

# Solving Linear Inequalities

## BIG Ideas

- Solve linear inequalities and graph in the coordinate plane.
- Solve absolute value equations and inequalities.
- Solve systems of linear inequalities by graphing.

## Key Vocabulary

compound inequality (p. 315)

intersection (p. 315)

set-builder notation (p. 295)

union (p. 316)

## Real-World Link

**Roller Coasters** Inequalities are used to represent various real-world situations in which a quantity must fall within a range of possible values. For example, a roller coaster must gain enough speed on the first hill to propel it through the entire ride.

## FOLDABLES™ Study Organizer

**Solving Linear Inequalities** Make this Foldable to record information about solving linear inequalities. Begin with two sheets of notebook paper.

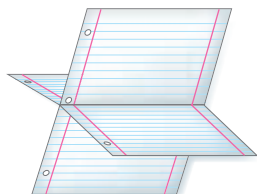
- 1** **Fold** one sheet in half along the width. Cut along the fold from the edges to margin.



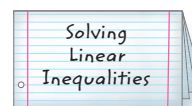
- 2** **Fold** the second sheet in half along the width. Cut along the fold between the margins.



- 3** **Insert** the first sheet through the second sheet and align the folds.



- 4** **Label** each page with a lesson number and title.



# GET READY for Chapter 6

**Diagnose Readiness** You have two options for checking Prerequisite Skills.

## Option 2



Take the Online Readiness Quiz at [algebra1.com](http://algebra1.com).

## Option 1

Take the Quick Check below. Refer to the Quick Review for help.

### QUICK Check

Solve each equation. (Lessons 2-4 and 2-5)

- $18 = 27 + f$
- $d - \frac{2}{3} = \frac{1}{2}$
- $5m + 7 = 4m - 12$
- $3y + 4 = 16$
- $\frac{1}{2}k - 4 = 7$
- $4.3b + 1.8 = 8.25$
- $6s - 12 = 2(s + 2)$
- $n - 3 = \frac{n + 1}{2}$

9. **NUMBER THEORY** Three times the lesser of two consecutive integers is 4 more than 2 times the greater number. Find the integers.

Find each value. (Prerequisite Skill)

- $|20|$
- $|-1.5|$
- $|14 - 7|$
- $|1 - 16|$
- $|2 - 3|$
- $|7 - 10|$

Graph each equation. (Lesson 3-3)

- $2x + 2y = 6$
- $x - 3y = -3$
- $y = 2x - 3$
- $y = -4$
- $x = -\frac{1}{2}y$
- $3x - 6 = 2y$
- $15 = 3(x + y)$
- $2 - x = 2y$

24. **CRAFTS** Rosa and Taylor are making scarves for the upcoming craft fair. Rosa can make  $x$  scarves per hour and Taylor can make  $y$  scarves per hour. Rosa can only work 8 hours and Taylor can only work 10. In total, they need to complete 25 scarves. Write a linear equation that relates Taylor's rate to Rosa's.

### QUICK Review

#### EXAMPLE 1

Solve  $\frac{1}{2} = \frac{1}{8}t + 1$ .

$$\frac{1}{2} = \frac{1}{8}t + 1$$

Original equation

$$-\frac{1}{2} = \frac{1}{8}t$$

Subtract 1 from each side.

$$8\left(-\frac{1}{2}\right) = 8\left(\frac{1}{8}t\right)$$

Multiply each side by 8.

$$-4 = t$$

Simplify.

#### EXAMPLE 2

Find the value of  $|15 - 20|$ .

$$\begin{aligned} |15 - 20| &= |-5| \\ &= 5 \end{aligned}$$

Evaluate inside the absolute value.  
 $-5$  is five units from zero in the negative direction.

#### EXAMPLE 3

Graph  $y - x = 1$ .

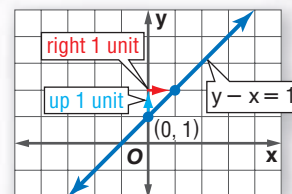
**Step 1** Put the equation in slope intercept form.

$$y - x = 1 \rightarrow y = x + 1$$

**Step 2** The  $y$ -intercept is 1. So, graph the point  $(0, 1)$ .

**Step 3** The slope is 1 or  $\frac{1}{1}$ .  $\frac{\text{rise}}{\text{run}}$   
From  $(0, 1)$ , move up 1 unit and right 1 unit. Draw a dot.

**Step 4** Draw a line connecting the points.



# Solving Inequalities by Addition and Subtraction

## GET READY for the Lesson

### Main Ideas

- Solve linear inequalities by using addition.
- Solve linear inequalities by using subtraction.

### New Vocabulary

set-builder notation

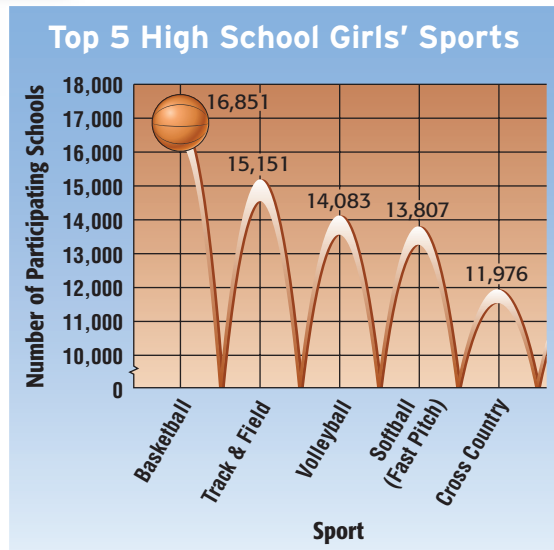
The data in the graph show that more high schools offer girls' track and field than girls' volleyball.

$$15,151 > 14,083$$

If 20 schools added girls' track and field and 20 schools added girls' volleyball, there would still be more schools offering girls' track and field than schools offering girls' volleyball.

$$15,151 + 20 \text{ ? } 14,083 + 20$$

$$15,171 > 14,103$$



Source: National Federation of State High School Associations

### Review Vocabulary

#### Inequality

An open sentence that contains the symbol  $<$ ,  $\leq$ ,  $>$ , or  $\geq$  is called an inequality.

Example:  $x \geq 4$   
(Lesson 1-3)

**Solve Inequalities by Addition** The sports application illustrates the **Addition Property of Inequalities**.

### KEY CONCEPT

#### Addition Property of Inequalities

**Words** If any number is added to each side of a true inequality, the resulting inequality is also true.

**Symbols** For all numbers  $a$ ,  $b$ , and  $c$ , the following are true.

1. If  $a > b$ , then  $a + c > b + c$ .
2. If  $a < b$ , then  $a + c < b + c$ .

This property is also true when  $>$  and  $<$  are replaced with  $\geq$  and  $\leq$ .

### EXAMPLE Solve by Adding

**1** Solve  $t - 45 \leq 13$ . Check your solution.

$$t - 45 \leq 13 \quad \text{Original inequality}$$

$$t - 45 + 45 \leq 13 + 45 \quad \text{Add 45 to each side.}$$

$$t \leq 58 \quad \text{Simplify.}$$

The solution is the set {all numbers less than or equal to 58}.

**CHECK** To check, substitute 58, a number less than 58, and a number greater than 58.

### CHECK Your Progress

Solve each inequality.

**1A.**  $22 > m - 8$

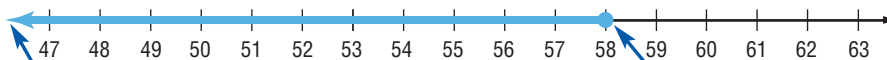
**1B.**  $d - 14 \geq -19$

## Reading Math

**Set-Builder Notation**  
 $\{t \mid t \leq 58\}$  is read *the set of all numbers  $t$  such that  $t$  is less than or equal to 58.*

The solution in Example 1 was expressed as a set. A more concise way of writing a solution set is to use **set-builder notation**. The solution in set-builder notation is  $\{t \mid t \leq 58\}$ .

The solution can also be graphed on a number line.



The heavy arrow pointing to the left shows that the inequality includes all numbers less than 58.

The dot at 58 shows that 58 is included in the inequality.

**Solve Inequalities by Subtraction** Subtraction can also be used to solve inequalities.

### KEY CONCEPT

### Subtraction Property of Inequalities

**Words** If any number is subtracted from each side of a true inequality, the resulting inequality is also true.

**Symbols** For all numbers  $a$ ,  $b$ , and  $c$ , the following are true.

1. If  $a > b$ , then  $a - c > b - c$ .
2. If  $a < b$ , then  $a - c < b - c$ .

This property is also true when  $>$  and  $<$  are replaced with  $\geq$  and  $\leq$ .



### Real-World EXAMPLE

### Solve by Subtracting

- 2 MUSIC** Josh added 19 more songs to his MP3 player, making the total number of songs more than 56. How many songs were originally on the player? Solve  $s + 19 > 56$ . Then graph the solution.

$$s + 19 > 56 \quad \text{Original inequality}$$

$$s + 19 - 19 > 56 - 19 \quad \text{Subtract 19 from each side.}$$

$$s > 37 \quad \text{Simplify.}$$

The solution set is  $\{s \mid s > 37\}$ . So, Josh had more than 37 songs originally on the music player.



### CHECK Your Progress

- 2. TEMPERATURE** The temperature  $t$  in a swimming pool increased  $4^\circ\text{F}$  since this morning. The temperature is now less than  $81^\circ\text{F}$ . What was the temperature this morning? Solve  $t + 4 < 81$ . Then graph the solution.

Terms with variables can also be subtracted from each side to solve inequalities.



## EXAMPLE Variables on Each Side

3 Solve  $5p + 7 > 6p$ . Then graph the solution.

$$5p + 7 > 6p \quad \text{Original inequality}$$

$$5p + 7 - 5p > 6p - 5p \quad \text{Subtract } 5p \text{ from each side.}$$

$$7 > p \quad \text{Simplify.}$$

Since  $7 > p$  is the same as  $p < 7$ , the solution set is  $\{p \mid p < 7\}$ .



### ✓ CHECK Your Progress

Solve each inequality. Graph the solution on a number line.

3A.  $9n - 1 < 10n$

3B.  $5h \leq 12 + 4h$

Here are some phrases that indicate inequalities in verbal problems.

Inequalities			
<	>	≤	≥
less than	greater than	at most	at least
fewer than	more than	no more than	no less than
		less than or equal to	greater than or equal to

## Real-World EXAMPLE Write an Inequality to Solve a Problem

4 **OLYMPICS** Irina Tchachina scored a total of 107.325 points in the four events of rhythmic gymnastics. Alina Kabaera scored a total of 81.300 in the clubs, hoop, and ball events. How many points did she need to score in the ribbon event to get ahead of Tchachina and win the gold medal?

<b>Words</b>	Kabaera's total	is greater than	Tchachina's total.
<b>Variable</b>	Let $r$ = Kabaera's score in the ribbon event.		
<b>Inequality</b>	$81.300 + r$	$>$	107.325

Solve the inequality.

$$81.300 + r > 107.325$$

**ESTIMATE:**  $110 - 80 = 30$

Original inequality

$$81.300 + r - 81.300 > 107.325 - 81.300 \quad \text{Subtract } 81.300 \text{ from each side.}$$

$$r > 26.025 \quad \text{Simplify.}$$

Kabaera needed to score more than 26.025 points to win the gold medal. The solution is close to the estimate, so the answer is reasonable.

### ✓ CHECK Your Progress

4. **SHOPPING** Terrell has \$65 to spend. He bought a T-shirt for \$18 and a belt for \$14. If Terrell still wants jeans, how much can he spend on the jeans?

Personal Tutor at [algebra1.com](http://algebra1.com)



### Real-World Link

Alina Kabaera of the Russian Federation won the gold medal in rhythmic gymnastics at the 2004 Summer Olympics in Athens, and her teammate Irina Tchachina won the silver medal.

Source:  
[www.athens2004.com](http://www.athens2004.com)

## CHECK Your Understanding

**Examples 1–3** Solve each inequality. Check your solution, and then graph it on a number line.  
(pp. 294–296)

- |                   |                     |
|-------------------|---------------------|
| 1. $t - 5 \geq 7$ | 2. $-7 \geq -2 + x$ |
| 3. $a + 4 < 2$    | 4. $9 \leq b + 4$   |
| 5. $10 > n - 1$   | 6. $k + 24 > -5$    |
| 7. $8r + 6 < 9r$  | 8. $7p \leq 6p - 2$ |

**Example 4** Define a variable, write an inequality, and solve each problem. Check your solution.  
(p. 296)

9. A number decreased by 8 is at most 14.
10. Twice a number is greater than  $-5$  plus the number.
11. **BIOLOGY** Adult Nile crocodiles weigh up to 2200 pounds. If a young Nile crocodile weighs 157 pounds, how many pounds might it be expected to gain in its lifetime?

## Exercises

HOMEWORK For Exercises	HELP See Examples
12–29	1–3
30–35	4

Solve each inequality. Check your solution, and then graph it on a number line.

- |                      |                       |                      |
|----------------------|-----------------------|----------------------|
| 12. $t + 14 \geq 18$ | 13. $d + 5 \leq 7$    | 14. $n - 7 < -3$     |
| 15. $-5 + s > -1$    | 16. $5 < 3 + g$       | 17. $13 > 18 + r$    |
| 18. $2 \leq -1 + m$  | 19. $-23 \geq q - 30$ | 20. $11 + m \geq 15$ |
| 21. $h - 26 < 4$     | 22. $8 \leq r - 14$   | 23. $-7 > 20 + c$    |
| 24. $2y > -8 + y$    | 25. $3f < -3 + 2f$    | 26. $3b \leq 2b - 5$ |
| 27. $4w \geq 3w + 1$ | 28. $6x + 5 \geq 7x$  | 29. $-9 + 2a < 3a$   |

Define a variable, write an inequality, and solve each problem. Check your solution.

30. The sum of a number and 13 is at least 27.
31. A number decreased by 5 is less than 33.
32. Twice a number is more than the sum of that number and 14.
33. The sum of two numbers is at most 18, and one of the numbers is  $-7$ .

For Exercises 34–36, define a variable, write an inequality, and solve each problem. Then interpret your solution.

34. **BIOLOGY** There are 3500 species of bees and more than 600,000 species of insects. How many species of insects are not bees?
35. **TECHNOLOGY** A recent survey found that more than 21 million people between the ages of 12 and 17 use the Internet. Of those online teens, about 16 million said they use the Internet at school. How many teens who are online do not use the Internet at school?

36. **ANALYZE TABLES** Chapa is limiting her fat intake to no more than 60 grams per day. Today, she has had two breakfast bars and a slice of pizza. How many more grams of fat can she have today?

Food	Grams of Fat
breakfast bar	3
slice of pizza	21



### Real-World Link

One common species of bees is the honeybee. A honeybee colony may have 60,000 to 80,000 bees.

**Source:** Penn State, Cooperative Extension Service

Solve each inequality. Check your solution, and then graph it on a number line.

37.  $y - (-2.5) > 8.1$

38.  $5.2r + 6.7 \geq 6.2r$

39.  $a + \frac{1}{4} > \frac{1}{8}$

40.  $\frac{3}{2}p - \frac{2}{3} \leq \frac{4}{9} + \frac{1}{2}p$

Define a variable, write an inequality, and solve each problem. Check your solution.

41. Thirty is no greater than the sum of a number and  $-8$ .

42. Four times a number is less than or equal to the sum of 3 times the number and  $-2$ .

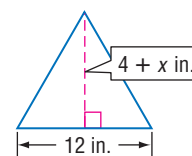
For Exercises 43 and 44, define a variable, write an inequality, and solve each problem. Then interpret your solution.

43. **MONEY** City Bank requires a minimum balance of \$1500 to maintain free checking services. If Mr. Hayashi is going to write checks for the amounts listed in the table, how much money should he have in his account before writing the checks in order to have free checking?

Check	Amount
750	\$1300
751	\$947

44. **SOCCER** The Centerville High School soccer team has a goal of winning at least 60% of their 18 games this season. In the first three weeks, the team has won 4 games. How many more games must the team win to meet their goal?

45. **GEOMETRY** The length of the base of the triangle is less than the height of the triangle. What are the possible values of  $x$ ? Formulate a linear inequality to solve the problem. Determine whether your answers are reasonable.



**EXTRA PRACTICE**  
See pages 728, 749.  
**Math** **online**  
Self-Check Quiz at [algebra1.com](http://algebra1.com)

46. If  $d + 5 \geq 17$ , then complete each inequality.

a.  $d \geq \underline{\quad ? \quad}$

b.  $d + \underline{\quad ? \quad} \geq 20$

c.  $d - 5 \geq \underline{\quad ? \quad}$

**H.O.T. Problems**

47. **REASONING** Compare and contrast the graphs of  $a < 4$  and  $a \leq 4$ .

48. **CHALLENGE** Determine whether each statement is *always*, *sometimes*, or *never* true. Explain.

a. If  $a < b$  and  $c < d$ , then  $a + c < b + d$ .

b. If  $a < b$  and  $c < d$ , then  $a + c \geq b + d$ .

c. If  $a < b$  and  $c < d$ , then  $a - c < b - d$ .

49. **OPEN ENDED** Formulate three linear inequalities that are equivalent to  $y < -3$ .

50. **Writing in Math** Use the information about sports on page 294 to explain how inequalities can be used to describe school sports. Include an inequality describing the number of schools needed to add girls' track and field so that the number is greater than the number of schools currently participating in girls' basketball.

51. Based on the graph below, which statement is true?



- A Maria started with 30 bottles of sports drinks.
- B On day 10, Maria will have drunk 15 bottles of sports drinks.
- C Maria will be out of sports drinks on day 14.
- D Maria drank 5 bottles in the first 2 days.

52. What is the solution set to the inequality  $7 + x < 5$ ?

- F  $x < 2$
- H  $x > 2$
- G  $x < -2$
- J  $x > -2$

53. **REVIEW** Miss Miller wants to calculate the cost of buying tile to cover her rectangular kitchen floor. She knows the cost per square foot of tile, and she knows the dimensions of the kitchen. Which formula should she use to find the cost of the tile that she needs?

- A  $A = \ell w$
- C  $P = 2\ell + 2w$
- B  $V = Bh$
- D  $c^2 = a^2 + b^2$

**Spiral Review**

Determine the best method to solve each system of equations. Then solve the system. (Lesson 5-5)

54.  $4x - 3y = -1$   
 $3x + 3y = 15$

55.  $x = 8y$   
 $2x + 3y = 38$

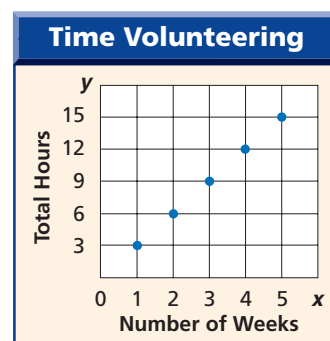
56.  $2a + 5b = -3$   
 $3a + 4b = -8$

57. **NUTRITION** The costs for various items at a mall food court are shown in the table. What is the cost of an iced tea and a vegetable wrap? (Lesson 5-4)

Number of Iced Teas	Number of Vegetable Wraps	Total Cost
2	2	\$10.50
3	4	\$19.50

**VOLUNTEERING** For Exercises 58 and 59, use the graph. It shows the total hours that Estella spent volunteering. (Lesson 3-5)

- 58. Write an equation in function notation for the relation.
- 59. What would be the total hours that Estella spent volunteering after 12 weeks?



**GET READY for the Next Lesson**

**PREREQUISITE SKILL** Solve each equation. (Lesson 2-3)

60.  $6g = 42$

61.  $3m = 435$

62.  $\frac{t}{9} = 14$

63.  $\frac{2}{3}y = -14$



# Algebra Lab

## Solving Inequalities

You can use algebra tiles to solve inequalities.

