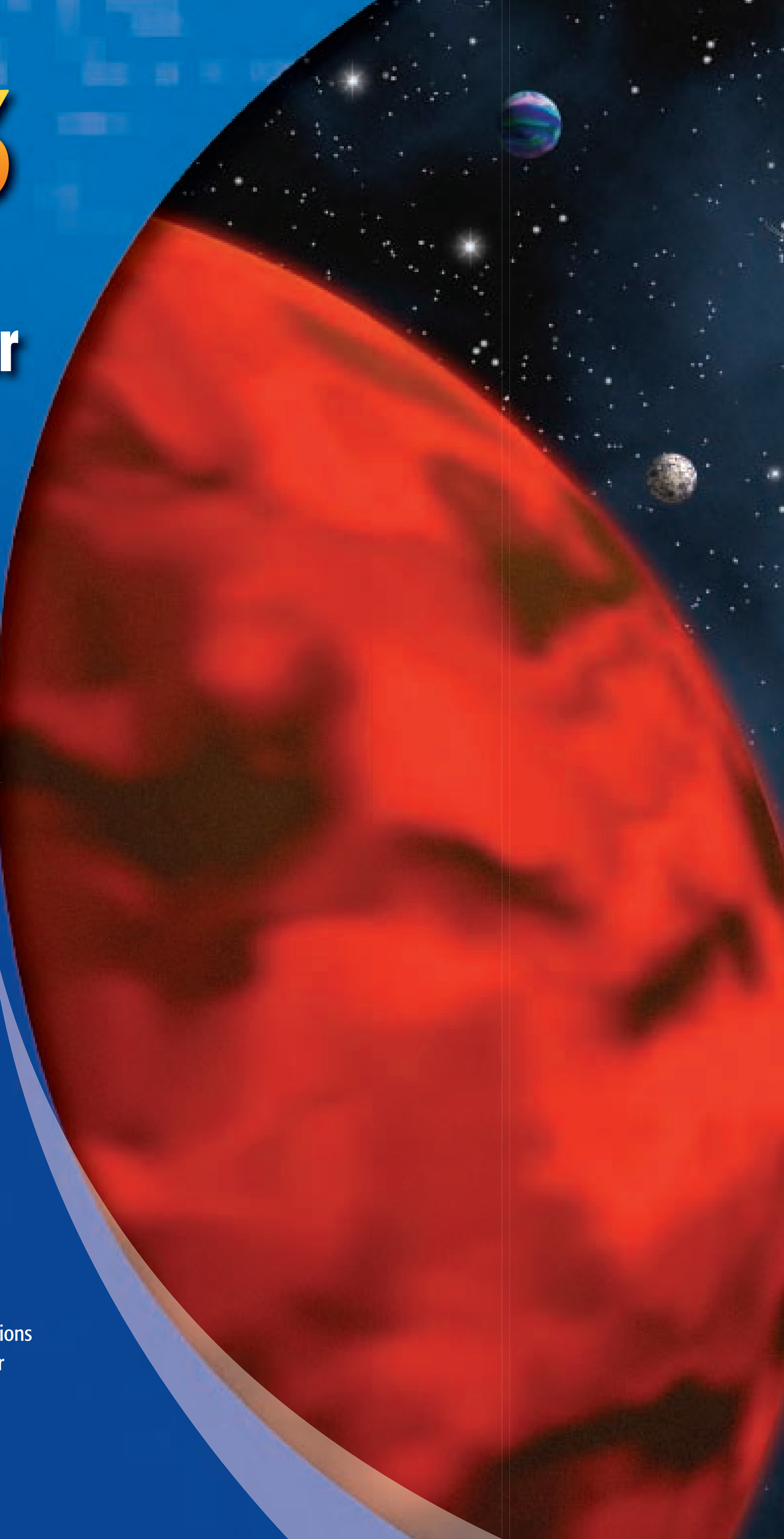
UNIT 3

Polynomials and Nonlinear Functions



Focus

Use quadratic and other nonlinear functions to represent and model problem situations and to analyze and interpret relationships.

CHAPTER 7 Polynomials

BIG Idea Understand there are situations modeled by functions that are not linear, and model the situations.

