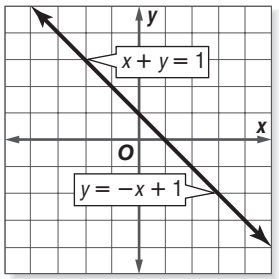

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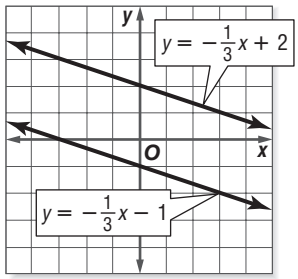
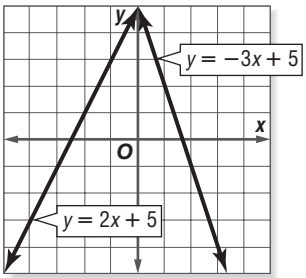


D



Use substitution or elimination to solve each system of equations.

5. $2x + 5y = 16$
$5x - 2y = 11$

6. $y + 2x = -1$
$y - 4 = -2x$

7. $2x + y = -4$
$5x + 3y = -6$

8. $y = 7 - x$
$x - y = -3$

9. $x = 2y - 7$
$y - 3x = -9$

10. $x + y = -10$
$x - y = -2$

11. $3x + y = 10$
$3x - 2y = 16$

12. $5x - 3y = 12$
$-2x + 3y = -3$

13. MULTIPLE CHOICE The units digit of a two-digit number exceeds twice the tens digit by 1. The sum of its digits is 10. Find the number.

F 7
G 19
H 37
J 39

14. GEOMETRY The difference between the length and width of a rectangle is 7 centimeters. Find the dimensions of the rectangle if its perimeter is 50 centimeters.

15. FINANCE Last year, Evelina invested $10,000, part at 6% annual interest and the rest at 8% annual interest. If she received $760 in interest at the end of the year, how much did she invest at each rate?

GEOMETRY For Exercises 16 and 17, use the graphs of $y = 2x + 6$, $3x + 2y = 19$, and $y = 2$, which contain the sides of a triangle.

16. Find the coordinates of the vertices of the triangle.

17. Find the area of the triangle.

18. MULTIPLE CHOICE At a movie theater, the costs for various amounts of popcorn and hot pretzels are shown below.

Boxes of Popcorn	Hot Pretzels	Total Cost
1	1	$6.25
2	4	$18.00

Which pair of equations can be used to find p, the cost of a box of popcorn, and z, the cost of a hot pretzel?

A $p + z = 6.25$
$2p + 2z = 18$

B $p + z = 6.25$
$4p + 4z = 18$

C $p + z = 6.25$
$2p + 4z = 18$

D $p + z = 6.25$
$4p + 2z = 18$

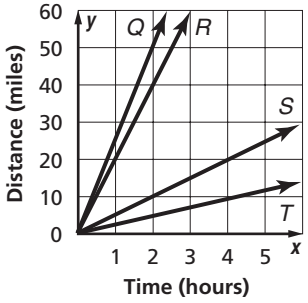
CHAPTER 5

Standardized Test Practice

Cumulative, Chapters 1–5

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. In the distance formula $d = rt$, r represents the rate of change, or slope. Which ray on the graph best represents a slope of 25 mph?



A Q
B R
C S
D T

2. Marla has a part-time job at a grocery store. The table shows the number of hours that Marla works in one week and the pay she receives.

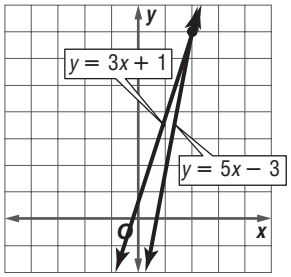
Hours Worked	Weekly Pay (Dollars)
5	60
15	140
20	180

Which equation best describes the relationship between Marla's total pay, p and the number of hours she works, h?