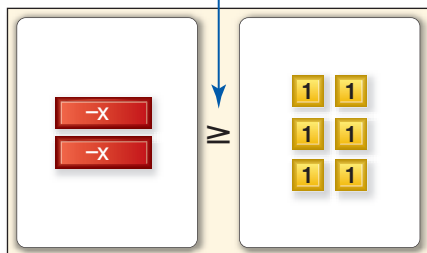
Concepts  
in Motion

Animation  
algebra1.com

**ACTIVITY** Solve  $-2x \geq 6$ .

**Step 1** Model the inequality.

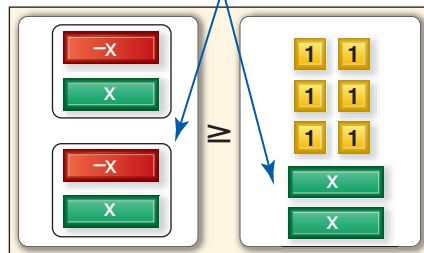
Use a self-adhesive note to cover the equals sign on the equation mat. Then write a  $\geq$  symbol on the note. Model the inequality.



$$-2x \geq 6$$

**Step 2** Remove the zero pairs.

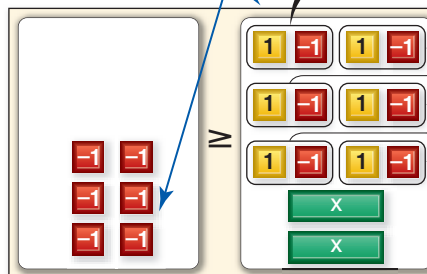
Since you do not want to solve for a negative  $x$ -tile, eliminate the negative  $x$ -tiles by adding 2 positive  $x$ -tiles to each side. Remove the zero pairs.



$$-2x + 2x \geq 6 + 2x$$

**Step 3** Remove the zero pairs.

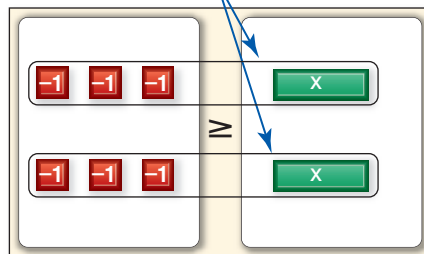
Add 6 negative 1-tiles to each side to isolate the  $x$ -tiles. Remove the zero pairs.



$$-6 \geq 2x$$

**Step 4** Group the tiles.

Separate the tiles into 2 groups.



$$-3 \geq x \text{ or } x \leq -3$$

### MODEL AND ANALYZE THE RESULTS

Use algebra tiles to solve each inequality.

1.  $-4x < 12$       2.  $-2x > 8$       3.  $-3x \geq -6$       4.  $-5x \leq -5$

- In Exercises 1–4, is the coefficient of  $x$  in each inequality *positive* or *negative*?
- Compare the inequality symbols and locations of the variable in Exercises 1–4 with those in their solutions. What do you find?
- Model the solution for  $2x \geq 6$ . What do you find? How is this different than solving  $-2x \geq 6$ ?

# Solving Inequalities by Multiplication and Division

## GET READY for the Lesson

### Main Ideas

- Solve linear inequalities by using multiplication.
- Solve linear inequalities by using division.

Isabel Franco is stacking cases of drinks to sell at a basketball game. A case of bottled water is 8 inches high. A case of sports drinks is 10 inches high. Notice that 8 inches is less than 10 inches, or  $8 \text{ in.} < 10 \text{ in.}$  How would the height of stacks of three cases of each compare?



8 in.

$$8 \text{ in.} < 10 \text{ in.}$$



10 in.

The height of the water is less than the height of the sports drinks.

$$8 \text{ in.} \times 3 < 10 \text{ in.} \times 3 \quad \text{Multiply to find the height of 3 cases of each.}$$

$$24 \text{ in.} < 30 \text{ in.} \quad \text{The height of 3 cases of water is less than the height of 3 cases of sports drinks.}$$

### Study Tip

#### Look Back

To review solving linear equations by multiplication, see Lesson 2-3.

**Solve Inequalities by Multiplication** If each side of an inequality is multiplied by a positive number, the inequality remains true.

$8 > 5$	Original inequality	$5 < 9$	Original inequality
$8(2) > 5(2)$	Multiply each side by 2.	$5(4) < 9(4)$	Multiply each side by 4.
$16 > 10$	Simplify.	$20 < 36$	Simplify.

This is *not* true when multiplying by negative numbers.

$5 > 3$	Original inequality	$-6 < 8$	Original inequality
$5(-2) < 3(-2)$	Multiply each side by $-2$ .	$-6(-5) < 8(-5)$	Multiply each side by $-5$ .
$-10 < -6$	Simplify.	$30 > -40$	Simplify.

If each side of an inequality is multiplied by a negative number, the direction of the inequality symbol changes. These examples illustrate the **Multiplication Property of Inequalities**.

### KEY CONCEPT

#### Multiplying by a Positive Number

**Words** If each side of a true inequality is multiplied by the same positive number, the resulting inequality is also true.

**Symbols** If  $a$  and  $b$  are any numbers and  $c$  is a positive number, the following are true.  
If  $a > b$ , then  $ac > bc$ , and if  $a < b$ , then  $ac < bc$ .

This property also holds for inequalities involving  $\geq$  and  $\leq$ .



**Real-World EXAMPLE** Write and Solve an Inequality

**1 BIOLOGY** Mt. Kinabalu in Malaysia has the greatest concentration of wild orchids on Earth. It contains more than 750 species, which is approximately one fourth of all orchid species in Malaysia. How many orchid species are there in Malaysia?

<b>Words</b>	One fourth	times	number of orchid species in Malaysia	is more than	750.
<b>Variable</b>	Let $n$ = the number of orchid species found in Malaysia.				
<b>Inequality</b>	$\frac{1}{4}$	$\times$	$n$	$>$	750

$\frac{1}{4}n > 750$  Original inequality

$(4)\frac{1}{4}n > (4)750$  Multiply each side by 4 and do not change the inequality's direction.

$n > 3000$  Simplify.

The solution set is  $\{n | n > 3000\}$ . This means that there are more than 3000 orchid species in Malaysia.

**Real-World Link**

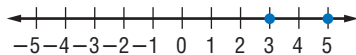
More than 30,000 different orchid species flower in the wild on every continent except Antarctica.

Source: [www.alohaorchid.com](http://www.alohaorchid.com)

**CHECK Your Progress**

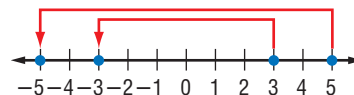
- 1A. SURVEYS** Of the students surveyed at Madison High School, fewer than eighty-four said they have never purchased an item online. This is about three eighths of those surveyed. How many students were surveyed?
- 1B. CANDY** Fewer than 42 employees at the factory stated that they preferred chocolate candy over fruit candy. This is about two thirds of the employees. How many employees are there?

Graph 3 and 5 on a number line.



Since 3 is to the left of 5,  $3 < 5$ .

Multiply each number by  $-1$ .



Since  $-3$  is to the right of  $-5$ ,  $-3 > -5$

Notice that the numbers being compared switched positions as a result of being multiplied by a negative number. In other words, their order reversed. This suggests the following property.

**KEY CONCEPT**

**Multiplying by a Negative Number**

**Words** If each side of a true inequality is multiplied by the same negative number, the direction of the inequality symbol must be *reversed* so that the resulting inequality is also true.

**Symbols** If  $a$  and  $b$  are any numbers and  $c$  is a negative number, the following are true.  
If  $a > b$ , then  $ac < bc$ , and if  $a < b$ , then  $ac > bc$ .

This property also holds for inequalities involving  $\geq$  and  $\leq$ .

**EXAMPLE** Multiply by a Negative Number

2 Solve  $-\frac{2}{5}p < -14$ .

$$-\frac{2}{5}p < -14$$
 Original inequality

$$\left(-\frac{5}{2}\right)\left(-\frac{2}{5}p\right) > \left(-\frac{5}{2}\right)(-14)$$
 Multiply each side by  $-\frac{5}{2}$  and change  $<$  to  $>$ .

$$p > 35$$
 Simplify.

The solution set is  $\{p | p > 35\}$ .**CHECK Your Progress** Solve each inequality.

2A.  $-\frac{n}{6} \leq 8$

2B.  $-\frac{4}{3}p > -10$

**Study Tip****Common Misconception**

You may be tempted to change the direction of the inequality when solving if there is a negative number anywhere in the inequality. Remember that you only change the direction of the inequality when multiplying or dividing by a negative.

**Solve Inequalities By Division** You can also solve an inequality by dividing each side by the same number. Consider the inequality  $6 < 15$ .

Divide each side by 3.

$$6 < 15$$

$$6 \div 3 \quad ? \quad 15 \div 3$$

$$2 < 5$$

The direction of the inequality symbol remains the same.

Divide each side by  $-3$ .

$$6 < 15$$

$$6 \div (-3) \quad ? \quad 15 \div (-3)$$

$$-2 > -5$$

The direction of the inequality symbol is reversed.

These examples illustrate the **Division Property of Inequalities**.

**KEY CONCEPT***Dividing by a Positive Number*

**Words** If each side of a true inequality is divided by the same positive number, the resulting inequality is also true.

**Symbols** If  $a$  and  $b$  are any numbers and  $c$  is a positive number, the following are true.

$$\text{If } a > b, \text{ then } \frac{a}{c} > \frac{b}{c}, \text{ and if } a < b, \text{ then } \frac{a}{c} < \frac{b}{c}.$$

**KEY CONCEPT***Dividing by a Negative Number*

**Words** If each side of a true inequality is divided by the same negative number, the direction of the inequality symbol must be *reversed* so that the resulting inequality is also true.

**Symbols** If  $a$  and  $b$  are any numbers and  $c$  is a negative number, the following are true.

$$\text{If } a > b, \text{ then } \frac{a}{c} < \frac{b}{c}, \text{ and if } a < b, \text{ then } \frac{a}{c} > \frac{b}{c}.$$

These properties also hold for inequalities involving  $\geq$  and  $\leq$ .



**EXAMPLE** Divide to Solve an Inequality**3** Solve each inequality.

a.  $14h > 91$

$14h > 91$  Original inequality

$\frac{14h}{14} > \frac{91}{14}$  Divide each side by 14 and do not change the direction of the inequality sign.

$h > 6.5$  Simplify.

The solution set is  $\{h|h > 6.5\}$ .

b.  $-5t \geq 275$

$-5t \geq 275$  Original inequality

$\frac{-5t}{-5} \leq \frac{275}{-5}$  Divide each side by  $-5$  and change  $\geq$  to  $\leq$ .

$t \leq -55$  Simplify.

The solution set is  $\{t|t \leq -55\}$ .**✓ CHECK Your Progress**

**3A.**  $9r < 27$

**3B.**  $-15 \geq 3t$

**3C.**  $32 < -8k$

**3D.**  $-5g \geq 40$

**STANDARDIZED TEST EXAMPLE** Write an Inequality**4** Homemade bracelets are for sale for \$4.75 each. Which inequality can be used to find how many bracelets Caitlin can buy for herself and her friends if she wants to spend no more than \$22?

**A**  $4.75b \geq 22$

**C**  $4.75 \leq 22b$

**B**  $4.75b \leq 22$

**D**  $4.75 \geq 22b$

**Read the Test Item**

You want to find the inequality that represents the number of bracelets that can be bought for \$22 or less.

**Solve the Test Item**If  $b$  represents the number of bracelets, then  $4.75b$  represents the total cost of  $b$  bracelets. So, you can eliminate Choices C and D.*No more than \$22* indicates less than or equal to 22, so the inequality that represents this situation is  $4.75b \leq 22$ . The answer is B.**✓ CHECK Your Progress****4.** Write an inequality for the sentence below.*Eighteen is greater than or equal to  $-9$  times a number.*

**F**  $-9n \geq 18$

**H**  $18 \geq -9n$

**G**  $-9 + n \geq 18$

**J**  $18 \geq n - 9$

 **Personal Tutor at [algebra1.com](http://algebra1.com)****Study Tip****Alternate Method**Since dividing is the same as multiplying by the reciprocal, you could also solve  $-5t \geq 275$  by multiplying each side by  $-\frac{1}{5}$ .**Test-Taking Tip****Clue Words**Always look for clue words like *no more than* in problems involving inequalities. They will indicate which symbol should be used.

## CHECK Your Understanding

**Examples 1, 2**  
(pp. 302–303)

Solve each inequality. Check your solution.

1.  $\frac{t}{9} < -12$       2.  $30 > \frac{1}{2}n$       3.  $-\frac{3}{4}r \leq -6$       4.  $-\frac{c}{8} \geq 7$

Define a variable, write an inequality, and solve each problem. Then check your solution.

5. The opposite of four times a number is more than 12.  
6. Half of a number is at least 26.

7. **FUND-RAISING** The Jefferson High School Band Boosters raised more than \$5500 for their Music Scholarship Fund. This money came from sales of their Marching Band Performances DVD, which sold for \$15. How many DVDs did they sell? Define a variable and write an inequality to solve the problem. Interpret your solution.

**Example 3**  
(pp. 303–304)

Solve each inequality. Check your solution.

8.  $7m \geq 42$       9.  $12x > -60$       10.  $75 < -15g$       11.  $-21 \leq -3s$

**Example 4**  
(p. 304)

12. **STANDARDIZED TEST PRACTICE** The area of the rectangle is less than 85 square feet. What is the width of the rectangle?



A  $w > 4\frac{1}{4}$  ft      B  $w \geq 4\frac{1}{4}$  ft      C  $w < 4\frac{1}{4}$  ft      D  $w \leq 4\frac{1}{4}$  ft

## Exercises

HOMEWORK HELP	
For Exercises	See Examples
13–20	1, 2
21–24	1
25–32	3
33, 34	4

Solve each inequality. Check your solution.

13.  $\frac{1}{4}m < -17$       14.  $\frac{b}{10} \leq 5$       15.  $-7 > -\frac{r}{7}$       16.  $-\frac{a}{11} > 9$   
17.  $\frac{5}{8}y \geq -15$       18.  $6 > \frac{2}{3}v$       19.  $-10 \geq \frac{x}{-2}$       20.  $-\frac{3}{4}q \leq -33$

Define a variable, write an inequality, and solve each problem. Then check your solution.

21. Seven times a number is greater than 28.  
22. Negative seven times a number is at least 14.  
23. Twenty-four is at most a third of a number.  
24. Two thirds of a number is less than  $-15$ .

Solve each inequality. Check your solution.

25.  $6g \leq 144$       26.  $84 < 7t$       27.  $-14d \geq 84$       28.  $14n \leq -98$   
29.  $32 > -2y$       30.  $-64 \geq -16z$       31.  $-26 < 26s$       32.  $-6x > -72$

For Exercises 33 and 34, define a variable, write an inequality, and solve each problem. Then interpret your solution.

33. **FUND-RAISING** The Middletown High School girls basketball team wants to make at least \$2000 on their annual mulch sale. The team makes \$2.50 on each bag of mulch sold. How many bags of mulch should the team sell?  
34. **EVENT PLANNING** Shaniqua is planning the prom. The hall does not charge a rental fee as long as at least \$4000 is spent on food. If she has chosen a buffet that costs \$28.95 per person, how many people must attend the prom to avoid a rental fee for the hall?

Solve each inequality. Check your solution.

35.  $-\frac{2}{3}b \leq -9$

36.  $25f \geq 9$

37.  $-2.5w < 6.8$

38.  $-0.8s > 6.4$

39.  $\frac{15c}{-7} > \frac{3}{14}$

40.  $\frac{4m}{5} < \frac{-3}{15}$

41. Solve  $-\frac{m}{9} \leq -\frac{1}{3}$ . Then graph the solution.

42. Solve  $\frac{x}{4} > \frac{3}{16}$ . Then graph the solution.

43. If  $2a \geq 7$ , then complete each inequality.

a.  $a \geq ?$

b.  $-4a \leq ?$

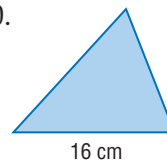
c.  $? a \leq -21$

Define a variable, write an inequality, and solve each problem. Check your solution.

44. Twenty-five percent of a number is greater than or equal to 90.

45. Forty percent of a number is less than or equal to 45.

46. **GEOMETRY** The area of the triangle is greater than 100 square centimeters. What is the height of the triangle? Estimate the height first, and then determine whether your solution is reasonable.



For Exercises 47–50, define a variable, write an inequality, and solve each problem. Then interpret your solution.

47. **LANDSCAPING** Morris is planning a circular flower garden with a low fence around the border. If he can use up to 38 feet of fence, what radius can he use for the garden? (*Hint:*  $C = 2\pi r$ )

48. **DRIVING** Average speed is calculated by dividing distance by time. If the speed limit on the interstate is 65 miles per hour, how far can a person travel legally in  $1\frac{1}{2}$  hours?

49. **ANALYZE TABLES** The annual membership to the San Diego Zoo for 2 adults and 2 children is \$128. The regular admission to the zoo is shown in the table. How many times should such a family plan to visit the zoo in a year to make a membership less expensive than paying regular admission?

Regular Admission	
Visitor	Price
adult	\$19.50
child	\$11.75

50. **CITY PLANNING** A city parking lot can have no more than 20% of the parking spaces limited to compact cars. If a certain parking lot has 35 spaces for compact cars, how many spaces must the lot have to conform to the code?

51. **REASONING** Explain why you can use either the Multiplication Property of Inequalities or the Division Property of Inequalities to solve  $-7r \leq 28$ .

52. **OPEN ENDED** Describe a real-life situation that can be represented by the inequality  $\frac{3}{4}a > 9$ .

53. **CHALLENGE** Give a counterexample to show that each statement is not always true.

a. If  $a > b$ , then  $a^2 > b^2$ .

b. If  $a < b$  and  $c < d$ , then  $ac < bd$ .



**Real-World Link**

Dr. Harry Wegeforth founded the San Diego Zoo in 1916 with just 50 animals. Today, the zoo has over 4000 animals.

Source:  
www.sandiegozoo.org

**EXTRA PRACTICE**  
See pages 729, 749.  
**Math online**  
Self-Check Quiz at [algebra1.com](http://algebra1.com)

**H.O.T. Problems**

54. **FIND THE ERROR** Ilonia and Zachary are solving  $-9b \leq 18$ . Who is correct? Explain your reasoning.

$$\begin{aligned} \text{Ilonia} \\ -9b &\leq 18 \\ \frac{-9b}{-9} &\geq \frac{18}{-9} \\ b &\geq -2 \end{aligned}$$

$$\begin{aligned} \text{Zachary} \\ -9b &\leq 18 \\ \frac{-9b}{-9} &\leq \frac{18}{-9} \\ b &\leq -2 \end{aligned}$$

55. **Writing in Math** Use the information about the cases of beverages on page 301 to explain how inequalities can be used in storage. Include an inequality representing a stack of cases of water or sports drinks that can be no higher than 3 feet and an explanation of how to solve the inequality.



### STANDARDIZED TEST PRACTICE

56. Juan's long-distance phone company charges 9¢ for each minute. Which inequality can be used to find how long can he talk to his friend if he does not want to spend more than \$2.50 on the call?

- A  $0.09 \geq 2.50m$   
 B  $0.09 \leq 2.50m$   
 C  $0.09m \geq 2.50$   
 D  $0.09m \leq 2.50$

57. **REVIEW** The table shows the results of a number cube being rolled. What is the experimental probability of rolling a 3?

Outcome	Frequency
1	4
2	8
3	2
4	0
5	5
6	1

- F  $\frac{2}{3}$     G  $\frac{1}{3}$     H 0.2    J 10%

### Spiral Review

Solve each inequality. Check your solution, and graph it on a number line. (Lesson 6-1)

58.  $s - 7 < 12$

59.  $g + 3 \leq -4$

60.  $7 > n + 2$

61. **GYMS** To join a gym, Cristina paid an initial fee of \$120, plus \$10 per month. Jackson pays \$20 per month without an initial fee. After how many months will they have paid the same amount? How much will it be? (Lesson 5-5)

Write an equation in standard form for a line that passes through each pair of points. (Lesson 4-4)

62.  $(-1, 3), (2, 4)$

63.  $(5, -2), (-1, -2)$

64.  $(3, 3), (-1, 2)$

### GET READY for the Next Lesson

**PREREQUISITE SKILL** Solve each equation. (Lessons 2-4 and 2-5)

65.  $5x - 3 = 32$

66.  $\frac{14g + 5}{6} = 9$

67.  $6y - 1 = 4y + 23$

68.  $2(p - 4) = 7(p + 3)$



# Solving Multi-Step Inequalities

## Main Ideas

- Solve linear inequalities involving more than one operation.
- Solve linear inequalities involving the Distributive Property.

## GET READY for the Lesson

The normal body temperature of a camel is  $97.7^{\circ}\text{F}$  in the morning. If it has had no water by noon, its body temperature can be greater than  $104^{\circ}\text{F}$ . If  $F$  represents temperature in degrees Fahrenheit, the inequality  $F > 104$  represents the temperature of a camel at noon.



If  $C$  represents degrees Celsius, then  $F = \frac{9}{5}C + 32$ . You can solve  $\frac{9}{5}C + 32 > 104$  to find the temperature in degrees Celsius of a camel at noon.

**Solve Multi-Step Inequalities** The inequality  $\frac{9}{5}C + 32 > 104$  involves more than one operation. It can be solved by undoing the operations in the same way you would solve a multi-step equation.

## Real-World EXAMPLE Multi-Step Inequality

- 1 SCIENCE** Find the body temperature in degrees Celsius of a camel that has had no water by noon.

$$\frac{9}{5}C + 32 > 104 \quad \text{Original inequality}$$

$$\frac{9}{5}C + 32 - 32 > 104 - 32 \quad \text{Subtract 32 from each side.}$$

$$\frac{9}{5}C > 72 \quad \text{Simplify.}$$

$$\left(\frac{5}{9}\right)\frac{9}{5}C > \left(\frac{5}{9}\right)(72) \quad \text{Multiply each side by } \frac{5}{9}.$$

$$C > 40 \quad \text{Simplify.}$$

The body temperature of a camel that has had no water by noon is greater than  $40^{\circ}\text{C}$ .