CHAPTER 8 Factoring

BIG Idea Use algebraic skills to simplify algebraic expressions, and solve equations and inequalities in problem situations.

CHAPTER 9 Quadratic and Exponential Functions

BIG Idea Understand there is more than one way to solve a quadratic equation and solve them using appropriate methods.

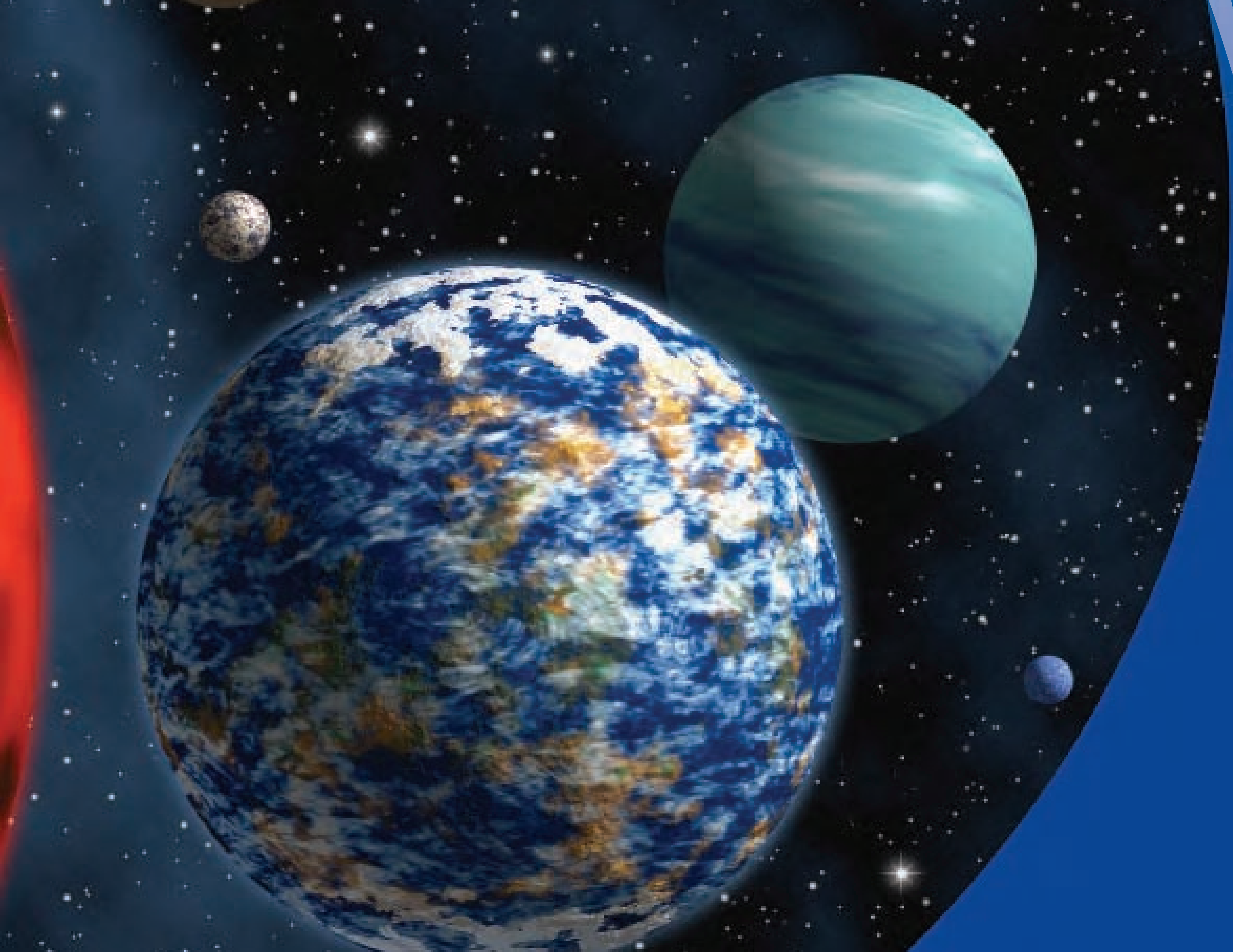
BIG Idea Understand there are situations modeled by functions that are neither linear nor quadratic, and model the situations.