F $p = 9h$
G $p = 8h + 20$
H $p = 10h - 20$
J $p = 15 + 8.25h$

3. GRIDDABLE Penny's Pizza Place estimates that 40% of their sales go toward labor costs. If the pizza place makes $2056.58 on Monday, approximately how much, in dollars, went to labor costs?

4. The graphs of the linear equations $y = 3x + 1$ and $y = 5x - 3$ are shown below.



If $3x + 1 = 5x - 3$, what is the value of x?

A -3
B 1
C 2
D 7

TEST-TAKING TIP

Question 4 When solving an equation, check to make sure that your answer works. For example, in Question 4, once you get a solution for x, substitute it back into the equation to make sure you get a true statement.

5. Katelyn is training for a marathon. This week she ran 6 less than 2 times the number of miles that she ran last week. In these two weeks she ran a total of 42 miles. Which system of equations can be used to find x, the number of miles she ran this week and y, the number of miles she ran last week?

F $x + y = 42$
$x = 2y - 6$
G $x + y = 42$
$y = 2x - 6$
H $x + y = 42$
$x = 2y + 6$
J $x + y = 42$
$y = 2x + 6$

Standardized Test Practice at algebra1.com

6. Crazy Rides Amusement Park sells adult tickets for $25 and children's tickets for $15. On opening day the park sold 15 more children's tickets than adult tickets and made $4625. Which system of equations could be used to find the number of adult tickets a, and children's tickets c, that the park sold that day?

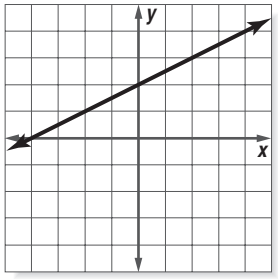
A $25a + 15c = 4625$
$a = c + 15$

B $25a + 15c = 15$
$c = a + 4625$

C $25a + 15c = 4625$
$15 = a + c$

D $25a + 15c = 4625$
$c = a + 15$

7. What are the x- and y-intercepts of the function graphed below?



F (2, 0) and (−4, 0)

G (2, 0) and (0, −4)

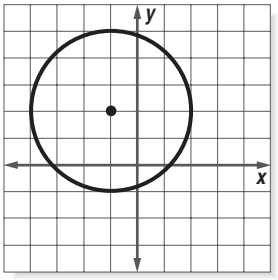
H (0, 2) and (−4, 0)

J (0, 2) and (0, −4)

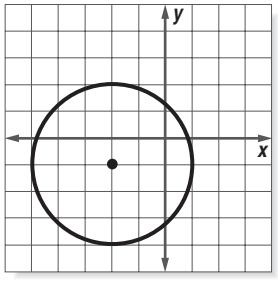
8. GRIDDABLE A rectangular prism has a volume of 400 cubic units. If the prism is dilated by a scale factor of 2, what is the volume of the resulting prism in cubic units?

9. Which circle has a center located at coordinates (−2, −1)?

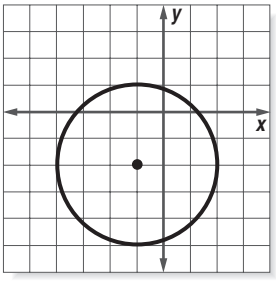
A



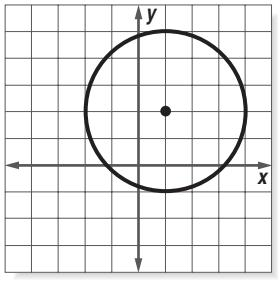
C



B



D



Pre-AP

Record your answers on a sheet of paper. Show your work.

10. The manager of a movie theater found that Saturday's ticket sales were $3675. He knew that a total of 650 tickets were sold. Adult tickets cost $7.50, and children's tickets cost $4.50.

a. Write equations to represent the number of tickets sold and the amount of money collected. Let a represent the number of adult tickets, and let c represent the number of children's tickets.

b. How many of each kind of ticket were sold? Show your work. Include all of the steps.

Need extra help?										
If You Missed Question...	1	2	3	4	5	6	7	8	9	10
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