## CHECK Your Progress

- 1. MONEY** ABC Cellphones advertises a plan with 400 minutes per month for less than the competition. The price includes the \$3.50 local tax. If the competition charges \$43.50, what does ABC Cellphones charge for each minute?

## EXAMPLE Inequality Involving a Negative Coefficient

2 Solve  $-7b + 19 < -16$ .

$$-7b + 19 < -16 \quad \text{Original inequality}$$

$$-7b + 19 - 19 < -16 - 19 \quad \text{Subtract 19 from each side.}$$

$$-7b < -35 \quad \text{Simplify.}$$

$$\frac{-7b}{-7} > \frac{-35}{-7} \quad \text{Divide each side by } -7 \text{ and change } < \text{ to } >.$$

$$b > 5 \quad \text{The solution set is } \{b \mid b > 5\}.$$

**CHECK Your Progress** Solve each inequality.

2A.  $23 \geq 10 - 2w$

2B.  $43 > -4y + 11$

## EXAMPLE Write and Solve an Inequality

3 Define a variable, write an inequality, and solve the problem below. Then check your solution. *Three times a number minus eighteen is at least five times the number plus twenty-one.*

Three times a number	minus	eighteen	is at least	five times the number	plus	twenty- one.
$3n$	$-$	$18$	$\geq$	$5n$	$+$	$21$

$$-2n - 18 \geq 21 \quad \text{Subtract } 5n \text{ from each side.}$$

$$-2n \geq 39 \quad \text{Add 18 to each side.}$$

$$n \leq -19.5 \quad \text{Divide each side by } -2 \text{ and change } \geq \text{ to } \leq.$$

The solution set is  $\{n \mid n \leq -19.5\}$ . Check your solution.

**CHECK Your Progress**

3. Write an inequality for the sentence below. Then solve the inequality. *Two more than half of a number is greater than twenty-seven.*

**Online Personal Tutor at algebra1.com**

You can solve an inequality in one variable by using a graphing calculator.

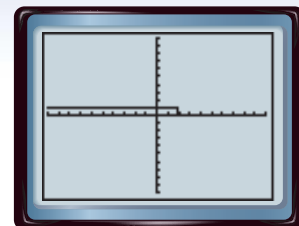
## GRAPHING CALCULATOR LAB

### Solving Inequalities

On a TI-83/84 Plus, clear the Y= list. Enter  $6x + 9 < -4x + 29$  as Y1. (The symbol  $<$  is item 5 on the TEST menu.) Press **GRAPH**.

#### THINK AND DISCUSS

- Describe what is shown on the screen.
- Use the TRACE function to scan the values along the graph. What do you notice about the values of  $y$  on the graph?
- Solve the inequality algebraically. How does your solution compare to the pattern you noticed in Exercise 2?



$[-10, 10]$  scl: 1 by  $[-10, 10]$  scl: 1

**Solve Inequalities Involving the Distributive Property** When solving equations that contain grouping symbols, first use the Distributive Property to remove the grouping symbols.

### EXAMPLE Distributive Property

4 Solve  $3d - 2(8d - 9) > -2d - 4$ .

$$3d - 2(8d - 9) > -2d - 4 \quad \text{Original inequality}$$

$$3d - 16d + 18 > -2d - 4 \quad \text{Distributive Property}$$

$$-13d + 18 > -2d - 4 \quad \text{Combine like terms.}$$

$$-13d + 18 + 13d > -2d - 4 + 13d \quad \text{Add } 13d \text{ to each side.}$$

$$18 > 11d - 4 \quad \text{Simplify.}$$

$$18 + 4 > 11d - 4 + 4 \quad \text{Add 4 to each side.}$$

$$22 > 11d \quad \text{Simplify.}$$

$$\frac{22}{11} > \frac{11d}{11} \quad \text{Divide each side by 11.}$$

$$2 > d \quad \text{Simplify.}$$

Since  $2 > d$  is the same as  $d < 2$ , the solution set is  $\{d \mid d < 2\}$ .

### CHECK Your Progress Solve each inequality.

4A.  $6(5z - 3) \leq 36z$

4B.  $2(h + 6) > -3(8 - h)$

Concepts  
in Motion  
Interactive Lab  
[algebra1.com](http://algebra1.com)

If solving an inequality results in a statement that is always true, the solution set is the set of all real numbers. This is written as  $\{x \mid x \text{ is a real number}\}$ . If solving an inequality results in a statement that is never true, the solution set is the empty set, written as the symbol  $\emptyset$ . The empty set has no members.

### EXAMPLE Empty Set and All Reals

5 Solve  $8(t + 2) - 3(t - 4) < 5(t - 7) + 8$ .

$$8(t + 2) - 3(t - 4) < 5(t - 7) + 8 \quad \text{Original inequality}$$

$$8t + 16 - 3t + 12 < 5t - 35 + 8 \quad \text{Distributive Property}$$

$$5t + 28 < 5t - 27 \quad \text{Combine like terms.}$$

$$5t + 28 - 5t < 5t - 27 - 5t \quad \text{Subtract } 5t \text{ from each side.}$$

$$28 < -27 \quad \text{Simplify.}$$

Since the inequality results in a false statement, the solution is the empty set  $\emptyset$ .

### CHECK Your Progress Solve each inequality.

5A.  $18 - 3(8c + 4) \geq -6(4c - 1)$  5B.  $46 \leq 8m - 4(2m + 5)$

**Examples 1, 2**  
(pp. 308–309)

Solve each inequality. Check your solution.

1.  $6h - 10 \geq 32$

2.  $-3 \leq \frac{2}{3}r + 9$

3.  $-4y - 23 < 19$

4.  $7b + 11 > 9b - 13$

5. **CANOEING** A certain canoe was advertised as having an “800 pound capacity,” meaning that it can hold at most 800 pounds. If four people plan to use the canoe and take 60 pounds of supplies, write and solve an inequality to find the average weight per person.

**Example 3**  
(p. 309)

6. Define a variable, write an inequality, and solve the problem below. Then check your solution.

*Seven minus two times a number is less than three times the number plus thirty-two.*

**Examples 4, 5**  
(p. 310)

Solve each inequality. Check your solution.

7.  $-6 \leq 3(5v - 2)$

8.  $-5(g + 4) > 3(g - 4)$

9.  $3 - 8x \geq 9 + 2(1 - 4x)$

**Exercises**

HOMEWORK HELP	
For Exercises	See Examples
10–19	1, 2
20–25	3
26–33	4, 5

Solve each inequality. Check your solution.

10.  $5b - 1 \geq -11$

11.  $21 > 15 + 2a$

12.  $-9 \geq \frac{2}{5}m + 7$

13.  $\frac{w}{8} - 13 > -6$

14.  $-3t + 6 \leq -3$

15.  $59 > -5 - 8f$

16.  $-2 - \frac{d}{5} < 23$

17.  $-\frac{3}{2}a + 4 > 10$

18.  $9r + 15 \leq 24 + 10r$

19.  $13k - 11 > 7k + 37$

Define a variable, write an inequality, and solve each problem. Then check your solution.

20. One eighth of a number decreased by five is at least thirty.

21. Two thirds of a number plus eight is greater than twelve.

22. Negative four times a number plus nine is no more than the number minus twenty-one.

23. Ten is no more than 4 times the sum of twice a number and three.

For Exercises 24 and 25, define a variable, write an inequality, and solve each problem. Then interpret your solution.

24. **SALES** A salesperson is paid \$22,000 a year plus 5% of the amount of sales made. What is the amount of sales needed to have an annual income greater than \$35,000?

25. **ANIMALS** Keith’s dog weighs 90 pounds. The veterinarian told him that a healthy weight for his dog would be less than 75 pounds. If Keith’s dog can lose an average of 1.25 pounds per week on a certain diet, how long will it take the dog to reach a healthy weight?

Solve each inequality. Check your solution.

26.  $5(2h - 6) > 4h$

27.  $21 \geq 3(a - 7) + 9$

28.  $2y + 4 > 2(3 + y)$

29.  $3(2 - b) < 10 - 3(b - 6)$

30.  $7 + t \leq 2(t + 3) + 2$

31.  $8a + 2(1 - 5a) \leq 20$

32. Solve  $4(t - 7) \leq 2(t + 9)$ . Show each step and justify your work.  
 33. Solve  $-5(k + 4) > 3(k - 4)$ . Show each step and justify your work.

**SCHOOL** For Exercises 34–36, use the following information.

Carmen's scores on three math tests are shown in the table. The fourth and final test of the grading period is tomorrow. She needs an average (mean) of at least 92 to receive an A for the grading period.

Test	Score
1	91
2	95
3	88

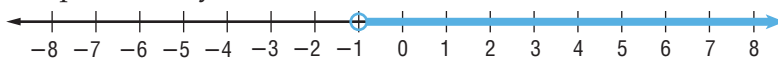
34. If  $s$  is her score on the fourth test, write an inequality to represent the situation.  
 35. If Carmen wants an A in math, what must she score on the test?  
 36. Is 150 a solution to the inequality that you wrote in Exercise 35? Is this a reasonable solution to the problem? Explain your reasoning.  
 37. **MONEY** Nicholas has \$13 to order a pizza. The pizza costs \$7.50 plus \$1.25 per topping. He plans to tip 15% of the total cost of the pizza. Write and solve an inequality to find how many toppings he can order.  
 38. **PHYSICAL SCIENCE** The melting point for an element is the temperature where the element changes from a solid to a liquid. If  $C$  represents degrees Celsius and  $F$  represents degrees Fahrenheit, then  $C = \frac{5(F - 32)}{9}$ . Refer to the information at the left to write and solve an inequality that can be used to find the temperatures in degrees Fahrenheit for which mercury is a solid.
39. Solve for  $x$  in each case.
- a.  $3x - 6 > 2 + 4(x - 2)$                       b.  $2(x - 4) \leq 2 + 3(x - 6)$   
 c.  $3x - 6 = 3(2 + 2(3 + x) + 4) - 1$         d.  $\frac{3}{3x - 6} = \frac{4}{2x + 4}$
40. **NUMBER THEORY** Find all sets of two consecutive positive odd integers with a sum no greater than 18.

Solve each inequality. Check your solution.

41.  $\frac{5b + 8}{3} < 3b$       42.  $3.1v - 1.4 \geq 1.3v + 6.7$       43.  $0.3(d - 2) - 0.8d > 4.4$   
 44. Define a variable, write an inequality, and solve the problem below. Then check your solution.  
*Three times the sum of a number and seven is greater than five times the number less thirteen.*

Use a graphing calculator to solve each inequality.

45.  $3x + 7 > 4x + 9$       46.  $13x - 11 \leq 7x + 37$       47.  $2(x - 3) < 3(2x + 2)$   
 48. **REASONING** Explain how you could solve  $-3p + 7 \geq 2$  without multiplying or dividing each side by a negative number.  
 49. **CHALLENGE** Create a multi-step inequality that has no solution and one that has infinitely many solutions. Investigate the best method for solving each one.  
 50. **OPEN ENDED** Create a multi-step inequality with the solution graphed below. Explain how you know.



**Real-World Link**

Mercury is a metal that is a liquid at room temperature. In fact, its melting point is  $-38^\circ\text{C}$ . Mercury is used in thermometers because it expands evenly as it is heated.

Source: World Book Encyclopedia

**EXTRA PRACTICE**  
 See pages 729, 749.  
**Math**  
 Self-Check Quiz at [algebra1.com](http://algebra1.com)

**Graphing Calculator**

**H.O.T. Problems**

51. **Which One Doesn't Belong?** Identify the inequality that does not belong with the other three. Explain.

$$4y + 9 > -3$$

$$3y - 4 > 5$$

$$2y + 1 < -5$$

$$-5y + 2 < -13$$

52. **Writing in Math** Use the information about camels on page 308 to explain how linear inequalities can be used in science.

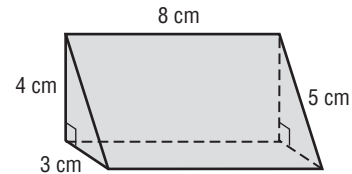


### STANDARDIZED TEST PRACTICE

53. What is the solution set of the inequality  $4t + 2 < 8t - (6t - 10)$ ?

- A  $t < -6.5$
- B  $t > -6.5$
- C  $t < 4$
- D  $t > 4$

54. **REVIEW** What is the volume of the triangular prism?



- F  $120 \text{ cm}^3$
- G  $96 \text{ cm}^3$
- H  $48 \text{ cm}^3$
- J  $30 \text{ cm}^3$

### Spiral Review

55. **BUSINESS** The charge per mile for a compact rental car is \$0.12. Mrs. Ludlow must rent a car for a business trip. She has a budget of \$50 for mileage charges. How many miles can she travel without going over her budget? (Lesson 6-2)

Solve each inequality. Check your solution, and then graph it on a number line. (Lesson 6-1)

56.  $d + 13 \geq 22$

57.  $t - 5 < 3$

58.  $4 > y + 7$

Write the standard form of an equation of the line that passes through the given point and has the given slope. (Lesson 4-5)

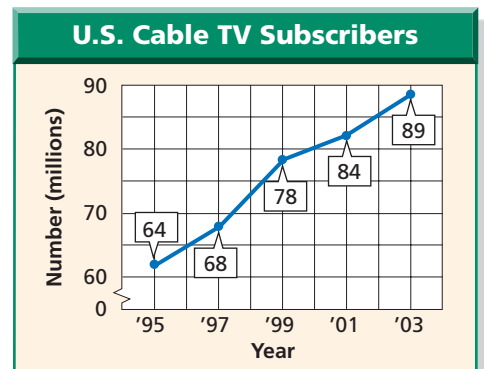
59.  $(1, -3), m = 2$

60.  $(-2, -1), m = -\frac{2}{3}$

61.  $(3, 6), m = 0$

**CABLE TV** For Exercises 62 and 63, use the graph at the right. (Lesson 4-1)

62. Find the rate of change for 2001–2003. Explain the meaning of the rate of change.
63. Without calculating, find a 2-year period that had a greater rate of change than 2001–2003. Explain.



### GET READY for the Next Lesson

**PREREQUISITE SKILL** Graph each set of numbers on a number line. (Lesson 1-8)

64.  $\{-1, 0, 3, 4\}$

65.  $\{-5, -4, -1, 1\}$

66.  $\{\text{integers less than } 5\}$

67.  $\{\text{integers between } 1 \text{ and } 6\}$

# READING MATH

## Compound Statements

Two simple statements connected by the words *and* or *or* form a compound statement. Before you can determine whether a compound statement is true or false, you must understand what the words *and* and *or* mean. Consider the statement below.

*A triangle has three sides, and a hexagon has five sides.*

For a compound statement connected by the word *and* to be true, both simple statements must be true. In this case, it is true that a triangle has three sides. However, it is false that a hexagon has five sides; it has six. Thus, the compound statement is false.

A compound statement connected by the word *or* may be *exclusive* or *inclusive*. For example, the statement “With your dinner, you may have soup *or* salad,” is exclusive. In everyday language, *or* means one or the other, but not both. However, in mathematics, *or* is inclusive. It means one or the other or both. Consider the statement below.

*A triangle has three sides, or a hexagon has five sides.*

For a compound statement connected by the word *or* to be true, at least one of the simple statements must be true. Since it is true that a triangle has three sides, the compound statement is true.



Triangle



Square



Pentagon



Hexagon



Octagon

## Reading to Learn

Determine whether each compound statement is *true* or *false*. Explain your answer.

1. A hexagon has six sides, *or* an octagon has seven sides.
2. An octagon has eight sides, *and* a pentagon has six sides.
3. A pentagon has five sides, *and* a hexagon has six sides.
4. A triangle has four sides, *or* an octagon does *not* have seven sides.
5. A pentagon has three sides, *or* an octagon has ten sides.
6. A square has four sides, *or* a hexagon has six sides.
7.  $5 < 4$  or  $8 < 6$
8.  $-1 > 0$  and  $1 < 5$
9.  $4 > 0$  and  $-4 < 0$
10.  $0 = 0$  or  $-2 > -3$
11.  $5 \neq 5$  or  $-1 > -4$
12.  $0 > 3$  and  $2 > -2$

# Solving Compound Inequalities

## Main Ideas

- Solve compound inequalities containing the word *and* and graph their solution sets.
- Solve compound inequalities containing the word *or* and graph their solution sets.

## New Vocabulary

compound inequality  
intersection  
union

## GET READY for the Lesson

The Mind Eraser Roller Coaster at Six Flags in Baltimore, Maryland, is an inverted steel track roller coaster. The trains are suspended underneath the tracks. To ride this coaster, you must be at least 52 inches tall and no more than 72 inches.

Let  $h$  represent the height of a rider. You can write two inequalities to represent the height restrictions.

at least 52 inches

$$h \geq 52$$

no more than 72 inches

$$h \leq 72$$

The inequalities  $h \geq 52$  and  $h \leq 72$  can be combined and written without using *and*.

$$52 \leq h \leq 72$$

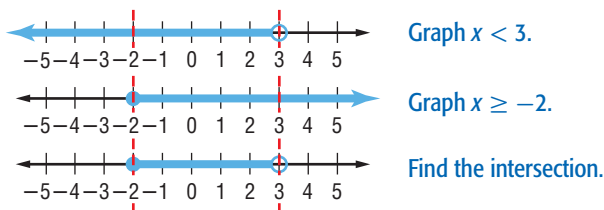


**Inequalities Containing *and*** When considered together, two inequalities such as  $w \geq 40$  and  $w \leq 250$  form a **compound inequality**. A compound inequality containing *and* is true only if both inequalities are true. Its graph is the **intersection** of the graphs of the two inequalities. In other words, the solution must be a solution of *both* inequalities.

The intersection can be found by graphing each inequality and then determining where the graphs overlap.

## EXAMPLE Graph an Intersection

- 1 Graph the solution set of  $x < 3$  and  $x \geq -2$ .



The solution set is  $\{x | -2 \leq x < 3\}$ . Note that the graph of  $x \geq -2$  includes the point  $-2$ . The graph of  $x < 3$  does *not* include 3.

## CHECK Your Progress

Graph the solution set of each compound inequality.

1A.  $a > -5$  and  $a < 0$

1B.  $p \leq 6$  and  $p > 2$

## Reading Math

### Compound Inequalities

The statement  $-2 \leq x < 3$  can be read *negative 2 is less than or equal to x, which is less than 3*.



## Reading Math

### Keywords

When solving problems involving inequalities,

- within is meant to be inclusive. Use  $\leq$  or  $\geq$ .
- between is meant to be exclusive. Use  $<$  or  $>$ .

## EXAMPLE Solve and Graph an Intersection

**2** Solve  $-5 < x - 4 < 2$ . Then graph the solution set.

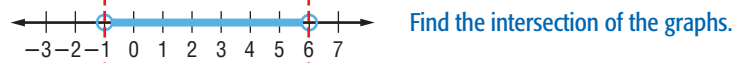
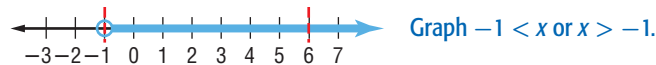
First express  $-5 < x - 4 < 2$  using *and*. Then solve each inequality.

$$-5 < x - 4 \quad \text{and} \quad x - 4 < 2 \quad \text{Write the inequalities.}$$

$$-5 + 4 < x - 4 + 4 \quad x - 4 + 4 < 2 + 4 \quad \text{Add 4 to each side.}$$

$$-1 < x \quad x < 6 \quad \text{Simplify.}$$

The solution set is  $\{x \mid -1 < x < 6\}$ . Now graph the solution set.



### CHECK Your Progress

Solve each compound inequality. Then graph the solution set.

**2A.**  $y - 3 \geq -11$  and  $y - 3 \leq -8$    **2B.**  $6 \leq r + 7 < 10$

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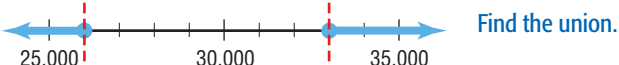
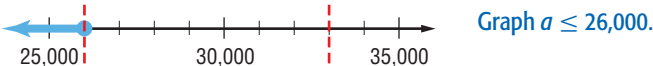
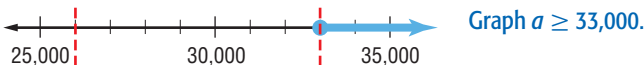
**Inequalities Containing *or*** Another type of compound inequality contains the word *or*. A compound inequality containing *or* is true if one or more of the inequalities is true. Its graph is the **union** of the graphs of the two inequalities. In other words, its solution is a solution of *either* inequality, not necessarily both. The union can be found by graphing each inequality.

## Real-World EXAMPLE Write and Graph a Compound Inequality

**3 AVIATION** A pilot flying at 30,000 feet is told by the control tower that he should increase his altitude to at least 33,000 feet or decrease his altitude to no more than 26,000 feet to avoid turbulence. Write and graph a compound inequality that describes the optimum altitude.