Cross-Curricular Project

Algebra and Physical Science

Out of this World You can probably name the planets in the solar system, but can you name planets outside of our system? In recent years, planets in other systems have been discovered. In August, 2004, a team of astronomers discovered a small planet orbiting a star known as 55 Cancri. Star 55 Cancri has three other planets, making it the first known four-planet system outside our system. In this project, you will examine how exponents, factors, and graphs are useful in presenting information about planets.

Math Online **Log on to algebra1.com to begin.**



CHAPTER 7

Polynomials

BIG Ideas

- Find products and quotients of monomials.
- Find the degree of a polynomial and arrange the terms in order.
- Add, subtract, and multiply polynomial expressions.
- Find special products of binomials.

Key Vocabulary

binomial (p. 376)
FOIL method (p. 399)
monomial (p. 358)
polynomial (p. 376)

Real-World Link

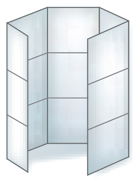
Running Polynomials can be used to model many real-world situations, such as the way that distance runners on a curved track should be staggered at the start of a race.

FOLDABLES™ Study Organizer

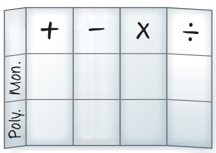
Polynomials Make this Foldable to help you organize information about polynomials. Begin with a sheet of 11″ by 17″ paper.

1. **Fold** in thirds lengthwise.

2. **Open** and fold a 2″ tab along the width. Then fold the rest in fourths.



3. **Draw** lines along folds and label as shown.



GET READY for Chapter 7

Diagnose Readiness You have two options for checking Prerequisite Skills.

Option 2

Math Online Take the Online Readiness Quiz at algebra1.com.

Option 1

Take the Quick Check below. Refer to the Quick Review for help.

QUICKCheck

Write each expression using exponents. (Lesson 1-1)

1. $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

2. $3 \cdot 3 \cdot 3 \cdot 3$

3. $5 \cdot 5$

4. $x \cdot x \cdot x$

5. $a \cdot a \cdot a \cdot a \cdot a \cdot a$

6. $x \cdot x \cdot y \cdot y \cdot y$

7. $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}$

8. $\frac{a}{b} \cdot \frac{a}{b} \cdot \frac{c}{d} \cdot \frac{c}{d} \cdot \frac{c}{d} \cdot \frac{c}{d}$

Evaluate each expression. (Lesson 1-1)

9. 3^2

10. 4^3

11. $(-6)^2$

12. $(-3)^3$

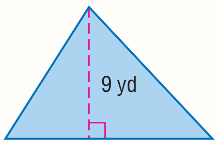
13. $\left(\frac{2}{3}\right)^4$

14. $\left(-\frac{7}{8}\right)^2$

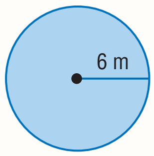
15. **PROBABILITY** The probability of correctly guessing the outcome of a flipped penny six times is $\left(\frac{1}{2}\right)^6$. Express this probability as a fraction without exponents.

Find the area or volume of each figure. (Prerequisite Skill)

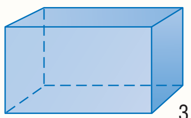
16.



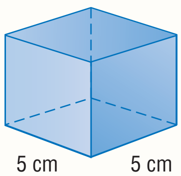
17.



18.



19.



QUICKReview

EXAMPLE 1

Express $6 \cdot 6 \cdot 6 \cdot x \cdot x + y \cdot y \cdot y \cdot y \cdot z$ using exponents.

3 factors of six is 6^3.

4 factors of y is y^4.

2 factors of x is x^2.

1 factor of z is z^1 or z.

So, $6 \cdot 6 \cdot 6 \cdot x \cdot x + y \cdot y \cdot y \cdot y \cdot z = 6^3x^2 + y^4z$.

EXAMPLE 2

Evaluate $\left(\frac{8}{11}\right)^2$.

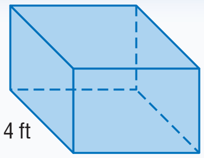
$\left(\frac{8}{11}\right)^2$ Original expression

$= \frac{8^2}{11^2}$ Power of a Quotient Rule

$= \frac{64}{121}$ Simplify.

EXAMPLE 3

Find the volume of the figure.



$V = \ell wh$ Volume formula

$= 3 \cdot 4 \cdot 2$ Substitute 3 for length, 4 for width, and 2 for height.

$= 24 \text{ ft}^3$ Evaluate volume.

The volume of the box is 24 cubic feet.

7-1 Multiplying Monomials

Main Ideas

- Multiply monomials.
- Simplify expressions involving powers of monomials.

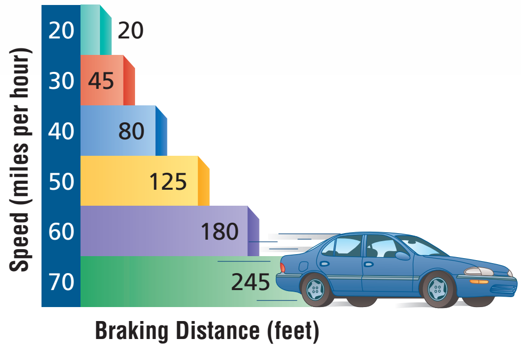
New Vocabulary

monomial

constant

GET READY for the Lesson

The table shows the braking distance for a vehicle at certain speeds. If s represents the speed in miles per hour, then the approximate number of feet that the driver must apply the brakes is $\frac{1}{20}s^2$. Notice that when speed is doubled, the braking distance is quadrupled.



Source: *British Highway Code*

Multiply Monomials A **monomial** is a number, a variable, or a product of a number and one or more variables like $\frac{1}{20}s^2$. An expression like $\frac{x}{2y}$, which involves the division of variables is not a monomial. Monomials that are real numbers are called **constants.**

EXAMPLE 1 Identify Monomials

Determine whether each expression is a monomial. Explain your reasoning.

	Expression	Monomial?	Reason
a.	-5	yes	-5 is a real number and an example of a constant.
b.	$p + q$	no	The expression involves the addition, not the product, of two variables.
c.	x	yes	Single variables are monomials.

CHECK Your Progress

1A. $-x + 5$ **1B.** $23abcd^2$ **1C.** $\frac{xyz^3}{2}$ **1D.** $\frac{ab}{c}$

Online Personal Tutor at algebra1.com

Recall that an expression of the form x^n is called a *power* and represents the product you obtain when x is used as a factor n times. The word *power* is also used to refer to the exponent itself. The number x is the *base*, and the number n is the *exponent.*

$$\underbrace{2^5}_{\text{base } 2,\ \text{exponent } 5} = \overbrace{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}^{5 \text{ factors}} \text{ or } 32$$

In the following examples, the definition of a power is used to find the products of powers. Look for a pattern in the exponents.

$$2^3 \cdot 2^5 = \underbrace{\overbrace{2 \cdot 2 \cdot 2}^{3\text{ factors}} \cdot \overbrace{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}^{5\text{ factors}}}_{3+5\text{ or }8\text{ factors}} \text{ or } 2^8$$

$$3^2 \cdot 3^4 = \underbrace{\overbrace{3 \cdot 3}^{2\text{ factors}} \cdot \overbrace{3 \cdot 3 \cdot 3 \cdot 3}^{4\text{ factors}}}_{2+4\text{ or }6\text{ factors}} \text{ or } 3^6$$

These examples suggest the property for multiplying powers.

KEY CONCEPT *Product of Powers*

Words To multiply two powers that have the same base, add their exponents.

Symbols For any number a and all integers m and n, $a^m \cdot a^n = a^{m+n}$.

Example $a^4 \cdot a^{12} = a^{4+12}$ or a^{16}

EXAMPLE Product of Powers

2 **Simplify each expression.**

a. $(5x^7)(x^6)$

$(5x^7)(x^6) = (5)(1)(x^7)(x^6)$ Group the coefficients and the variables.

$= (5 \cdot 1)(x^{7+6})$ Product of Powers

$= 5x^{13}$ Simplify.

b. $(4ab^6)(-7a^2b^3)$

$(4ab^6)(-7a^2b^3) = (4)(-7)(a \cdot a^2)(b^6 \cdot b^3)$ Group the coefficients and the variables.