<b>Words</b>	The plane's altitude	is at least	33,000 feet	or	the altitude	is no more than	26,000 feet.
<b>Variable</b>	Let $a$ be the plane's altitude.						
<b>Inequality</b>	$a$	$\geq$	33,000	or	$a$	$\leq$	26,000

Now, graph the solution set.



The compound inequality  $a \geq 33,000$  or  $a \leq 26,000$  is graphed above.



### Real-World Career

#### Pilot

A pilot uses math to calculate the altitude that will provide the smoothest flight.

### Math Online

For more information, go to [algebra1.com](http://algebra1.com).

**CHECK Your Progress**

- 3. SHOPPING** A store is offering a \$30 mail-in rebate on all color printers. Luisana is looking at different color printers that range in price from \$175 to \$260. How much can she expect to spend after the mail-in rebate?

 **Online Personal Tutor at [algebra1.com](http://algebra1.com)**

**EXAMPLE Solve and Graph a Union**

- 4** Solve  $-3h + 4 < 19$  or  $7h - 3 > 18$ . Then graph the solution set.

$$-3h + 4 < 19 \quad \text{or} \quad 7h - 3 > 18 \quad \text{Write the inequalities.}$$

$$-3h + 4 - 4 < 19 - 4 \quad 7h - 3 + 3 > 18 + 3 \quad \text{Add or subtract.}$$

$$-3h < 15 \quad 7h > 21 \quad \text{Simplify.}$$

$$\frac{-3h}{-3} > \frac{15}{-3} \quad \frac{7h}{7} > \frac{21}{7} \quad \text{Divide.}$$

$$h > -5 \quad h > 3 \quad \text{Simplify.}$$



Notice that the graph of  $h > -5$  contains every point in the graph of  $h > 3$ . So, the union is the graph of  $h > -5$ . The solution set is  $\{h \mid h > -5\}$ .

**Concepts in Motion**  
Animation  
[algebra1.com](http://algebra1.com)

**CHECK Your Progress**

Solve each compound inequality. Then graph the solution set.

**4A.**  $a + 1 < 4$  or  $a - 1 \geq 3$

**4B.**  $x \leq 9$  or  $2 + 4x < 10$

**CHECK Your Understanding**

**Example 1**  
(p. 315)

Graph the solution set of each compound inequality.

**1.**  $a \leq 6$  and  $a \geq -2$

**2.**  $y < 12$  and  $y > 9$

**Examples 2, 4**  
(pp. 316–317)

Solve each compound inequality. Then graph the solution set.

**3.**  $6 < w + 3$  and  $w + 3 < 11$

**4.**  $n - 7 \leq -5$  or  $n - 7 \geq 1$

**5.**  $3z + 1 < 13$  or  $z \leq 1$

**6.**  $-8 < x - 4 \leq -3$

**Example 3**  
(pp. 316–317)

- 7. BIKES** The recommended air pressure for the tires of a mountain bike is at least 35 pounds per square inch (psi), but no more than 80 pounds per square inch. If a bike's tires have 24 pounds per square inch, what increase in air pressure is needed so the tires are in the recommended range?



# Exercises

## HOMEWORK HELP

For Exercises	See Examples
8–13	1
14–23	2, 4
24–25	3

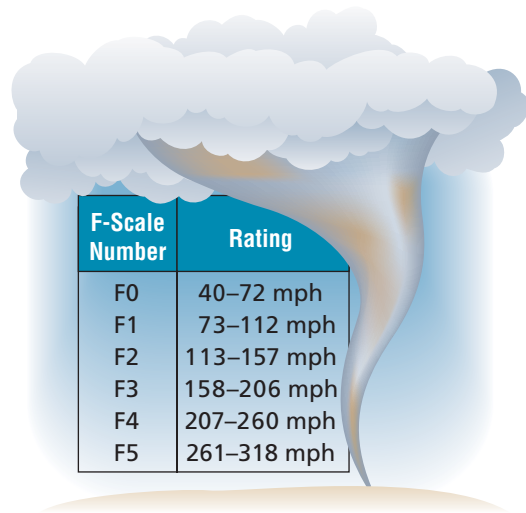
Graph the solution set of each compound inequality.

8.  $x > 5$  and  $x \leq 9$       9.  $s < -7$  and  $s \leq 0$   
 10.  $r < 6$  or  $r > 6$       11.  $m \geq -4$  or  $m > 6$   
 12.  $7 < d < 11$       13.  $-1 \leq g < 3$

Solve each compound inequality. Then graph the solution set.

14.  $k + 2 > 12$  and  $k + 2 \leq 18$       15.  $f + 8 \leq 3$  and  $f + 9 \geq -4$   
 16.  $d - 4 > 3$  or  $d - 4 \leq 1$       17.  $h - 10 < -21$  or  $h + 3 < 2$   
 18.  $3 < 2x - 3 < 15$       19.  $4 < 2y - 2 < 10$   
 20.  $3t - 7 \geq 5$  and  $2t + 6 \leq 12$       21.  $8 > 5 - 3q$  and  $5 - 3q > -13$   
 22.  $-1 + x \leq 3$  or  $-x \leq -4$       23.  $3n + 11 \leq 13$  or  $-3n \geq -12$

24. **ANALYZE TABLES** The Fujita Scale (F-scale) is the official classification system for tornado damage. One factor used to classify a tornado is wind speed. Use the information in the table to write an inequality for the range of wind speeds of an F3 tornado.



25. **BIOLOGY** Each type of fish thrives in a specific range of temperatures. The optimum temperatures for sharks range from  $18^{\circ}\text{C}$  to  $22^{\circ}\text{C}$ , inclusive. Write an inequality to represent temperatures where sharks will *not* thrive. (*Hint:* The word *inclusive* means that  $18^{\circ}\text{C}$  and  $22^{\circ}\text{C}$  are included in the optimum temperature range.)

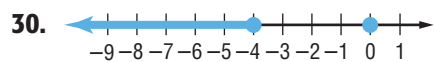
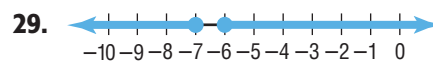
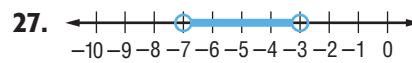
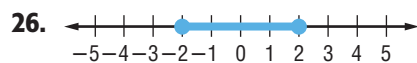


### Real-World Link

The average life span of a shark is 25 years, but some sharks can live to be 100.

Source: about.com

Write a compound inequality for each graph.



Solve each compound inequality. Then graph the solution set.

32.  $2p - 2 \leq 4p - 8 \leq 3p - 3$       33.  $3g + 12 \leq 6 - g \leq 3g - 18$   
 34.  $4c < 2c - 10$  or  $-3c < -12$       35.  $0.5b > -6$  or  $3b + 16 < -8 + b$

36. **HEALTH** About 20% of the time you sleep is spent in rapid eye movement (REM) sleep, which is associated with dreaming. If an adult sleeps 7 to 8 hours, how much time is spent in REM sleep?



### Real-World Link

A dog's sense of smell is about 1000 times better than a human's, but its eyesight is not as good.

Source: about.com



### Graphing Calculator

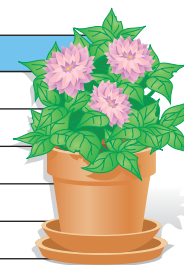
### H.O.T. Problems

**EXTRA PRACTICE**  
See pages 729, 749.

**Math online**  
Self-Check Quiz at [algebra1.com](http://algebra1.com)

- 37. FUND-RAISING** Rashid is selling potted flowers for his school's fund-raiser. He can earn prizes depending on how much he sells. So far, he has sold \$70 worth of flowers. How much more does he need to sell to earn a prize in category D?

Sales (\$)	Prize
0–25	A
26–60	B
61–120	C
121–180	D
180+	E



Define a variable, write an inequality, and solve each problem. Then check your solution.

- 38.** Eight less than a number is no more than 14 and no less than 5.  
**39.** The sum of 3 times a number and 4 is between  $-8$  and 10.  
**40.** The product of  $-5$  and a number is greater than 35 or less than 10.  
**41.** One half a number is greater than 0 and less than or equal to 1.

**HEARING** For Exercises 42–44, use the table.

What Humans and Dogs Can Hear	
Species	Sound Waves (hertz)
humans	20–20,000
dogs	15–50,000

- 42.** Write a compound inequality for the hearing range of humans and one for the hearing range of dogs.  
**43.** What is the union of the two graphs? the intersection?  
**44.** Write an inequality or inequalities for the range of sounds that dogs can hear, but humans cannot.  
**45. RESEARCH** Use the Internet or other resource to find the altitudes in miles of the layers of Earth's atmosphere, troposphere, stratosphere, mesosphere, thermosphere, and exosphere. Write inequalities for the range of altitudes for each layer.  
**46.** In Lesson 6-3, you learned how to use a graphing calculator to find the values of  $x$  that make a given inequality true. You can also use this method to test compound inequalities. The words *and* and *or* can be found in the LOGIC submenu of the TEST menu of a TI-83/84 Plus. Use this method to solve each of the following compound inequalities using your graphing calculator.  
**a.**  $x + 4 < -2$  or  $x + 4 > 3$       **b.**  $x - 3 \leq 5$  and  $x + 6 \geq 4$   
**47. OPEN ENDED** Create an example of a compound inequality containing *and* that has no solution.  
**48. REASONING** Formulate a compound inequality to represent  $\$7$  is less than  $t$ , which is less than  $\$12$ . Interpret what the solution means.  
**49. CHALLENGE** Select compound inequalities that represent the values of  $x$  which make the following expressions *false*.  
**a.**  $x < 5$  or  $x > 8$       **b.**  $x \leq 6$  and  $x \geq 1$   
**50. Writing in Math** Use the information about the roller coaster on page 315 to explain how compound inequalities can be used to describe weight restrictions at amusement parks. Include a compound inequality describing a possible height restriction for riders of the roller coaster. Describe what this represents.

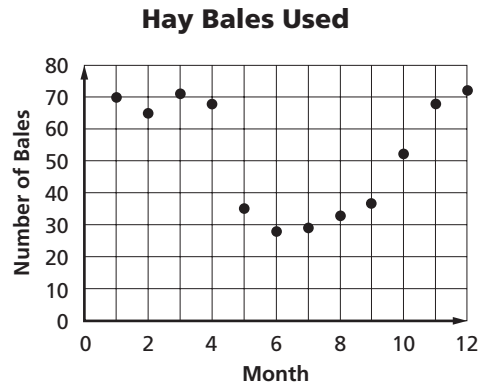
**51. REVIEW** Ten pounds of fresh tomatoes make about 15 cups of cooked tomatoes. How many cups does one pound of tomatoes make?

- A  $1\frac{1}{2}$  cups
- B 5 cups
- C 3 cups
- D 4 cups

**52.** What is the solution set of the inequality  $-7 < x + 2 < 4$ ?

- F  $-5 < x < 6$
- G  $-9 < x < 2$
- H  $-5 < x < 2$
- J  $-9 < x < 6$

**53. REVIEW** The scatterplot below shows the number of hay bales used by the Crosley farm during the last year.



Which is an invalid conclusion?

- A The Crosleys used less hay in the summer than they did in the winter.
- B The Crosleys used a total of 629 bales of hay.
- C The Crosleys used about 52 bales each month.
- D The Crosleys used the most hay in February.

**Spiral Review**

**54. MONEY** In the summer, Richard earns \$200 per month at his part-time job at a restaurant, plus an average of \$18 for each lawn that he mows. If his goal is to earn at least \$280 this month, how many lawns will he have to mow?

(Lesson 6-3)

Solve each inequality. Check your solution. (Lesson 6-2)

55.  $18d \geq 90$

56.  $-7v < 91$

57.  $\frac{t}{13} < 13$

58.  $-\frac{3}{8}b > 9$

Solve. Assume that  $y$  varies directly as  $x$ . (Lesson 5-2)

59. If  $y = -8$  when  $x = -3$ , find  $x$  when  $y = 6$ .

60. If  $y = 2.5$  when  $x = 0.5$ , find  $y$  when  $x = 20$ .

**GET READY for the Next Lesson**

**PREREQUISITE SKILL** Solve each equation. (Lesson 2-4)

61.  $3n - 14 = 1$

62.  $2t + 5 = 7$

63.  $8w - 13 = 3$

64.  $5d + 9 = -6$

65.  $12 = 3n + 15$

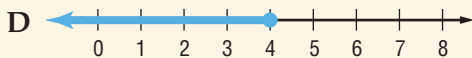
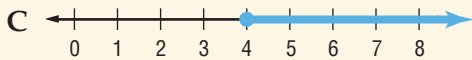
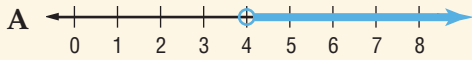
66.  $17 = 4p - 3$

67.  $-3 = 4x + 5$

68.  $-14 = 4 + 3w$

1. **MULTIPLE CHOICE** Which graph represents all the values of  $m$  such that  $m + 3 > 7$ ?

(Lesson 6-1)



Solve each inequality. Check your solution, then graph it on a number line. (Lesson 6-1)

2.  $8 + x < 9$                       3.  $h - 16 > -13$   
 4.  $r + 3 \leq -1$                     5.  $4 \geq p + 9$   
 6.  $-3 < a - 5$                       7.  $7g \leq 6g - 1$

8. **MULTIPLE CHOICE** Which inequality does NOT have the same solution as  $-14w < 14$ ? (Lesson 6-2)

- F  $8w > -8$                       H  $\frac{w}{5} > -\frac{1}{5}$   
 G  $11 > -11w$                     J  $-\frac{3}{2} < \frac{2}{3}w$

Solve each inequality. Check your solution.

(Lesson 6-2)

9.  $12 \geq 3a$                       10.  $15z \geq 105$   
 11.  $\frac{v}{5} < 4$                         12.  $-\frac{3}{7}q > 15$   
 13.  $-156 < 12r$                 14.  $-\frac{2}{5}w \leq -\frac{1}{2}$

15. **MONEY** Javier is saving \$175 each month to buy a used all-terrain vehicle. How long will it take him to save at least \$2900? Define a variable and write an inequality to solve the problem. Interpret your solution. (Lesson 6-2)

Solve each inequality. Check your solution.

(Lesson 6-3)

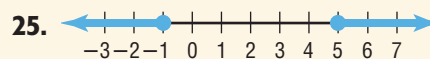
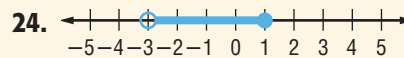
16.  $5 - 4b > -23$             17.  $\frac{1}{2}n + 3 \geq -5$   
 18.  $3(t + 6) < 9$             19.  $9x + 2 > 20$   
 20.  $2m + 5 \leq 4m - 1$     21.  $a < \frac{2a - 15}{3}$

22. **MULTIPLE CHOICE** What is the first step in solving  $\frac{y - 5}{9} \geq 13$ ? (Lesson 6-3)

- A Add 5 to each side.  
 B Subtract 5 from each side.  
 C Divide each side by 9.  
 D Multiply each side by 9.

23. **PHYSICAL SCIENCE** Chlorine is a gas for all temperatures greater than  $-31^\circ\text{F}$ . If  $F$  represents temperature in degrees Fahrenheit, the inequality  $F > -31$  represents the temperatures for which chlorine is a gas. Solve  $\frac{9}{5}C + 32 > -31$  to find the temperatures in degrees Celsius for which chlorine is a gas. (Lesson 6-3)

Write a compound inequality for each graph. (Lesson 6-4)



26. **MULTIPLE CHOICE** Some parts of a state get less than 9 inches of annual rainfall. Other parts of the state get more than 57 inches. Which inequality represents this situation? (Lesson 6-4)

- F  $9 < r < 57$   
 G  $9 > r > 57$   
 H  $r < 9$  or  $r > 57$   
 J  $r < 9$  and  $r > 57$

Solve each compound inequality. Then graph the solution set. (Lesson 6-4)

27.  $x - 2 < 7$  and  $x + 2 > 5$   
 28.  $2b + 5 \leq -1$  or  $b - 4 \geq -4$   
 29.  $4m - 5 > 7$  or  $4m - 5 < -9$   
 30.  $a - 4 < 1$  and  $a + 2 > 1$

# Solving Open Sentences Involving Absolute Value

## Main Ideas

- Solve absolute value equations.
- Graph absolute value functions.

## New Vocabulary

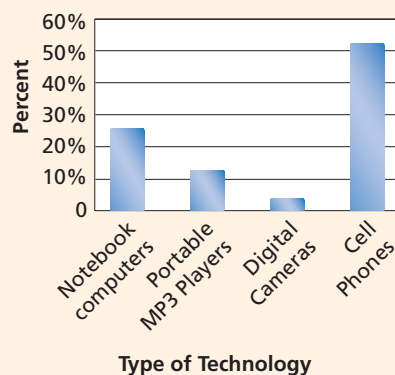
absolute value  
absolute value function  
piece wise function

### GET READY for the Lesson

In an international survey of students from 25 high schools in 9 different countries, 52% of those surveyed chose cell phones as the technology that is most important to them.

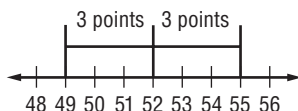
Suppose the survey had a 3-point margin of error. This means that the result may be 3 percentage points higher or lower. So, the number of students favoring cell phones over other technology may be as high as 55% or as low as 49%.

### Which Technology is Most Important to You?



Source: [www.ucdsb.on.ca/athens/surveys.index.htm](http://www.ucdsb.on.ca/athens/surveys.index.htm)

**Absolute Value Equations** The margin of error of the data in the bar graph is an example of absolute value. The distance between 52 and 55 on a number line is the same as the distance between 49 and 52.

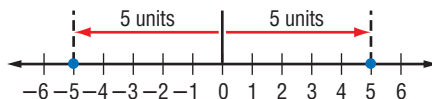


## Study Tip

### Absolute Value

Since distance cannot be less than zero, absolute values are always greater than or equal to zero.

The **absolute value** of any number  $n$  is its distance from zero on a number line. The absolute value of  $n$  is written as  $|n|$ . There are three types of open sentences involving absolute value. They are  $|x| = n$ ,  $|x| < n$ , and  $|x| > n$ . Consider the first type.  $|x| = 5$  means the distance between 0 and  $x$  is 5 units.



If  $|x| = 5$ , then  $x = -5$  or  $x = 5$ . The solution set is  $\{-5, 5\}$ .

## KEY CONCEPT

### Solving Absolute Value Equations

When solving equations that involve absolute value, there are two cases to consider.

**Case 1** The expression inside the absolute value symbols is positive.

**Case 2** The expression inside the absolute value symbols is negative.

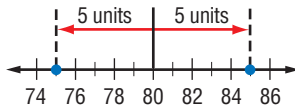
**Real-World EXAMPLE**

**Solve an Absolute Value Equation**

- 1 a. SNAKES** The temperature of an enclosure for a pet snake should be about 80°F, give or take 5°. Solve  $|a - 80| = 5$  to find the maximum and minimum of the temperatures.

**Method 1** Graphing

$|a - 80| = 5$  means that the distance between  $a$  and 80 is 5 units. To find  $a$  on the number line, start at 80 and move 5 units in either direction.



The distance from 80 to 75 is 5 units.  
The distance from 80 to 85 is 5 units.  
The solution set is  $\{75, 85\}$ .

**Method 2** Compound Sentence

Write  $|a - 80| = 5$  as  $a - 80 = 5$  or  $a - 80 = -5$ .

**Case 1**

$$a - 80 = 5$$

$$a - 80 + 80 = 5 + 80 \quad \text{Add 80 to each side.} \quad a - 80 + 80 = -5 + 80$$

$$a = 85 \quad \text{Simplify.}$$

**Case 2**

$$a - 80 = -5$$

$$a = 75$$

The solution set is  $\{75, 85\}$ . The maximum and minimum temperatures are 85°F and 75°F.

- b. Solve  $|b - 1| = -3$ .**

$|b - 1| = -3$  means that the distance between  $b$  and 1 is  $-3$ . Since distance cannot be negative, the solution is the empty set  $\emptyset$ .

**Study Tip**

**Absolute Value**

Since  $|a| = 3$  means  $a = 3$  or  $-a = 3$ , the second equation can be written as  $a = -3$ . So,  $|a - 4| = 3$  means  $a - 4 = 3$  or  $-(a - 4) = 3$ . These can be written as  $a - 4 = 3$  or  $a - 4 = -3$ .

**CHECK Your Progress**

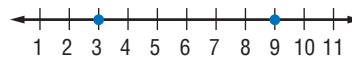
Solve each open sentence. Then graph the solution set.

**1A.**  $|y + 2| = 4$

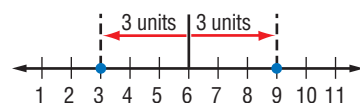
**1B.**  $|3n - 4| = -1$

**EXAMPLE** Write an Absolute Value Equation

- 2** Write an open sentence involving absolute value for the graph.



Find the point that is the same distance from 3 and from 9. This is the midpoint between 3 and 9, which is 6.