$= -28(a^{1+2})(b^{6+3})$ Product of Powers

$= -28a^3b^9$ Simplify.

Study Tip

Power of 1

A variable with no exponent indicated can be written as a power of 1, for example, $x = x^1$ and $ab = a^1b^1$.

CHECK Your Progress

2A. $(3y^4)(7y^5)$ **2B.** $(-4rs^2t^3)(-6r^5s^2t^3)$

Powers of Monomials You can also look for a pattern to discover the property for finding the power of a power.

$$(4^2)^5 = \overbrace{(4^2)(4^2)(4^2)(4^2)(4^2)}^{5\text{ factors}}$$
$$= 4^{2+2+2+2+2}$$
$$= 4^{10}$$

Apply rule for Product of Powers.

$$(z^8)^3 = \overbrace{(z^8)(z^8)(z^8)}^{3\text{ factors}}$$
$$= z^{8+8+8}$$
$$= z^{24}$$

These examples suggest the property for finding the power of a power.

KEY CONCEPT *Power of a Power*

Words To find the power of a power, multiply the exponents.

Symbols For any number a and all integers m and n, $(a^m)^n = a^{m \cdot n}$.

Example $(k^5)^9 = k^{5 \cdot 9}$ or k^{45}

EXAMPLE 3 Power of a Power

Simplify $[(3^2)^3]^2$.

$[(3^2)^3]^2 = (3^{2 \cdot 3})^2$ — Power of a Power

$= (3^6)^2$ — Simplify.

$= 3^{6 \cdot 2}$ — Power of a Power

$= 3^{12}$ or 531,441 — Simplify.

CHECK Your Progress

3. Simplify $[(2^2)^2]^4$.

Look for a pattern in these examples.

$$(xy)^4 = (xy)(xy)(xy)(xy) = (x \cdot x \cdot x \cdot x)(y \cdot y \cdot y \cdot y) = x^4y^4$$

$$(6ab)^3 = (6ab)(6ab)(6ab) = (6 \cdot 6 \cdot 6)(a \cdot a \cdot a)(b \cdot b \cdot b) = 6^3a^3b^3 \text{ or } 216a^3b^3$$

These examples suggest the following property.

> **Study Tip**
>
> **Powers of Monomials**
>
> Sometimes the rules for the Power of a Power and the Power of a Product are combined into one rule.
> $(a^mb^n)^p = a^{mp}b^{np}$

KEY CONCEPT — *Power of a Product*

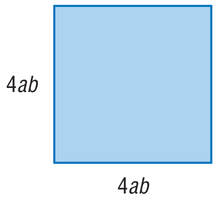
Words To find the power of a product, find the power of each factor and multiply.

Symbols For all numbers a and b and any integer m, $(ab)^m = a^mb^m$.

Example $(-2xy)^3 = (-2)^3x^3y^3$ or $-8x^3y^3$

EXAMPLE 4 Power of a Product

GEOMETRY Express the area of the square as a monomial.



Area $= s^2$ — Formula for the area of a square

$= (4ab)^2$ — Replace s with $4ab$.

$= 4^2a^2b^2$ — Power of a Product

$= 16a^2b^2$ — Simplify.

The area of the square is $16a^2b^2$ square units.

CHECK Your Progress

4. Express the area of a square with sides of length $2xy^2$ as a monomial.

CONCEPT SUMMARY — *Simplifying Expressions*

To *simplify* an expression involving monomials, write an equivalent expression in which:

- each base appears exactly once,
- there are no powers of powers, and
- all fractions are in simplest form.

EXAMPLE Simplify Expressions

5 **Simplify $(3xy^4)^2[(-2y)^2]^3$.**

$(3xy^4)^2[(-2y)^2]^3$	$= (3xy^4)^2(-2y)^6$	Power of a Power
	$= (3)^2x^2(y^4)^2(-2)^6y^6$	Power of a Product
	$= 9x^2y^8(64)y^6$	Power of a Power
	$= 9(64)x^2 \cdot y^8 \cdot y^6$	Commutative Property
	$= 576x^2y^{14}$	Product of Powers

CHECK Your Progress

5. Simplify $\left(\frac{1}{2}a^2b^2\right)^3[(-4b)^2]^2$.

CHECK Your Understanding

Example 1 (p. 358)

Determine whether each expression is a monomial. Write *yes* or *no*. Explain.

1. $5 - 7d$ **2.** $\frac{4a}{3b}$ **3.** n

Examples 2, 3 (pp. 359–360)

Simplify.

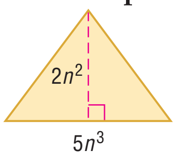
4. $x(x^4)(x^6)$ **5.** $(4a^4b)(9a^2b^3)$ **6.** $[(2^3)^2]^3$

7. $[(3^2)^2]^2$ **8.** $(3y^5z)^2$ **9.** $(-2f^2g)^3$

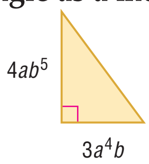
Example 4 (p. 360)

GEOMETRY Express the area of each triangle as a monomial.

10.



11.



Example 5 (p. 361)

Simplify.

12. $(-2v^3w^4)^3(-3vw^3)^2$ **13.** $(5x^2y)^2(2xy^3z)^3(4xyz)$

Exercises

Determine whether each expression is a monomial. Write *yes* or *no*. Explain.

14. 12 **15.** $4x^3$ **16.** $a - 2b$

17. $4n + 5m$ **18.** $\frac{x}{y^2}$ **19.** $\frac{1}{5}abc^{14}$

HOMEWORK HELP

For Exercises	See Examples
14–19	1
20–23	2
24–28	3
29–30	4
31–34	5

Simplify.

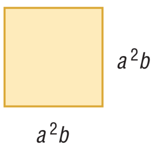
20. $(ab^4)(ab^2)$ **21.** $(p^5q^4)(p^2q)$ **22.** $(-7c^3d^4)(4cd^3)$

23. $(-3j^7k^5)(-8jk^8)$ **24.** $(9pq^7)^2$ **25.** $(7b^3c^6)^3$

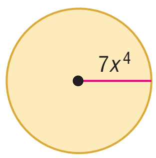
26. $[(3^2)^4]^2$ **27.** $[(4^2)^3]^2$ **28.** $[(-2xy^2)^3]^2$

GEOMETRY Express the area of each figure as a monomial.

29.



30.



Simplify.

31. $(4cd)^2(-3d^2)^3$

32. $(-2x^5)^3(-5xy^6)^2$

33. $(2ag^2)^4(3a^2g^3)^2$

34. $(2m^2n^3)^3(3m^3n)^4$

35. Simplify the expression $(-2b^3)^4 - 3(-2b^4)^3$.

36. Simplify the expression $2(-5y^3)^2 + (-3y^3)^3$.

Cross-Curricular Project

Math Online You can use powers to write and compare the distances of the planets to the Sun. Visit algebra1.com.

37. CHEMISTRY Lemon juice is 10^2 times as acidic as tomato juice. Tomato juice is 10^3 times as acidic as egg whites. How many times as acidic is lemon juice as egg whites? Write as a monomial.

38. GEOLOGY The seismic waves of a magnitude 6 earthquake are 10^2 times as great as a magnitude 4 earthquake. The seismic waves of a magnitude 4 earthquake are 10 times as great as a magnitude 3 earthquake. How many times as great are the seismic waves of a magnitude 6 earthquake as those of a magnitude 3 earthquake? Write as a monomial.

Simplify.

39. $(5a^2b^3c^4)(6a^3b^4c^2)$

40. $(10xy^5z^3)(3x^4y^6z^3)$

41. $(0.5x^3)^2$

42. $(0.4h^5)^3$

43. $\left(-\frac{3}{4}c\right)^3$

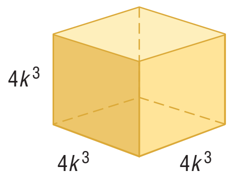
44. $\left(\frac{4}{5}a^2\right)^2$

45. $(8y^3)(-3x^2y^2)\left(\frac{3}{8}xy^4\right)$

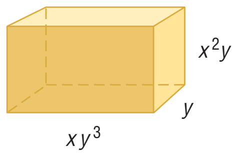
46. $\left(\frac{4}{7}m\right)^2(49m)(17p)\left(\frac{1}{34}p^5\right)$

GEOMETRY **Express the volume of each solid as a monomial.**

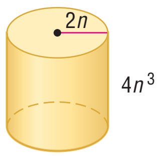
47.