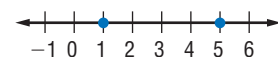


The distance from 6 to 3 is 3 units.  
The distance from 6 to 9 is 3 units.

So, an equation is  $|x - 6| = 3$ .

**CHECK Your Progress**

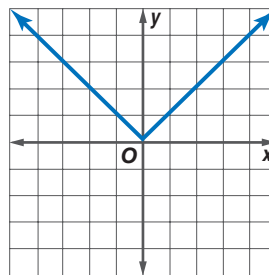
- 2.** Write an open sentence involving absolute value for the graph.





**Graphing Absolute Value Functions** An **absolute value function** is a function written as  $f(x) = |x|$ , when  $f(x) \geq 0$  for all values of  $x$ . To graph  $f(x) = |x|$ , make a table of values and plot the ordered pairs on a coordinate plane. Notice that for negative values of  $x$ , the slope of the line is  $-1$ . When the  $x$ -values are positive, the slope of the line is 1. An absolute value function can be written using two or more expressions. This is an example of a **piecewise function**.

$f(x) =  x $	
$x$	$f(x)$
-3	3
-2	2
-1	1
-0	0
-1	1
-2	2
-3	3



The absolute value function  $f(x) = |x|$  can be written as  $f(x) = \begin{cases} -x & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases}$ .

### EXAMPLE Graphing Absolute Value Functions

#### Study Tip

##### Absolute Value

When plotting points on a graph, choose values of  $x$  that are less than and that are greater than the minimum point of the graph.

3 Graph  $f(x) = |x - 4|$ .

First, find the minimum point on the graph. Since  $f(x)$  cannot be negative, the minimum point of the graph is where  $f(x) = 0$ .

$$f(x) = |x - 4| \quad \text{Original function}$$

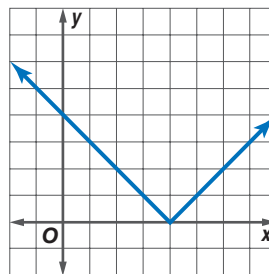
$$0 = x - 4 \quad \text{Set } f(x) = 0.$$

$$4 = x \quad \text{Add 4 to each side.}$$

The minimum point of the graph is at  $(4, 0)$ .

Next fill out a table of values. Include values for  $x > 4$  and  $x < 4$ .

$f(x) =  x - 4 $	
$x$	$f(x)$
-2	6
0	4
2	2
4	0
5	1
6	2
7	3
8	4



#### CHECK Your Progress

3. Graph  $f(x) = |2x + 1|$ .

# CHECK Your Understanding

**Example 1**  
(p. 323)

Solve each open sentence. Then graph the solution set.

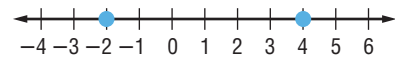
1.  $|r + 3| = 10$

2.  $|2x - 8| = 6$

3.  $|4n - 1| = -6$

**Example 2**  
(p. 323)

4. Write an open sentence involving absolute value for the graph.



**Examples 3**  
(p. 324)

Graph each function.

5.  $f(x) = |x - 3|$

6.  $g(x) = |2x + 4|$

## Exercises

HOMEWORK HELP	
For Exercises	See Examples
7-18	1
19-22	2
23-30	3

Solve each open sentence. Then graph the solution set.

7.  $|x - 5| = 8$

8.  $|b + 9| = 2$

9.  $|v - 2| = -5$

10.  $|2p - 3| = 17$

11.  $|5c - 8| = 12$

12.  $|6y - 7| = -1$

13.  $|\frac{1}{2}x + 5| = -3$

14.  $|-2x + 6| = 6$

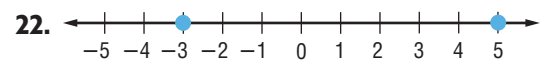
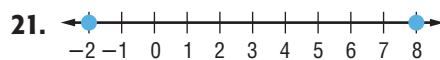
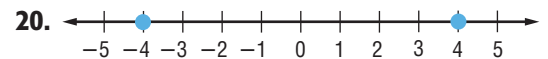
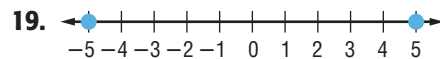
15.  $|\frac{3}{4}x - 3| = 9$

16.  $|\frac{1}{2}x - 2| = 10$

17.  $|-4x + 6| = 12$

18.  $|5x - 3| = 12$

Write an open sentence involving absolute value for each graph.



Graph each function.

23.  $f(x) = |2x - 1|$

24.  $f(x) = |x + 5|$

25.  $g(x) = |-3x - 5|$

26.  $g(x) = |-x - 3|$

27.  $f(x) = |\frac{1}{2}x - 2|$

28.  $f(x) = |\frac{1}{3}x + 2|$

29.  $g(x) = |x + 2| + 3$

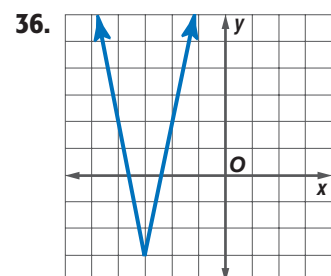
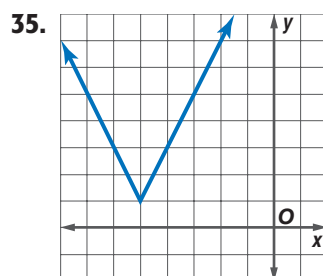
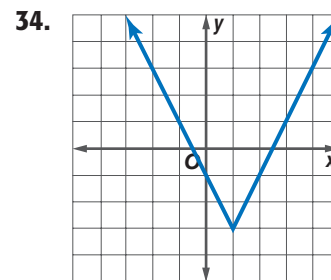
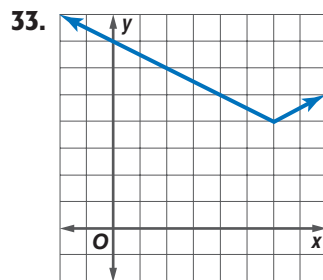
30.  $g(x) = |2x - 3| + 1$

Solve for  $x$ .

31.  $2|x| - 3 = 8$

32.  $4 - 3|x| = 10$

Determine the domain and range for each absolute value function.



- 37. ANALYZE GRAPHS** The circle graph at the right shows the results of a survey that asked teens “When do you think humans will be able to live on the moon?” If the margin of error is  $\pm 3$  percentage points, what is the range of the percent of teens who say humans will live on the moon in 2100?



- 38. PHYSICS** As part of a physics lab, Tiffany and Curtis determined that an object was traveling at 25 miles per hour. If the margin of error is 6%, determine the slowest and the fastest rate of the object.

**PING PONG** For Exercises 39 and 40, use the following information.

Esmerelda dropped a ping pong ball from a height of 4 feet. She tracked the height of the ball and the elapsed time. The ball hit the floor at 2 seconds and then bounced. At 4 seconds, the height was 4 feet.

- 39.** Draw a graph of the path of the ping pong ball. Let the  $x$ -axis represent the time and the  $y$ -axis represent the height.
- 40.** Write a piecewise function to describe the path of the ping pong ball.
- 41. OPEN ENDED** Describe a real-world situation that could be represented by the absolute value equation  $|x - 4| = 10$ .

**REASONING** Determine whether the following statements are *sometimes*, *always*, or *never* true, where  $c$  is a whole number. Explain your reasoning.

- 42.** The value of the expression  $|x + 1|$  is greater than zero.
- 43.** The solution of the equation  $|x + c| = 0$  is greater than 0.
- 44.** The inequality  $|x| + c < 0$  has no solution.
- 45.** The value of the expression  $|x + c| + c$  is greater than zero.
- 46. CHALLENGE** Use the sentence  $x = 3 \pm 1.2$ .
- Describe the values of  $x$ .
  - Translate the sentence into an expression involving absolute value.
- 47. FIND THE ERROR** Leslie and Holly are solving  $|x + 3| = 2$ . Who is correct? Explain your reasoning.

Leslie	
$x + 3 = 2$	or $x + 3 = -2$
$x + 3 - 3 = 2 - 3$	$x + 3 - 3 = -2 - 3$
$x = -2$	$x = -5$

Holly	
$x + 3 = 2$	or $x - 3 = 2$
$x + 3 - 3 = 2 - 3$	$x + 3 + 3 = 2 + 3$
$x = -1$	$x = 5$

- 48. Writing in Math** Use the data about the technology survey on page 322 to explain how absolute value is used in surveys.

**EXTRA PRACTICE**  
See pages 729, 749.  
**Math**  
Self-Check Quiz at [algebra1.com](http://algebra1.com)

**H.O.T. Problems**

49. Assume  $n$  is an integer and solve for  $n$ .

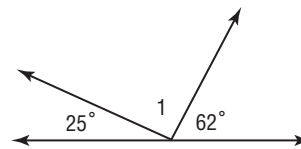
$$|2n - 3| = 5$$

- A  $\{-4, -1\}$                       C  $\{-1, 4\}$   
 B  $\{1, 1\}$                             D  $\{4, 4\}$

50. If  $p$  is an integer, which of the following is the solution set for  $2|p| = 16$ ?

- F  $\{0, 8\}$   
 G  $\{-8, 8\}$   
 H  $\{-8, 0\}$   
 J  $\{-8, 0, 8\}$

51. **REVIEW** What is the measure of  $\angle 1$  in the figure shown below?



- A  $83^\circ$   
 B  $87^\circ$   
 C  $90^\circ$   
 D  $93^\circ$

Spiral Review

52. **FITNESS** To achieve the maximum benefits from aerobic activity, your heart rate should be in your target zone. Your target zone is the range between 60% and 80% of your maximum heart rate. If Rafael's maximum heart rate is 190 beats per minute, what is his target zone? (Lesson 6-4)

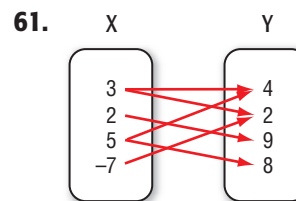
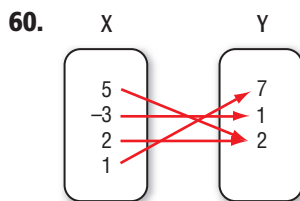
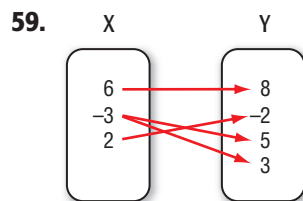
Solve each inequality. Check your solution. (Lesson 6-3)

53.  $2m + 7 > 17$                       54.  $-2 - 3x \geq 2$                       55.  $\frac{2}{3}w - 3 \leq 7$

Find the slope and  $y$ -intercept of each equation. (Lesson 4-4)

56.  $2x + y = 4$                       57.  $2y - 3x = 4$                       58.  $\frac{1}{2}x + \frac{3}{4}y = 0$

Express the relation shown in each mapping as a set of ordered pairs. Then state the domain, range, and inverse. (Lesson 4-3)



Solve each equation for the variable specified. (Lesson 2-8)

62.  $I = prt$ , for  $r$                       63.  $ex - 2y = 3z$ , for  $x$                       64.  $\frac{a + 5}{3} = 7x$ , for  $x$

GET READY for the Next Lesson

**PREREQUISITE SKILL** Solve each inequality. Check your solution. (Lesson 6-1)

65.  $m + 4 \geq 6$                       66.  $p - 3 < 2$                       67.  $z + 5 \leq 11$   
 68.  $8 + w < -12$                       69.  $4 - r \geq -3$                       70.  $6 - v \geq -2$

# Graphing Calculator Lab

## Graphing Absolute Value Functions

The absolute value function  $y = |x|$  is the parent function of the family of absolute value functions. The graphs of absolute value functions are similar to the graphs of linear functions.

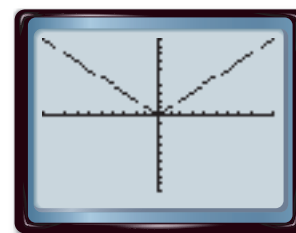
### ACTIVITY 1

Graph  $y = |x|$  in the standard viewing window.

Enter the equation in the Y= list. Then graph the equation.

**KEYSTROKES:**  $\boxed{Y=}$   $\boxed{\text{MATH}}$   $\boxed{\blacktriangleright}$   $\boxed{1}$   $\boxed{X,T,\theta,n}$   $\boxed{)}$   $\boxed{\text{ZOOM}}$   $\boxed{6}$

- 1A. How does the graph of  $y = |x|$  compare to the graph of  $y = x$ ?
- 1B. What are the domain and range of the function  $y = |x|$ ? Explain.



$[-10, 10]$  scl: 1 by  $[-10, 10]$  scl: 1

The graphs of absolute value functions are affected by changes in parameters in a way similar to the way changes in parameters affect the graphs of linear functions.

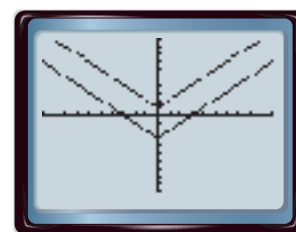
### ACTIVITY 2

Graph  $y = |x| - 3$  and  $y = |x| + 1$  in the standard viewing window.

Enter the equations in the Y= list. Then graph.

**KEYSTROKES:**  $\boxed{Y=}$   $\boxed{\text{MATH}}$   $\boxed{\blacktriangleright}$   $\boxed{1}$   $\boxed{X,T,\theta,n}$   $\boxed{)}$   $\boxed{-}$   $\boxed{3}$   $\boxed{\text{ENTER}}$   $\boxed{\text{MATH}}$   $\boxed{\blacktriangleright}$   $\boxed{1}$   $\boxed{X,T,\theta,n}$   $\boxed{)}$   $\boxed{+}$   $\boxed{1}$   $\boxed{\text{ZOOM}}$   $\boxed{6}$

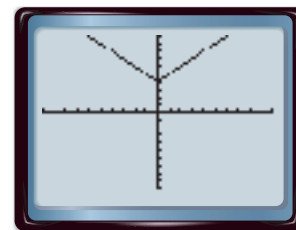
- 2A. Compare and contrast the graphs to the graph of  $y = |x|$ .
- 2B. How does the value of  $c$  affect the graph of  $y = |x| + c$ ?



$[-10, 10]$  scl: 1 by  $[-10, 10]$  scl: 1

### ANALYZE THE RESULTS

1. Write the function shown in the graph.
2. Graph  $y = -|x|$  in the standard viewing window. How is this graph related to the graph of  $y = |x|$ ?
3. **MAKE A CONJECTURE** Describe the transformation of the parent graph  $y = |x + c|$ . Use a graphing calculator with different values of  $c$  to test your conjecture.
4. Determine whether the following statement is *always*, *sometimes*, or *never* true. Justify your answer. The  $y$ -values of the function  $y = -|x - 1| - 1$  are negative.



$[-10, 10]$  scl: 1 by  $[-10, 10]$  scl: 1

# 6-6

## Solving Inequalities Involving Absolute Value

### GET READY for the Lesson

#### Main Ideas

- Solve absolute value inequalities.
- Apply absolute value inequalities in real-world problems.

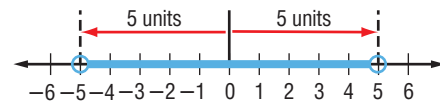
To make baby carrots for snacks, long carrots are sliced into 2-inch sections and then peeled. If the machine that slices the carrots is accurate to within  $\frac{1}{8}$  of an inch, the length of a baby carrot ranges from  $1\frac{7}{8}$  inch to  $2\frac{1}{8}$  inch.



**Absolute Value Inequalities** Consider the inequality  $|x| < n$ .  $|x| < 5$  means that the distance from 0 to  $x$  is less than 5 units.

Therefore,  $x > -5$  and  $x < 5$ . The solution set is  $\{x \mid -5 < x < 5\}$ .

When solving inequalities of the form  $|x| < n$ , consider the two cases.



**Case 1** The expression inside the absolute value symbols is positive.

**Case 2** The expression inside the absolute value symbols is negative.

To solve, find the *intersection* of the solutions of these two cases.

### EXAMPLE Solve an Absolute Value Inequality (<)

1 Solve each open sentence. Then graph the solution set.

a.  $|t + 5| < 9$

Write  $|t + 5| < 9$  as  $t + 5 < 9$  and  $t + 5 > -9$ .

**Case 1**  $t + 5$  is positive.

**Case 2**  $t + 5$  is negative.

$$t + 5 < 9$$

$$-(t + 5) < 9$$

$$t + 5 - 5 < 9 - 5$$

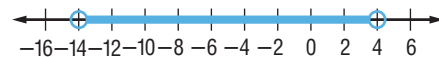
$$t + 5 > -9$$

$$t < 4$$

$$t + 5 - 5 > -9 - 5$$

$$t > -14$$

The solution set is  $\{t \mid -14 < t < 4\}$ .



b.  $|x + 2| < -1$

Since  $|x + 2|$  cannot be negative,  $|x + 2|$  cannot be less than  $-1$ . So, the solution set is the empty set  $\emptyset$ .

### Study Tip

#### Less Than

When an absolute value is on the left and the inequality symbol is  $<$  or  $\leq$ , the compound sentence uses *and*.

### CHECK Your Progress

1A.  $|n - 8| \leq 2$

1B.  $|2c - 5| < -3$



Consider the inequality  $|x| > n$ .  $|x| > 5$  means that the distance from 0 to  $x$  is greater than 5 units.



Therefore,  $x < -5$  or  $x > 5$ . The solution set is  $\{x \mid x < -5 \text{ or } x > 5\}$ .

When solving inequalities of the form  $|x| > n$ , consider the two cases.

**Case 1** The expression inside the absolute value symbols is positive.

**Case 2** The expression inside the absolute value symbols is negative.

To solve, find the *union* of the solutions of these two cases.

### EXAMPLE Solve an Absolute Value Inequality ( $>$ )

2 Solve each open sentence. Then graph the solution set.

a.  $|2x + 8| \geq 6$

**Case 1**  $2x + 8$  is positive.

$$2x + 8 \geq 6$$

$$2x + 8 - 8 \geq 6 - 8 \quad \text{Subtract 8 from each side.}$$

$$2x \geq -2 \quad \text{Simplify.}$$

$$\frac{2x}{2} \geq \frac{-2}{2} \quad \text{Divide each side by 2.}$$

$$x \geq -1 \quad \text{Simplify.}$$

**Case 2**  $2x + 8$  is negative.

$$-(2x + 8) \geq 6$$

$$2x + 8 \leq -6 \quad \text{Divide each side by } -1 \text{ and reverse the symbol.}$$

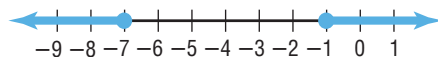
$$2x + 8 - 8 \leq -6 - 8 \quad \text{Subtract 8 from each side.}$$

$$2x \leq -14 \quad \text{Simplify.}$$

$$\frac{2x}{2} \leq \frac{-14}{2} \quad \text{Divide each side by 2.}$$

$$x \leq -7 \quad \text{Simplify.}$$

The solution set is  $\{x \mid x \leq -7 \text{ or } x \geq -1\}$ .



b.  $|2y - 1| \geq -4$

Since  $|2y - 1|$  is always greater than or equal to 0, the solution set is  $\{y \mid y \text{ is a real number}\}$ .

Its graph is the entire number line.



### Study Tip

#### Greater Than

When the absolute value is on the left and the inequality symbol is  $>$  or  $\geq$ , the compound sentence uses *or*.



2A.  $|2k + 1| > 7$

2B.  $|r - 6| \geq -5$

Online Personal Tutor at [algebra1.com](http://algebra1.com)

In general, there are three rules to remember when solving equations and inequalities involving absolute value.

**CONCEPT SUMMARY** *Absolute Value Equations and Inequalities*

- If  $|x| = n$ , then  $x = -n$  or  $x = n$ .
- If  $|x| < n$ , then  $x < n$  and  $x > -n$ .
- If  $|x| > n$ , then  $x > n$  or  $x < -n$ .

These properties are also true when  $>$  or  $<$  is replaced with  $\geq$  or  $\leq$ .

**Applying Absolute Value Inequalities** Many situations can be represented using an absolute value inequality.

**Real-World EXAMPLE**

**3 BIOLOGY** The pH is a measure of the acidity of a solution. The pH of a healthy human stomach is about 2.5 and is within 0.5 pH of this value. Find the range of pH levels of a healthy stomach.

The difference between the actual pH of a stomach and the ideal pH of a stomach is less than or equal to 0.5. Let  $x$  be the actual pH of a stomach. Then  $|x - 2.5| \leq 0.5$ .

Solve each case of the inequality.

<b>Case 1</b>	<b>Case 2</b>
$x - 2.5 \leq 0.5$	$-(x - 2.5) \leq 0.5$
$x - 2.5 + 2.5 \leq 0.5 + 2.5$	$x - 2.5 \geq -0.5$
$x \leq 3.0$	$x - 2.5 + 2.5 \geq -0.5 + 2.5$
	$x \geq 2.0$

The range of pH levels of a healthy stomach is  $\{x \mid 2.0 \leq x \leq 3.0\}$ .