48.



49.



Real-World Link

In a demolition derby, the winner is not the car that finishes first but the last car still moving under its own power.

Source: *Smithsonian Magazine*

TELEPHONES **For Exercises 50 and 51, use the following information.**
The first transatlantic telephone cable has 51 amplifiers along its length. Each amplifier strengthens the signal on the cable 10^6 times.

50. After it passes through the second amplifier, the signal has been boosted $10^6 \cdot 10^6$ times. Simplify this expression.

51. Represent the number of times the signal has been boosted after it has passed through the first four amplifiers as a power of 10^6. Then simplify the expression.

DEMOLITION DERBY **For Exercises 52 and 53, use the following information.**
When a car hits an object, the damage is measured by the collision impact. For a certain car, the collision impact I is given by $I = 2s^2$, where s represents the speed in kilometers per minute.

52. What is the collision impact if the speed of the car is 1 kilometer per minute? 2 kilometers per minute? 4 kilometers per minute?

53. As the speed doubles, explain what happens to the collision impact.

EXTRA PRACTICE
See pages 730, 750.
Math Online
Self-Check Quiz at algebra1.com

TESTING **For Exercises 54 and 55, use the following information.**

A history test covers two chapters. There are 2^{12} ways to answer the 12 true-false questions on the first chapter and 2^{10} ways to answer the 10 true-false questions on the second chapter.

54. How many ways are there to answer all 22 questions on the test?

55. If a student guesses on each question, what is the probability of answering all questions correctly?

H.O.T. Problems

56. **OPEN ENDED** Write three different expressions that are equivalent to x^6.

CHALLENGE Determine whether each statement is *true* or *false*. If true, explain your reasoning. If false, give a counterexample.

57. For any real number a, $(-a)^2 = -a^2$.

58. For all real numbers a and b, and all integers m, n, and p, $(a^m b^n)^p = a^{mp} b^{np}$.

59. For all real numbers a, b, and all integers n, $(a + b)^n = a^n + b^n$.

60. **FIND THE ERROR** Nathan and Poloma are simplifying $(5^2)(5^9)$. Who is correct? Explain your reasoning.

Nathan	Poloma
$(5^2)(5^9) = (5 \cdot 5)^{2+9}$ $= 25^{11}$	$(5^2)(5^9) = 5^{2+9}$ $= 5^{11}$

61. **REASONING** Compare each pair of monomials. Explain why each pair is or is not equivalent.

a. $5m^2$ and $(5m)^2$

b. $(yz)^4$ and y^4z^4

c. $-3a^2$ and $(-3a)^2$

d. $2(c^7)^3$ and $8c^{21}$

62. ***Writing in Math*** Use the data about braking distances on page 358 to explain why doubling speed quadruples braking distance.

STANDARDIZED TEST PRACTICE

63. The length of a rectangle is three times the width of the rectangle. If the width of the rectangle is y units, what is the area of the rectangle?

A $3y$ units^2

B $3y^2$ units^2

C $y + 3$ units^2

D $3y(y + 3)$ units^2

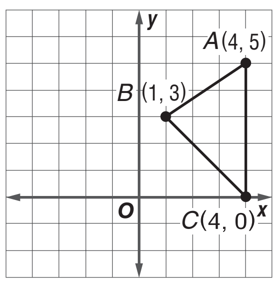
64. **REVIEW** $\triangle ABC$ has coordinates $A(4, 5)$, $B(1, 3)$, and $C(4, 0)$. What will the coordinates of A' be if the triangle is translated 3 units down and 2 units to the left?