**CHECK Your Progress**

**3. CHEMISTRY** The melting point of ice is  $0^\circ$  Celsius. During a chemistry experiment, Jill observed ice melting within 2 degrees. Write the range of temperatures that Jill observed ice melting.

**CHECK Your Understanding**

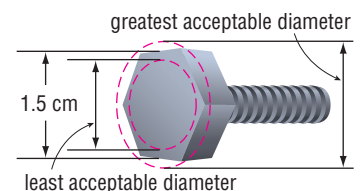
**Examples 1, 2**  
(p. 330)

Solve each open sentence. Then graph the solution set.

- |                    |                      |                       |
|--------------------|----------------------|-----------------------|
| 1. $ c - 2  < 6$   | 2. $ x + 5  \leq 3$  | 3. $ m - 4  \leq -3$  |
| 4. $ 10 - w  > 15$ | 5. $ 2g + 5  \geq 7$ | 6. $ 3p + 2  \geq -8$ |

**Example 3**  
(p. 331)

**7. MANUFACTURING** A manufacturer produces bolts which must have a diameter within 0.001 centimeter of 1.5 centimeters. What are the acceptable measurements for the diameter of the bolts?





# Exercises

## HOMework HELP

For Exercises	See Examples
8–6	1, 2
17, 18, 31–34	3

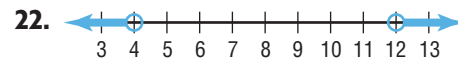
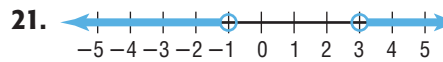
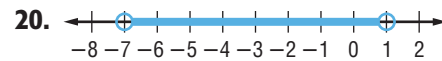
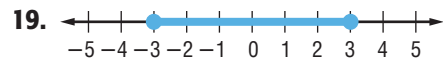
Solve each open sentence. Then graph the solution set.

8.  $|z - 2| \leq 5$       9.  $|t + 8| < 2$       10.  $|6 - d| \leq -4$   
 11.  $|v + 3| > 1$       12.  $|w - 6| \geq 3$       13.  $|3a - 9| > -2$   
 14.  $|3k + 4| \geq 8$       15.  $|2n + 1| < 9$       16.  $|4q + 7| \leq -13$

17. **SCUBA DIVING** The pressure of a typical scuba tank should be within 500 pounds per square inch (psi) of 2500 psi. Write the range of optimum pressures for scuba tanks.

18. **ANIMALS** A sheep's normal body temperature is  $39^\circ\text{C}$ . However, a healthy sheep may have body temperatures  $1^\circ\text{C}$  above or below this temperature. What is the range of body temperatures for a sheep?

Write an open sentence involving absolute value for each graph.



Solve each open sentence. Then graph the solution set.

23.  $\left| \frac{5h + 2}{6} \right| = 7$       24.  $\left| \frac{2 - 3x}{5} \right| \geq 2$   
 25.  $|3s + 2| > -7$       26.  $|6r + 8| < -4$

Express each statement using an inequality involving absolute value. Do *not* solve.

27. The pH of a swimming pool must be within 0.3 of a pH of 7.5.  
 28. The temperature inside a refrigerator should be within 1.5 degrees of  $38^\circ\text{F}$ .  
 29. Ramona's bowling score was within 6 points of her average score of 98.  
 30. The cruise control of a car set at 55 miles per hour should keep the speed within 3 miles per hour of 55.

31. **DRIVING** Tires should be kept within 2 pounds per square inch (psi) of the manufacturer's recommended tire pressure. If the recommendation for a tire is 30 psi, what is the range of acceptable pressures?

32. **PHYSICAL SCIENCE** Li-Cheng must add 3.0 milliliters of sodium chloride to a solution. The sodium chloride must be within 0.5 milliliter of the required amount. How much sodium chloride can she add and obtain the correct results?

33. **MINIATURE GOLF** Ginger played miniature golf. Her score was within 5 strokes of her average score of 52. Determine the range of scores for Ginger's game.

34. **MUSIC DOWNLOADS** Carlos is allowed to download \$10 worth of music each month. This month he has spent within \$3 of his allowance. What is the range of money he has spent on music downloads this month?



### Real-World Link

Always inflate your tires to the pressure that is recommended by the manufacturer. The pressure stamped on the tire is the *maximum* pressure and should only be used under certain circumstances.

Source: [www.etires.com](http://www.etires.com)

### EXTRA PRACTICE

See pages 730, 749.

Math  

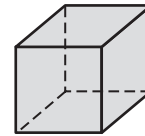
Self-Check Quiz at [algebra1.com](http://algebra1.com)

**H.O.T. Problems**

- 35. REASONING** Compare and contrast the solution of  $|x - 2| > 6$  and the solution of  $|x - 2| < 6$ .
- 36. OPEN ENDED** Formulate an absolute value inequality to represent a real-world situation and graph its solution set. Interpret the solution.
- 37. CHALLENGE** Translate the inequality  $x < 2 \pm 0.3$  into an absolute value inequality.
- 38. Writing in Math** Refer to the information on page 329. Describe how the definition of absolute value can be applied to manufacturing baby carrots. Write an absolute value inequality to represent the range of lengths for a baby carrot.

**STANDARDIZED TEST PRACTICE**

- 39.** What is the solution to the inequality  $-6 < |x| < 6$ ?
- A  $-x \geq 0$                       C  $-x < 6$   
 B  $x \leq 0$                          D  $-x > 6$
- 40.** Which inequality *best* represents the statement below?  
*A jar contains 832 gumballs. Amanda's guess was within 46 pieces.*
- F  $|g - 832| \leq 46$      H  $|g - 832| \geq 46$   
 G  $|g + 832| \leq 46$      J  $|g + 832| \geq 46$
- 41. REVIEW** An 84-centimeter piece of wire is cut into equal segments and then attached at the ends to form the edges of a cube. What is the volume of the cube?



- A  $294 \text{ cm}^3$                       C  $1158 \text{ cm}^3$   
 B  $343 \text{ cm}^3$                       D  $2744 \text{ cm}^3$

**Spiral Review**

Solve each open sentence. Then graph the solution set. (Lesson 6-5)

42.  $|x + 3| = 5$                       43.  $|2x + 3| = -4$                       44.  $|3x - 2| = 4$

- 45. SHOPPING** A catalog company varies the costs to ship merchandise based on the amount of the order. The cost for shipping is shown in the table. Write a compound inequality for each shipping cost. (Lesson 6-4)

Shipping Costs	
Merchandise	Shipping
\$0-\$25	\$5
\$25.01-\$50	\$8

Write an equation in slope-intercept form of the line with the given slope and *y*-intercept. (Lesson 4-3)

46. slope:  $-3$ , *y*-intercept:  $4$                       47. slope:  $\frac{1}{2}$ , *y*-intercept:  $\frac{3}{4}$

**GET READY for the Next Lesson**

**PREREQUISITE SKILL** Graph each equation. (Lesson 3-3)

48.  $y = 3x + 4$                       49.  $x + y = 3$                       50.  $y - 2x = -1$                       51.  $2y - x = -6$

# Graphing Inequalities in Two Variables

## Main Ideas

- Graph inequalities on the coordinate plane.
- Solve real-world problems involving linear inequalities.

## New Vocabulary

half-plane  
boundary

### GET READY for the Lesson

Hannah budgets \$30 a month for lunch. On most days, she brings her lunch. She can also buy lunch at the cafeteria or at a fast-food restaurant. She spends an average of \$3 for lunch at the cafeteria and an average of \$4 for lunch at a restaurant. How many times a month can Hannah buy her lunch and remain within her budget?

My Monthly Budget	
Lunch (school days)	\$30
Entertainment	\$55
Clothes	\$50
Fuel	\$60



<b>Words</b>	The cost of eating in the cafeteria	plus	the cost of eating in a restaurant	is less than or equal to	\$30.
<b>Variables</b>	Let $x$ = the number of days she buys lunch at the cafeteria.		Let $y$ = the number of days she buys lunch at a restaurant.		
<b>Inequality</b>	$3x$	+	$4y$	$\leq$	30

There are many solutions for this inequality. Each solution represents a different combination of lunches bought in the cafeteria and in a restaurant.

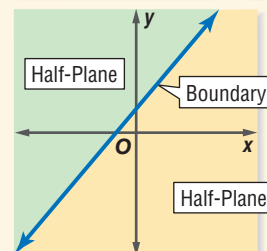
**Graph Linear Inequalities** The solution set for an inequality in two variables contains many ordered pairs when the domain and range are the set of real numbers. The graphs of all of these ordered pairs fill a region on the coordinate plane called a **half-plane**. An equation defines the **boundary** or edge for each half-plane.

### KEY CONCEPT

### Half-Planes and Boundaries

**Words** Any line in the plane divides the plane into two regions called half-planes. The line is called the boundary of each of the two half-planes.

**Model**



The boundary may or may not be included in the graph of an inequality. Graphing the boundary is the first step in graphing a linear inequality.

## Study Tip

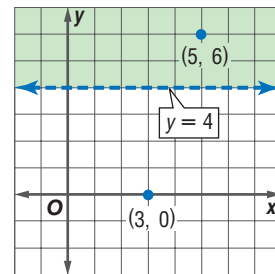
### Dashed Line

Like a circle on a number line, a dashed line on a coordinate plane indicates that the boundary is not part of the solution set.

### Solid Line

Like a dot on a number line, a solid line on a coordinate plane indicates that the boundary is included.

Consider the graph of  $y > 4$ . First determine the boundary by graphing  $y = 4$ , the equation you obtain by replacing the inequality sign with an equals sign. Since the inequality involves  $y$ -values greater than 4, but not equal to 4, the line should be dashed. The boundary divides the coordinate plane into two half-planes.



To determine which half-plane contains the solution, choose a point from each half-plane and test it in the inequality.

Try (3, 0).

Try (5, 6).

$$y > 4 \quad y = 0$$

$$y > 4 \quad y = 6$$

$$0 > 4 \quad \text{false}$$

$$6 > 4 \quad \text{true}$$

The half-plane that contains (5, 6) contains the solution. Shade that half-plane.

## EXAMPLE Graph an Inequality

### 1 Graph $y - 2x \leq -4$ .

**Step 1** Solve for  $y$  in terms of  $x$ .

$$y - 2x \leq -4 \quad \text{Original inequality}$$

$$y - 2x + 2x \leq -4 + 2x \quad \text{Add } 2x \text{ to each side.}$$

$$y \leq 2x - 4 \quad \text{Simplify.}$$

**Step 2** Graph  $y = 2x - 4$ . Since  $y \leq 2x - 4$  means  $y < 2x - 4$  or  $y = 2x - 4$ , the boundary is included in the solution set. The boundary should be drawn as a solid line.

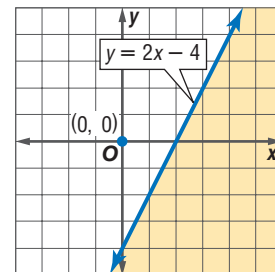
**Step 3** Select a point in one of the half-planes and test it. Let's use (0, 0).

$$y - 2x \leq -4 \quad \text{Original inequality}$$

$$0 - 2(0) \leq -4 \quad x = 0, y = 0$$

$$0 \leq -4 \quad \text{false}$$

Since the statement is false, the half-plane containing the origin is not part of the solution. Shade the other half-plane.



**CHECK** Test a point in the other half plane, for example, (3, -3).

$$y - 2x \leq -4 \quad \text{Original inequality}$$

$$-3 - 2(3) \leq -4 \quad x = 3, y = -3$$

$$-9 \leq -4 \quad \checkmark \quad \text{Simplify.}$$

Since the statement is true, the half-plane containing (3, -3) should be shaded. The graph of the solution is correct.

## CHECK Your Progress Graph each inequality.

1A.  $x \leq -1$

1B.  $y > \frac{1}{2}x + 3$

## Study Tip

### Origin as the Test Point

Use the origin as a standard test point because the values often simplify calculations.



**Solve Real-World Problems** When solving real-world inequalities, the domain and range of the inequality are often restricted to nonnegative numbers or whole numbers.

**Real-World EXAMPLE Write and Solve an Inequality**

**2 ADVERTISING** Rosa Padilla sells radio advertising in 30-second and 60-second time slots. During every hour, there are up to 15 minutes available for commercials. How many commercial slots can she sell for one hour of broadcasting?

**Explore** You know the length of the time slots in seconds and the number of minutes each hour available for commercials.

**Plan** Let  $x$  = the number of 30-second commercials. Let  $y$  = the number of 60-second or 1-minute commercials. Write an open sentence representing this situation.

$$\frac{1}{2} \text{ min times } \overset{\text{the number of}}{30\text{-s commercials}} \text{ plus } 1 \text{ min times } \overset{\text{the number of}}{1\text{-min commercials}} \text{ is up to } 15 \text{ min.}$$

$$\frac{1}{2} \cdot x + 1 \cdot y \leq 15$$

**Solve** Solve for  $y$  in terms of  $x$ .

$$\frac{1}{2}x + y \leq 15 \quad \text{Original inequality}$$

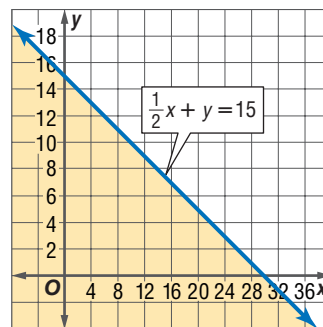
$$\frac{1}{2}x + y - \frac{1}{2}x \leq 15 - \frac{1}{2}x \quad \text{Subtract } \frac{1}{2}x \text{ from each side.}$$

$$y \leq 15 - \frac{1}{2}x \quad \text{Simplify.}$$

Since the open sentence includes the equation, graph  $y = 15 - \frac{1}{2}x$  as a solid

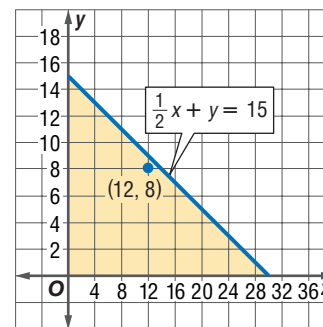
line. Test a point in one of the half-planes, for example  $(0, 0)$ . Shade the half-plane containing  $(0, 0)$  since

$$0 \leq 15 - \frac{1}{2}(0) \text{ is true.}$$



**Check** Examine the solution.

- Rosa cannot sell a negative number of commercials. Therefore, the domain and range contain only nonnegative numbers.
- She also cannot sell half of a commercial. Thus, only points in the shaded half-plane with  $x$ - and  $y$ -coordinates that are whole numbers are possible solutions.



One solution is  $(12, 8)$ . This represents twelve 30-second commercials and eight 60-second commercials in a one hour period.



**Real-World Link**

A typical one-hour program on television contains 40 minutes of the program and 20 minutes of commercials. During peak periods, a 30-second commercial can cost an average of \$2.3 million.

Source: [www.superbowl-ads.com](http://www.superbowl-ads.com)

## CHECK Your Progress

2. **MARATHONS** Neil wants to run a marathon at a pace of at least 6 miles per hour. Write an inequality for the miles  $y$  he will run in  $x$  hours and graph the solution set.



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## CHECK Your Understanding

### Example 1 (p. 335)

Graph each inequality.

1.  $y \geq 4$

2.  $y \leq 2x - 3$

3.  $y > x + 3$

4.  $4 - 2x < -2$

5.  $1 - y > x$

6.  $x + 2y \leq 5$

### Example 2 (pp. 336–337)

7. **ENTERTAINMENT** Coach Washington wants to take her softball team out for pizza and soft drinks after the last game of the season. She doesn't want to spend more than \$60. Write an inequality that represents this situation and graph the solution set.



## Exercises

HOMEWORK HELP	
For Exercises	See Examples
8–17	1
18–21	2

Graph each inequality.

8.  $y < -3$

9.  $x \geq 2$

10.  $5x + 10y > 0$

11.  $y < x$

12.  $2y - x \leq 6$

13.  $6x + 3y > 9$

14.  $3y - 4x \geq 12$

15.  $y \leq -2x - 4$

16.  $8x - 6y < 10$

17.  $3x - 1 \geq y$

**POSTAGE** For Exercises 18 and 19, use the following information.

The U.S. Postal Service limits the size of packages. The length of the longest side plus the distance around the thickest part must be less than or equal to 108 inches.

18. Write an inequality that represents this situation.  
19. Are there any restrictions on the domain or range? Explain.

**ANALYZE TABLES** For Exercises 20–22, use the table.

A delivery truck with a 4000-pound weight limit is transporting televisions and microwaves.

Item	Weight (lb)
television	77
microwave	55

20. Define variables and write an inequality for this situation.  
21. Will the truck be able to deliver 35 televisions and 25 microwaves at once?  
22. Write two possible solutions to the inequality. Are there solutions that make the inequality mathematically true, but are not reasonable in the context of the problem? Explain.

**Cross-Curricular Project**

**Math** A linear inequality can be used to represent trends in Olympic times. Visit [algebra1.com](http://algebra1.com) to continue work on your project.

Determine which ordered pairs are part of the solution set for each inequality.

23.  $y \leq 3 - 2x$ ,  $\{(0, 4), (-1, 3), (6, -8), (-4, 5)\}$

24.  $y < 3x$ ,  $\{(-3, 1), (-3, 2), (1, 1), (1, 2)\}$

25.  $x + y < 11$ ,  $\{(5, 7), (-13, 10), (4, 4), (-6, -2)\}$

26.  $2x - 3y > 6$ ,  $\{(3, 2), (-2, -4), (6, 2), (5, 1)\}$

Match each inequality with its graph.

27.  $2y + x \leq 6$

28.  $\frac{1}{2}x - y > 4$

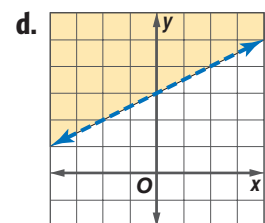
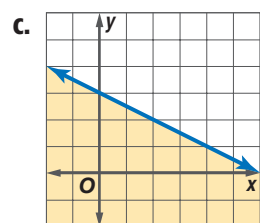
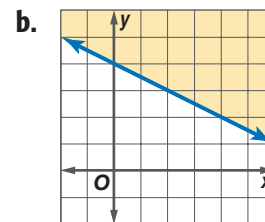
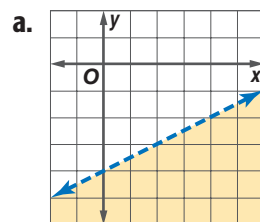
29.  $y > 3 + \frac{1}{2}x$

30.  $4y + 2x \geq 16$

Determine which ordered pairs are part of the solution set for each inequality.

31.  $|x - 3| \geq y$ ,  $\{(6, 4), (-1, 8), (-3, 2), (5, 7)\}$

32.  $|y + 2| < x$ ,  $\{(2, -4), (-1, -5), (6, -7), (0, 0)\}$



Graph each inequality.

33.  $3(x + 2y) > -18$

34.  $\frac{1}{2}(2x + y) < 2$

**MUSEUMS** For Exercises 35 and 36, use the table at the right.

A Cub Scout troop plans to visit a flight museum. The troop leaders can spend up to \$96 on admission.

Flight Museum	
Visitor	Admission Cost
adult	\$8.00
children 6–12 years old	\$4.00

35. Write an inequality for this situation.

36. Will the entire troop be able to go to the museum if there are 3 adults and 16 Cub Scouts who are all under 12 years old? Explain.


37. **REASONING** Compare and contrast the graph of  $y = x + 2$  and the graph of  $y < x + 2$ .

38. **OPEN ENDED** Create a linear inequality in two variables and graph it.

39. **REASONING** Explain why it is usually only necessary to test one point when graphing an inequality.

40. **CHALLENGE** Graph the intersection of the graphs of  $y \leq x - 1$  and  $y \geq -x$ .

41. *Writing in Math* Use the information about budgets on page 334 to explain how inequalities are used in finances. Include an explanation of the restrictions placed on the domain and range of the inequality that describes the number of times Hannah can buy lunch. Describe three possible solutions of the inequality.

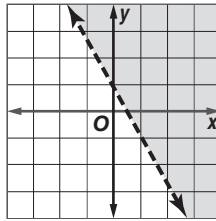
**EXTRA PRACTICE**  
See pages 730, 749.  
**Math**   
Self-Check Quiz at [algebra1.com](http://algebra1.com)

**H.O.T. Problems**



## STANDARDIZED TEST PRACTICE

42. Which inequality is shown on the graph at the right?



- A  $2x + y < 1$   
 B  $2x + y > 1$   
 C  $2x + y \leq 1$   
 D  $2x + y \geq 1$

43. **REVIEW** The perimeters of two similar polygons are 250 centimeters and 300 centimeters, respectively. What is the scale factor of the two polygons?

- F  $\frac{5}{6}$   
 G  $\frac{3}{4}$   
 H  $\frac{1}{2}$   
 J  $\frac{1}{4}$

## Spiral Review

**BIOLOGY** For Exercises 44 and 45, use the following information.

The *average* length of a human pregnancy is 280 days. However, a healthy, full-term pregnancy can be 14 days longer or shorter. (Lesson 6-6)

44. If  $d$  is the length in days, write an absolute value inequality for the length of a full-term pregnancy.  
 45. Solve the inequality for the length of a full-term pregnancy.

Solve each open sentence. Then graph the solution. (Lesson 6-4)

46.  $|y + 0.5| = 6.5$

47.  $|m - 0.5| = 2.5$

Write an equation in slope-intercept form of the line that passes through the given point and is parallel to the graph of each equation. (Lesson 4-7)

48.  $(1, -3); y = 3x - 2$

49.  $(0, 4); x + y = -3$

50.  $(-1, 2); 2x - y = 1$

Find the next two terms in each sequence. (Lesson 3-4)

51. 7, 13, 19, 25, ...

52. 243, 81, 27, 9, ...

53. 3, 6, 12, 24, ...

State whether each percent of change is a percent of *increase* or *decrease*.

Then find the percent of change. Round to the nearest whole percent. (Lesson 2-7)

54. original: 200  
new: 172

55. original: 100  
new: 142

56. original: 53  
new: 75

## GET READY for the Next Lesson

**PREREQUISITE SKILL** Graph each equation. (Lesson 4-3)

57.  $y = 3x + 1$

58.  $x - y = -4$

59.  $5x + 2y = 6$



# Graphing Calculator Lab

## Graphing Inequalities

To graph inequalities, graphing calculators shade between two functions. Enter a lower boundary as well as an upper boundary for each inequality.

### ACTIVITY 1 Graph two different inequalities on your graphing calculator.

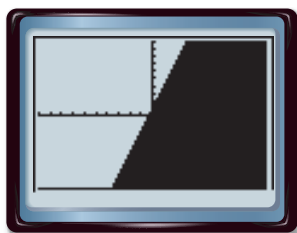
**Step 1** Graph  $y \leq 3x + 1$ .

- Clear all functions from the Y= list.

KEYSTROKES:  $\boxed{Y=}$   $\boxed{\text{CLEAR}}$

- Graph  $y \leq 3x + 1$  in the standard window.

KEYSTROKES:  $\boxed{2\text{nd}}$   $\boxed{[\text{DRAW}]}$   $\boxed{7}$   $\boxed{-10}$   $\boxed{,}$   $\boxed{3}$   
 $\boxed{X,T,\theta,n}$   $\boxed{+}$   $\boxed{1}$   $\boxed{)}$   $\boxed{\text{ENTER}}$



$[-10, 10]$  scl: 1 by  $[-10, 10]$  scl: 1

The lower boundary is Ymin or  $-10$ . The upper boundary is  $y = 3x + 1$ . All ordered pairs for which  $y$  is *less than or equal to*  $3x + 1$  lie *below or on* the line and are solutions.

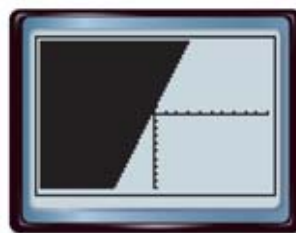
**Step 2** Graph  $y - 3x \geq 1$ .

- Clear the drawing that is displayed.

KEYSTROKES:  $\boxed{2\text{nd}}$   $\boxed{[\text{DRAW}]}$   $\boxed{1}$

- Rewrite  $y - 3x \geq 1$  as  $y \geq 3x + 1$  and graph it.

KEYSTROKES:  $\boxed{2\text{nd}}$   $\boxed{[\text{DRAW}]}$   $\boxed{7}$   $\boxed{3}$   $\boxed{X,T,\theta,n}$   $\boxed{+}$   $\boxed{1}$   
 $\boxed{,}$   $\boxed{10}$   $\boxed{)}$   $\boxed{\text{ENTER}}$



$[-10, 10]$  scl: 1 by  $[-10, 10]$  scl: 1

The lower boundary is  $y = 3x + 1$ . The upper boundary is Ymax or  $10$ . All ordered pairs for which  $y$  is *greater than or equal to*  $3x + 1$  lie *above or on* the line and are solutions.

### EXERCISES

1. Compare and contrast the two graphs shown above.
2. Graph the inequality  $y \geq -2x + 4$  in the standard viewing window.
  - a. What functions do you enter as the lower and upper boundaries?
  - b. Using your graph, name four solutions of the inequality.
3. Suppose student movie tickets cost \$4 and adult movie tickets cost \$8. You would like to buy at least 10 tickets, but spend no more than \$80.
  - a. Let  $x$  = number of student tickets and  $y$  = number of adult tickets. Write two inequalities, one representing the total number of tickets and the other representing the total cost of the tickets.
  - b. Which inequalities would you use as the lower and upper boundaries?
  - c. Graph the inequalities. Use the viewing window  $[0, 20]$  scl: 1 by  $[0, 20]$  scl: 1.
  - d. Name four possible combinations of student and adult tickets.

# Graphing Systems of Inequalities

## Main Ideas

- Solve systems of inequalities by graphing.
- Solve real-world problems involving systems of inequalities.

## New Vocabulary

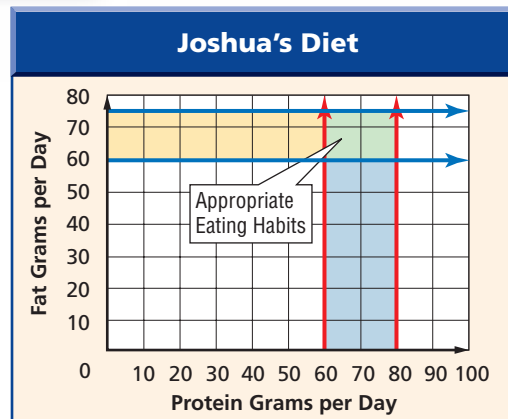
system of inequalities

## GET READY for the Lesson

Joshua's doctor recommends the following.

- Get between 60 and 80 grams of protein per day.
- Keep daily fat intake between 60 and 75 grams.

The green section of the graph indicates the appropriate amounts of protein and fat for Joshua.



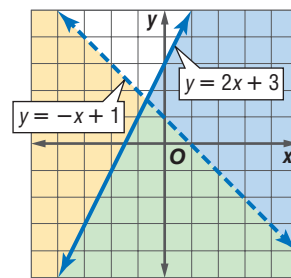
**Systems of Inequalities** A **system of inequalities** is a set of two or more inequalities with the same variables. To solve a system of inequalities like the one above, find the ordered pairs that satisfy all the inequalities. The solution set is represented by the intersection, or overlap, of the graphs.

## EXAMPLE Solve by Graphing

1 Solve each system of inequalities by graphing.

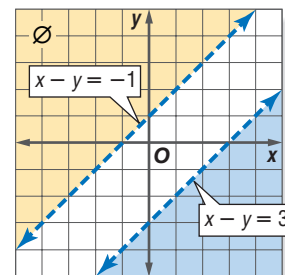
a.  $y < -x + 1$   
 $y \leq 2x + 3$

The solution includes the ordered pairs in the intersection of the graphs of  $y < -x + 1$  and  $y \leq 2x + 3$ . This region is shaded in green at the right. The graph of  $y = -x + 1$  is dashed and is *not* included in the graph of  $y < -x + 1$ . The graph of  $y = 2x + 3$  is solid and *is* included in the graph of  $y \leq 2x + 3$ .



b.  $x - y < -1$   
 $x - y > 3$

The graphs of  $x - y = -1$  and  $x - y = 3$  are parallel lines. Because the two regions have no points in common, the system of inequalities has no solution.



Concepts in Motion  
 Animation [algebra1.com](http://algebra1.com)

## CHECK Your Progress

1A.  $y \leq 3$   
 $x + y \geq 1$

1B.  $2x + y \geq 2$   
 $2x + y < 4$

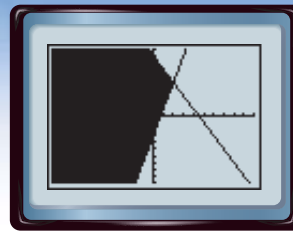
Personal Tutor at [algebra1.com](http://algebra1.com)

You can use a TI-83/84 Plus to solve systems of inequalities.

## GRAPHING CALCULATOR LAB

### Graphing Systems of Inequalities

To graph the system  $y \geq 4x - 3$  and  $y \leq -2x + 9$  on a TI-83/4 Plus, select the **SHADE** feature in the **DRAW** menu. Enter the function that is the lower boundary of the region to be shaded, followed by the upper boundary. (Note that inequalities that have  $>$  or  $\geq$  are lower boundaries and inequalities that have  $<$  or  $\leq$  are upper boundaries.)



$[-10, 10]$  scl: 1 by  $[-10, 10]$  scl: 1

### THINK AND DISCUSS

- To graph the system  $y \leq 3x + 1$  and  $y \geq -2x - 5$  on a graphing calculator, which function should you enter first?
- Use a graphing calculator to graph the system  $y \leq 3x + 1$  and  $y \geq -2x - 5$ .
- Explain how you could use a graphing calculator to graph the system  $2x + y \geq 7$  and  $x - 2y \geq 5$ . Then use a graphing calculator to graph the system.

**Real-World Problems** In real-world problems involving systems of inequalities, sometimes only whole-number solutions make sense.



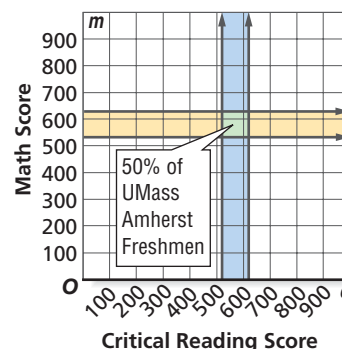
### Real-World EXAMPLE Use a System of Inequalities

- COLLEGE** The middle 50% of first-year students attending the University of Massachusetts at Amherst scored between 520 and 630, inclusive, on the math portion of the SAT. They scored between 510 and 620, inclusive, on the critical reading portion of the test. Graph the scores that a student would need to be in the middle 50% of first year students.

**Words** The math score is between 520 and 630, inclusive. The critical reading score is between 510 and 620, inclusive.

**Variables** Let  $m$  = the math score and let  $c$  = the critical reading score.

**Inequalities**  $520 \leq m \leq 630$   
 $510 \leq c \leq 620$



The solution is the set of all ordered pairs that are in the intersection of the graphs of these inequalities. However, since SAT scores are whole numbers, only whole-number solutions make sense in this problem.



### CHECK Your Progress

- HEALTH** The LDL or “bad” cholesterol of a teenager should be less than 110. The HDL or “good” cholesterol of a teenager should be between 35 and 59. Make a graph showing appropriate levels of cholesterol for a teenager.



### Real-World Link

The average first-year student at University of Massachusetts Amherst had a high school GPA of 3.38.

Source: [umass.edu/oir](http://umass.edu/oir)

## CHECK Your Understanding

**Example 1**  
(p. 341)

Solve each system of inequalities by graphing.

- |  |   |
|--|---|
| <p>1. <math>x &gt; 5</math><br/><math>y \leq 4</math></p> <p>3. <math>y \leq -x + 3</math><br/><math>y \leq x + 3</math></p> | <p>2. <math>y &gt; 3</math><br/><math>y &gt; -x + 4</math></p> <p>4. <math>2x + y \geq 4</math><br/><math>y \leq -2x - 1</math></p> |
|--|---|

**Example 2**  
(p. 342)

**HEALTH** For Exercises 5 and 6, use the following information.

Natasha exercises every day by walking and jogging at least 3 miles. Natasha walks at a rate of 4 miles per hour and jogs at a rate of 8 miles per hour. Suppose she only has a half-hour to exercise today.

5. Draw a graph showing the possible amount of time she can spend walking and jogging.
6. List three possible solutions.

## Exercises

HOMEWORK HELP	
For Exercises	See Examples
7–18	1
19–22	2

Solve each system of inequalities by graphing.

- |  |   |  |
|--|---|--|
| <p>7. <math>y &lt; 0</math><br/><math>x \geq 0</math></p> <p>10. <math>x \geq 2</math><br/><math>y + x \leq 5</math></p> <p>13. <math>y &lt; 2x + 1</math><br/><math>y \geq -x + 3</math></p> <p>16. <math>2x + y \leq 4</math><br/><math>3x - y \geq 6</math></p> | <p>8. <math>x &gt; -4</math><br/><math>y \leq -1</math></p> <p>11. <math>x \leq 3</math><br/><math>x + y &gt; 2</math></p> <p>14. <math>y - x &lt; 1</math><br/><math>y - x &gt; 3</math></p> <p>17. <math>3x - 4y &lt; 1</math><br/><math>x + 2y \leq 7</math></p> | <p>9. <math>y \geq -2</math><br/><math>y - x &lt; 1</math></p> <p>12. <math>y \geq 2x + 1</math><br/><math>y \leq -x + 1</math></p> <p>15. <math>y - x &lt; 3</math><br/><math>y - x \geq 2</math></p> <p>18. <math>x + y &gt; 4</math><br/><math>-2x + 3y &lt; -12</math></p> |
|--|---|--|

**MANUFACTURING** For Exercises 19 and 20, use the following information.

The Natural Wood Company has machines that sand and varnish desks and tables. The table below gives the time requirements of the machines.

Machine	Hours per Desk	Hours per Table	Total Hours Available Each Week
Sanding	2	1.5	31
Varnishing	1.5	1	22

19. Make a graph showing the number of desks and the number of tables that can be made in a week.
20. List three possible solutions.

**ART** For Exercises 21 and 22, use the following information.

A painter has 32 units of yellow dye and 54 units of blue dye for mixing to make two shades of green. The units needed to make a gallon of light green and a gallon of dark are shown in the table.

Color	Units of Yellow Dye	Units of Blue Dye
light green	4	1
dark green	1	6

21. Make a graph showing the numbers of gallons of the two greens she can make.
22. List three possible solutions.

**Real-World Career**

**Visual Artist**

A visual artist uses math to create art to communicate ideas. The work of fine artists is made for display. Illustrators and graphic designers produce art for clients.



For more information, go to [algebra1.com](http://algebra1.com).

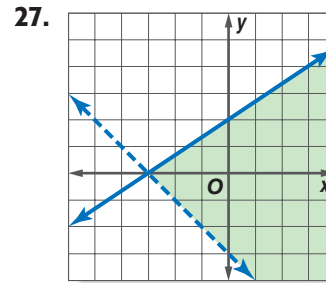
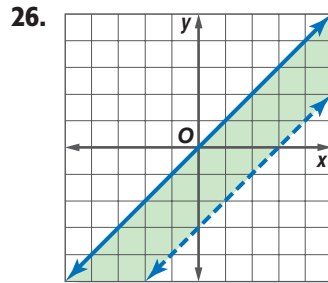
Solve each system of inequalities by graphing.

23.  $x \geq 0$   
 $x - 2y \leq 2$   
 $3x + 4y \leq 12$

24.  $y \leq x + 3$   
 $2x - 7y \leq 4$   
 $3x + 2y \leq 6$

25.  $x < 2$   
 $4y > x$   
 $2x - y < -9$   
 $x + 3y < 9$

Write a system of inequalities for each graph.



**EXTRA PRACTICE**  
 See pages 730, 749.  
**Math**  
 Self-Check Quiz at [algebra1.com](http://algebra1.com)

**AGRICULTURE** For Exercises 28 and 29, use the following information.

To ensure a growing season of sufficient length, Mr. Hobson has at most 16 days left to plant his corn and soybean crops. He can plant corn at a rate of 250 acres per day and soybeans at a rate of 200 acres per day.

28. If he has at most 3500 acres available, make a graph showing how many acres of each type of crop he can plant.  
 29. Name one solution and explain what it means.



**Graphing Calculator**

Use a graphing calculator to solve each system of inequalities.

30.  $y \leq x + 9$   
 $y \geq -x - 4$

31.  $y \leq 2x + 10$   
 $y \geq 7x + 15$

32.  $3x - y \leq 6$   
 $x - y \geq -1$

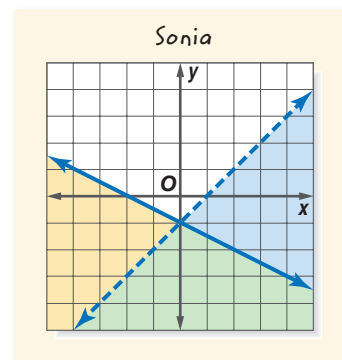
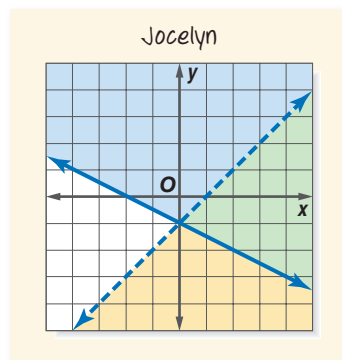
Sketch the region in the plane that satisfies both inequalities.

33.  $3y - x \geq 6$   
 $y < -2x - 1$

34.  $3y - x \leq 9$   
 $4y + x \leq 12$

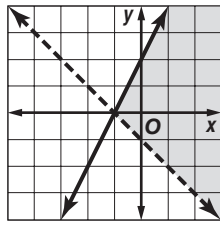
**H.O.T. Problems**

35. **OPEN ENDED** Draw the graph of a system of inequalities that has no solution.  
 36. **CHALLENGE** Create a system of inequalities equivalent to  $|x| \leq 4$ .  
 37. **FIND THE ERROR** Jocelyn and Sonia are solving the system of inequalities  $x + 2y \geq -2$  and  $x - y > 1$ . Who is correct? Explain your reasoning.



38. **Writing in Math** Use the information about nutrition on page 341 to explain how you can use a system of inequalities to plan a sensible diet. Include two appropriate Calorie and fat intakes for a day and the system of inequalities that is represented by the graph.

39. Which system of inequalities is *best* represented by the graph?



- A  $y \leq 2x + 2$   
 $y > -x - 1$
- B  $y \geq 2x + 2$   
 $y < -x - 1$
- C  $y < 2x + 2$   
 $y \leq -x - 1$
- D  $y > 2x + 2$   
 $y \leq -x - 1$

40. **REVIEW** The table shows the cost of organic wheat flour, depending on the amount purchased. Which conclusion can be made based on the information in the table?

Number of Pounds	Cost (\$)
2	3.40
10	16.00
25	37.50
50	75.00

- F The cost of 4 pounds of flour would be more than \$7.
- G The cost of 100 pounds of flour would be less than \$150.
- H The cost of flour is always more than \$1.60 per pound.
- J The cost of flour is always less than \$1.50 per pound.

### Spiral Review

Graph each inequality. (Lesson 6-6)

41.  $y < \frac{1}{2}x - 4$

42.  $2y + x \leq 6$

43.  $x + 2y \geq 8$

44. **FISH** The temperature of a freshwater tropical fish tank should be within 1.5 degrees of 76.5°F. Express this using an inequality involving absolute value. (Lesson 6-6)

Use elimination to solve each system of equations. (Lesson 5-4)

45.  $2x + 3y = 1$   
 $4x - 5y = 13$

46.  $5x - 2y = -3$   
 $3x + 6y = -9$

47.  $-3x + 2y = 12$   
 $2x - 3y = -13$

48.  $6x - 2y = 4$   
 $5x - 3y = -2$

Determine whether each relation is a function. (Lesson 3-2)

49.  $y = -15$

50.  $x = 5$

51.  $\{(1, 0), (1, 4), (-1, 1)\}$

52.  $\{(6, 3), (5, -2), (2, 3)\}$

### Cross-Curricular Project

#### Math and Science

**The Spirit of the Games** It's time to complete your project. Use the information and data you have gathered about the Olympics to prepare a portfolio or Web page. Be sure to include graphs and/or tables in the presentation.

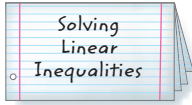


Cross-Curricular Project at [algebra1.com](http://algebra1.com)

FOLDABLES™  
Study Organizer

## GET READY to Study

Be sure the following  
Key Concepts are noted  
in your Foldable.

**Key Concepts****Solving Inequalities by Adding,  
Subtracting, Multiplying,  
or Dividing** (Lessons 6-1 and 6-2)

- If any number is added to or subtracted from each side of a true inequality, the resulting inequality is also true.
- If each side of a true inequality is multiplied or divided by the same positive number, the resulting inequality is also true.

**Multi-Step and Compound  
Inequalities** (Lessons 6-3 and 6-4)

- If each side of a true inequality is multiplied or divided by the same negative number, the direction of the inequality symbol must be *reversed* so that the resulting inequality is also true.

**Absolute Value Equations and  
Inequalities** (Lessons 6-5 and 6-6)

- The absolute value of any number  $n$  is its distance from zero on a number line and is written as  $|n|$ .
- If  $|x| = n$ , then  $x = -n$  or  $x = n$ .  
If  $|x| < n$ , then  $x < n$  or  $x > -n$ .  
If  $|x| > n$ , then  $x > n$  or  $x < -n$ .

**Inequalities in Two Variables** (Lesson 6-7)

- Any line in the plane divides the plane into two regions called *half-planes*. The line is called the boundary of each of the two half-planes.

**Systems of Inequalities** (Lesson 6-8)

- A system of inequalities is a set of two or more inequalities with the same variables.

**Key Vocabulary**

- boundary (p. 334)
- compound inequality (p. 315)
- half-plane (p. 334)
- intersection (p. 315)
- piecewise function (p. 324)
- set-builder notation (p. 295)
- system of inequalities (p. 341)
- union (p. 316)

**Vocabulary Check**

State whether each sentence is *true* or *false*.  
If *false*, replace the underlined word or  
phrase to make a true sentence.

1. The edge of a half-plane is called a boundary.
2. The symbol  $\emptyset$  means intersection.
3. The phrase at least is represented by the same symbol as the phrase greater than.
4. To solve a system of inequalities, find the ordered pairs that satisfy all the inequalities involved.
5. The union can be found by graphing each inequality and then determining where the graphs overlap.
6. The solution  $\{x \mid x < 5\}$  is written in set-builder notation.
7. A compound inequality containing and is true if one or more of the inequalities is true.
8. When solving  $4x > -12$ , the direction of the inequality symbol should be reversed.
9. The graph of  $y > 3x - 6$  is a dashed line.
10. A union is formed when a line in the plane divides the plane into two regions.



## Lesson-by-Lesson Review

6-1

### Solving Inequalities by Addition and Subtraction (pp. 298-303)

Solve each inequality. Check your solution, and then graph it on a number line.

11.  $x - 9 < 16$       12.  $-11 \geq -5 + p$

13.  $12w + 4 \leq 13w$       14.  $8g > 7g - 1$

For Exercises 15 and 16, define a variable, write an inequality, and solve each problem. Check your solution.

15. Sixteen is less than the sum of a number and 31.

16. **TOMATOES** There are more than 10,000 varieties of tomatoes. One seed company produces seed packages for 200 varieties of tomatoes. For how many varieties do they not provide seeds?

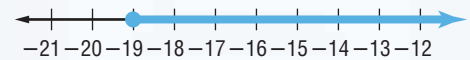
**Example 1** Solve  $-2 \leq h + 17$ . Check your solution, and then graph it on a number line.

$$-2 \leq h + 17 \quad \text{Original inequality}$$

$$-2 - 17 \leq h + 17 - 17 \quad \text{Subtract 17 from each side.}$$

$$-19 \leq h \quad \text{Simplify.}$$

Since  $-19 \leq h$  is the same as  $h \geq -19$ , the solution set is  $\{h \mid h \geq -19\}$ .



6-2

### Solving Inequalities by Multiplication and Division (pp. 305-311)

Solve each inequality. Check your solution.

17.  $15v > 60$       18.  $3 \leq -\frac{d}{13}$

19.  $-9m < 99$       20.  $-15 \geq \frac{3}{5}k$

For Exercises 21 and 22, define a variable, write an inequality, and solve the problem. Check your solution.

21. Eighty percent of a number is greater than or equal to 24.

22. **FISHING** About 41.6 million tons of fish were caught in China in a recent year. If this is over 35% of the world's catch, how many fish were caught in the world that year?

**Example 2** Solve  $-14g \geq 126$ .

$$-14g \geq 126 \quad \text{Original inequality}$$

$$\frac{-14g}{-14} \leq \frac{126}{-14} \quad \text{Divide each side by } -14 \text{ and change } \geq \text{ to } \leq.$$

$$g \leq -9 \quad \text{Simplify.}$$

The solution set is  $\{g \mid g \leq -9\}$ .

**Example 3** Solve  $\frac{3}{4}w < 15$ .

$$\frac{3}{4}w < 15 \quad \text{Original inequality}$$

$$\left(\frac{4}{3}\right)\frac{3}{4}w < \left(\frac{4}{3}\right)15 \quad \text{Multiply each side by } \frac{4}{3}.$$

$$w < 20 \quad \text{Simplify.}$$

The solution set is  $\{w \mid w < 20\}$ .



**6-3 Solving Multi-Step Inequalities** (pp. 312–317)

Solve each inequality. Check your solution.

23.  $5 - 6y > -19$

24.  $\frac{1-7n}{5} \geq 10$

25.  $-5x + 3 \leq 3x + 19$

26.  $7(g + 8) < 3(g + 2) + 4g$

For Exercises 27 and 28, define a variable, write an inequality, and solve the problem. Check your solution.

27. Two thirds of a number decreased by 27 is at least 9.

28. **CATS** Dexter has \$20 to spend at the pet store. He plans to buy a toy for his cat that costs \$3.75 and several bags of cat food. If each bag of cat food costs \$2.99, what is the greatest number of bags that he can buy?

**Example 4** Solve  $4(n - 1) < 7n + 8$ .

$$4(n - 1) < 7n + 8 \quad \text{Original Inequality}$$

$$4n - 4 < 7n + 8 \quad \text{Distributive Property}$$

$$4n - 4 - 7n < 7n + 8 - 7n \quad \text{Subtract } 7n \text{ from each side.}$$

$$-3n - 4 < 8 \quad \text{Simplify.}$$

$$-3n - 4 + 4 < 8 + 4 \quad \text{Add 4 to each side.}$$

$$-3n < 12 \quad \text{Simplify.}$$

$$\frac{-3n}{-3} > \frac{12}{-3} \quad \text{Divide and change } < \text{ to } > .$$

$$n > -4 \quad \text{Simplify.}$$

The solution set is  $\{n \mid n > -4\}$ .

**6-4 Solving Compound Inequalities** (pp. 319–324)

Graph the solution set of each compound inequality.

29.  $10 - 2y > 12$  and  $7y < 4y + 9$

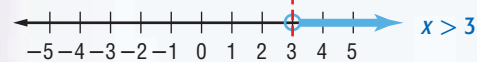
30.  $a - 3 \leq 8$  or  $a + 5 \geq 21$

31.  $3w + 8 \leq 2$  or  $w + 12 \geq 2 - w$

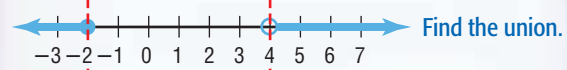
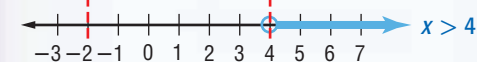
32.  $-1 < p + 3 < 5$

33. **FAIRS** A vendor at the state fair is trying to guess Martin's age within 2 years. The vendor guesses that Martin is 35 years old. If  $m$  represents Martin's age, write a compound inequality that represents the possible range of  $m$  if the vendor is correct. Then graph the solution set.

**Example 5** Graph the solution set of  $x \geq -1$  and  $x > 3$ .



**Example 6** Graph the solution set of  $x \leq -2$  or  $x > 4$ .



**6-5**

**Solving Open Sentences Involving Absolute Value** (pp. 322-327)

Solve each open sentence. Then graph the solution set.

34.  $|x + 4| = -3$       35.  $|2x - 3| = 5$

36. **TESTS** Kent has an A in math class. If his score on the next test is 98%, plus or minus 2 percentage points, he will maintain an A average. Write an open sentence to find the highest and lowest scores he can earn on the next test.

37. **CARS** The stated capacity of a fuel tank in a passenger car is accurate within 3%. Write an open sentence to find the greatest and least capacity for a fuel tank if the stated capacity is 13.6 gallons.

**Example 7** Solve  $|x + 6| = 15$ . Then graph the solution set.

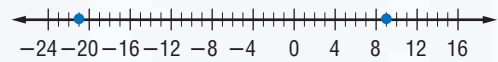
$|x + 6| = 15$  is  $x + 6 = 15$  or  $x + 6 = -15$ .

$x + 6 = 15$                        $x + 6 = -15$

$x + 6 - 6 = 15 - 6$        $x + 6 - 6 = -15 - 6$

$x = 9$                                        $x = -21$

The solution set is  $\{-21, 9\}$ .



**6-6**

**Solving Inequalities Involving Absolute Value** (pp. 329-333)

Solve each open sentence. Then graph the solution set.

38.  $|3d + 8| < 23$       39.  $|g + 2| \geq -9$

40.  $|m - 1| > -6$       41.  $|2x - 5| \geq 7$

42.  $|4h - 3| < 13$       43.  $|w + 8| \leq 11$

44. **AIRPLANES** For the average commercial airplane to take off from the runway, its speed should be within 10 miles per hour of 170 miles per hour. Define a variable, write an open sentence, and find this range of takeoff speeds.

**Example 8** Solve  $|2x - 3| < 5$ . Then graph the solution set.

$|2x - 3| < 5$  is  $2x - 3 < 5$  or  $2x - 3 > -5$ .

$2x - 3 < 5$                        $2x - 3 > -5$

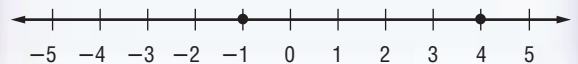
$2x - 3 + 3 < 5 + 3$        $2x - 3 + 3 > -5 + 3$

$2x < 8$                                        $2x > -2$

$\frac{2x}{2} < \frac{8}{2}$                                        $\frac{2x}{2} > \frac{-2}{2}$

$x < 4$                                        $x > -1$

The solution set is  $\{x | -1 < x < 4\}$ .



## 6-7

## Graphing Inequalities in Two Variables (pp. 334–339)

Graph each inequality.

45.  $y - 2x < -3$

46.  $x + 2y \geq 4$

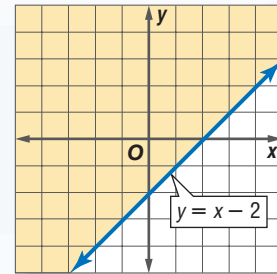
47.  $y \leq 5x + 1$

48.  $2x - 3y > 6$

49. **MOVING** A moving company charges \$95 an hour and \$0.08 a mile to move items from Brianna's old apartment to her new house. If Brianna has only \$500 for moving expenses, write an inequality for this situation. Can she afford to hire this moving company if she knows it will take 5 hours and the distance between houses is 75 miles?

**Example 9** Graph  $y \geq x - 2$ .

Since the boundary is included in the solution, draw a solid line.



Test the point  $(0, 0)$ .

$$y \geq x - 2 \quad \text{Original inequality}$$

$$0 \geq 0 - 2 \quad x = 0, y = 0$$

$$0 \geq -2 \quad \text{true}$$

The half-plane that contains  $(0, 0)$  should be shaded.

## 6-8

## Graphing Systems of Inequalities (pp. 341–345)

Solve each system of inequalities by graphing.

50.  $y < 3x$

51.  $y > -x - 1$

$$x + 2y \geq -21$$

$$y \leq 2x + 1$$

52.  $2x + y < 9$

53.  $y \geq 1$

$$x + 11y < -6$$

$$y + x \leq 3$$

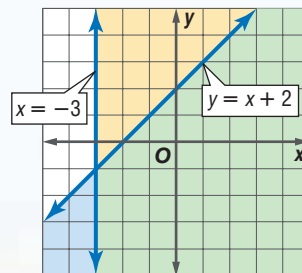
54. **TREES** Justin wants to plant peach and apple trees in his backyard. He can fit at most 12 trees. Each peach tree costs \$60, and each apple tree costs \$75. If he only has \$800 to spend, make a graph showing the number of each kind of tree that he can buy. Then list three possible solutions.

**Example 10** Solve the system of inequalities by graphing.

$$x \geq -3$$

$$y \leq x + 2$$

The solution includes the ordered pairs in the intersection of the graphs  $x \geq -3$  and  $y \leq x + 2$ . This region is shaded in green. The graphs of  $x \geq -3$  and  $y \leq x + 2$  are boundaries of this region.



Solve each inequality. Check your solution.

1.  $-23 \geq g - 6$

2.  $9p < 8p - 18$

3.  $4m - 11 \geq 8m + 7$

4.  $3(k - 2) > 12$

5. **REAL ESTATE** A homeowner is selling her house. She must pay 7% of the selling price to her real estate agent after the house is sold. Define a variable and write and solve an inequality to find what the selling price of her house must be to have at least \$140,000 after the agent is paid. Round to the nearest dollar.

6. Solve  $6 + |r| = 3$ .

7. Solve  $|d| > -2$ .

Solve each compound inequality. Then graph the solution set.

8.  $r + 3 > 2$  and  $4r < 12$

9.  $3n + 2 \geq 17$  or  $3n + 2 \leq -1$

Solve each open sentence. Then graph the solution set.

10.  $|4x + 3| = 9$

11.  $|6 - 4m| = 8$

12.  $|2a - 5| < 7$

13.  $|7 - 3s| \geq 2$

For Exercises 14–17, define a variable, write an inequality, and solve each problem. Check your solution.

14. One fourth of a number is no less than
- $-3$
- .

15. Three times a number subtracted from 14 is less than two.

16. Five less than twice a number is between 13 and 21.

17. **TRAVEL** Mary's car gets the gas mileage shown in the table. If her car's tank holds 15 gallons, what is the range of distance that Mary can drive her car on one tank of gasoline?

Gas Mileage	Miles Per Gallon
minimum	18
maximum	21

Graph each inequality.

18.  $y \geq 3x - 2$

19.  $2x + 3y < 6$

20.  $x - 2y > 4$

21. **MULTIPLE CHOICE** Ricardo purchased  $x$  bottles of paint and  $y$  paint brushes. He spent less than \$20, not including tax. If  $3x + 2y < 20$  represents this situation, which point represents a reasonable number of bottles of paint and paint brushes that Ricardo could have purchased?

Product	Cost
bottle of paint	\$3.00
paint brush	\$2.00

A (2, 7)

B (5, 4)

C (2, 8)

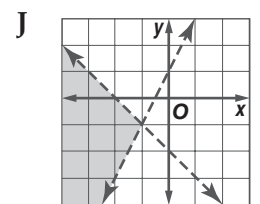
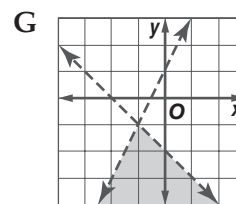
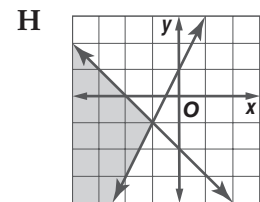
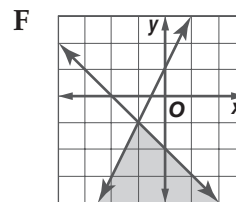
D (5, 2)

Solve each system of inequalities by graphing.

22.  $y > -4$   
 $y < -1$

23.  $y \leq 3$   
 $y > -x + 2$

24. **MULTIPLE CHOICE** Which graph represents  $y > 2x + 1$  and  $y < -x - 2$ ?



# Standardized Test Practice

Cumulative, Chapters 1–6

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. Perry measured the distance that a stick floated down a stream and the time that it took to float that distance. He recorded this in the table below.

Time, $x$ (minutes)	Distance, $y$ (meters)
0	0
2	6
3	9
5	15
10	30

Which equation best represents the relationship between distance floated,  $y$ , and the time,  $x$ ?

- A  $y = 3x$                       C  $y = 3x^2$   
 B  $y = \frac{3}{x}$                       D  $y = -3x$

### TEST-TAKING TIP

**QUESTION 1** When you write an equation, check that the given values make a true statement. For example, in Question 1, substitute the values of the coordinates of one of the points into your equation to check.

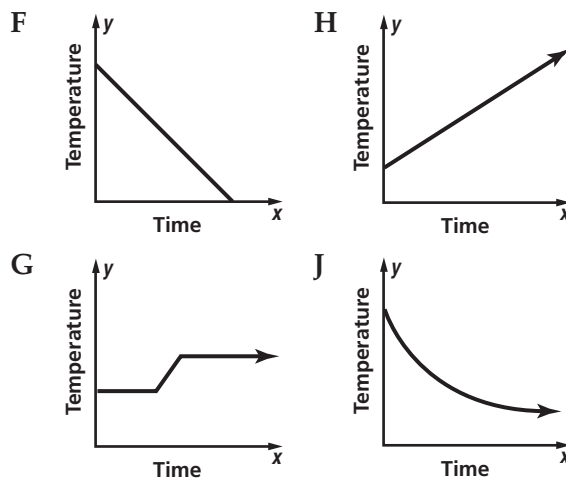
2. Mr. Carter's history class earned \$500 to visit the local museum. The class needed to purchase tickets and rent a bus. The tickets cost \$8 each and the bus costs \$50 to rent. Which inequality represents the number of students  $x$ , that can go on the trip?

- F  $50 - 8x \leq 500$             H  $50x + 8 \geq 500$   
 G  $50x - 8 \geq 500$             J  $50 + 8x \leq 500$

3. **GRIDDABLE** The drama club sold 180 tickets to their first performance. They charged \$5 for each adult ticket and \$4 for each student ticket. They made a total of \$798. How many student tickets did they sell?

4. If the variables  $a$  and  $b$  are related so that  $a + b > a - b$ , which statement must be true about  $a$  and  $b$ ?
- A The variable  $a$  is greater than the variable  $b$ .  
 B The variable  $a$  is a negative number.  
 C The variable  $b$  is a negative number.  
 D The variable  $b$  is a positive number.

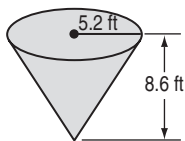
5. Which graph best represents the temperature of a glass of water after an ice cube is placed in it?



6. Jonah has 80 sports trading cards. The number of baseball trading cards is 16 less than twice the number of basketball trading cards. Which system of equations can be used to find how many baseball trading cards,  $x$ , and basketball cards,  $y$ , Jonah has?

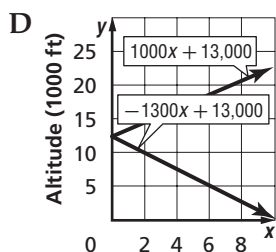
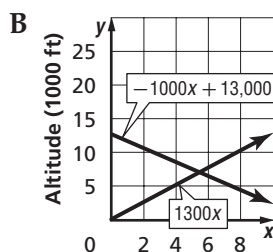
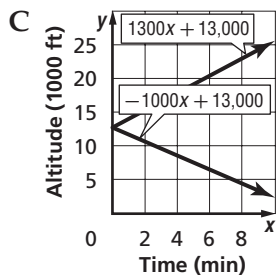
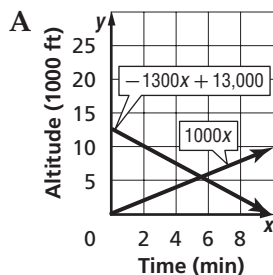
- A  $x + y = 80$   
 $y = 2x + 16$   
 B  $x + y = 80$   
 $x = 2y - 16$   
 C  $y - x = 80$   
 $x = 2y - 16$   
 D  $y - 80 = x$   
 $x = 2y$

7. Find the volume of the figure shown. Round to the nearest tenth.



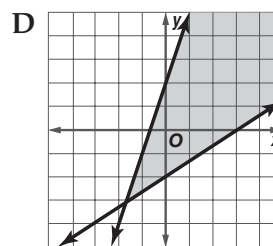
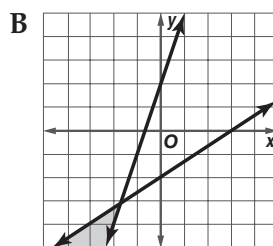
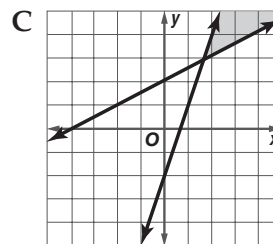
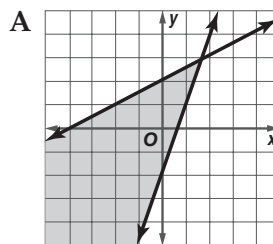
- F  $243.5 \text{ ft}^3$   
 G  $421.5 \text{ ft}^3$   
 H  $487.0 \text{ ft}^3$   
 J  $730.6 \text{ ft}^3$

8. Airplane A is descending from an altitude of 13,000 feet at a rate of 1300 feet per minute. Airplane B is ascending from the ground at a rate of 1000 feet per minute. Which graph below accurately represents the point when the airplanes will reach the same altitude?



9. Which graph best represents the solution to this system of inequalities?

$$\begin{aligned} 3x &\geq y + 2 \\ -2x &\geq -4y + 8 \end{aligned}$$



**Pre-AP**

**Record your answers on a sheet of paper. Show your work.**

10. The Carlson family is building a house on a lot that is 91 feet long and 158 feet wide.
- Town law states that the sides of a house cannot be closer than 10 feet to the edges of a lot. Write an inequality for the possible lengths of the Carlson family's house, and solve the inequality.
  - The Carlson family wants their house to be at least 2800 square feet and no more than 3200 square feet. They also want their house to have the maximum possible length. Write an inequality for the possible widths of their house, and solve the inequality. Round your answer to the nearest whole number of feet.

**NEED EXTRA HELP?**

**If You Missed Question...**

**Go to Lesson or Page...**

1	2	3	4	5	6	7	8	9	10
4-4	6-3	5-4	6-1	1-9	5-1	708	5-5	6-8	6-1, 6-2, 6-4