F (1, 3)

G (2, 2)

H (7, 7)

J (8, 0)



Spiral Review

Solve each system of inequalities by graphing. (Lesson 6-8)

65. $y \leq 2x + 2$
$y \geq -x - 1$

66. $y \geq x - 2$
$y < 2x - 1$

67. $x > -2$
$y < x + 3$

Determine which ordered pairs are part of the solution set for each inequality. (Lesson 6-7)

68. $y \leq 2x$, $\{(1, 4), (-1, 5), (5, -6), (-7, 0)\}$

69. $y < 8 - 3x$, $\{(-4, 2), (-3, 0), (1, 4), (1, 8)\}$

Solve each compound inequality. Then graph the solution set. (Lesson 6-4)

70. $4 + h \leq -3$ or $4 + h \geq 5$

71. $4 < 4a + 12 < 24$

72. $14 < 3h + 2 < 2$

73. $2m - 3 > 7$ or $2m + 7 > 9$

Use elimination to solve each system of equations. (Lesson 5-4)

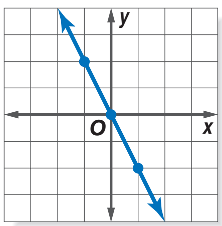
74. $-4x + 5y = 2$
$x + 2y = 6$

75. $3x + 4y = -25$
$2x - 3y = 6$

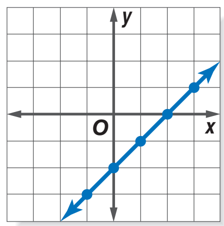
76. $x + y = 20$
$4 = 0.4x + 0.15y$

Write an equation in function notation for each relation. (Lesson 3-5)

77.



78.

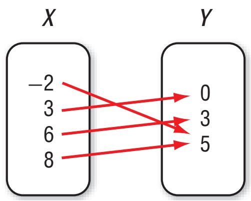


Express the relation shown in each table, mapping, or graph as a set of ordered pairs. Then write the inverse of the relation. (Lesson 3-1)

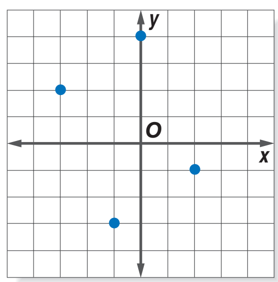
79.

x	y
−5	2
−2	3
0	5
4	9

80.



81.



82. TRANSPORTATION Two trains leave York at the same time, one traveling north, the other south. The northbound train travels at 40 miles per hour and the southbound at 30 miles per hour. In how many hours will the trains be 245 miles apart? (Lesson 2-9)

GET READY for the Next Lesson

PREREQUISITE SKILL **Simplify.** (Pages 694–695)

83. $\frac{2}{6}$

84. $\frac{3}{15}$

85. $\frac{10}{5}$

86. $\frac{27}{9}$

87. $\frac{14}{36}$

88. $\frac{9}{48}$

89. $\frac{44}{32}$

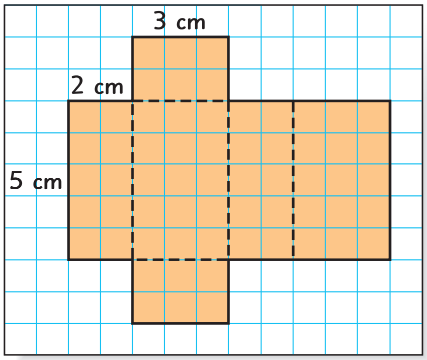
90. $\frac{45}{18}$

EXTEND 7-1

Algebra Lab

Investigating Surface Area and Volume

ACTIVITY



- Cut out the pattern shown from a sheet of centimeter grid paper. Fold along the dashed lines and tape the edges together to form a rectangular prism.
- Find the surface area SA of the prism by counting the squares on all the faces of the prism or by using the formula $SA = 2w\ell + 2wh + 2\ell h$, where w is the width, ℓ is the length, and h is the height of the prism.
- Find the volume V of the prism by using the formula $V = \ell wh$.
- Now construct another prism with dimensions that are 2 times each of the dimensions of the first prism, or 4 centimeters by 10 centimeters by 6 centimeters.
- Finally, construct a third prism with dimensions that are 3 times each of the dimensions of the first prism.

Analyze the Results

1. Copy and complete the table using the prisms you made.

Prism	Dimensions	Surface Area (cm^2)	Volume (cm^3)	Surface Area Ratio $\left(\frac{SA \text{ of New}}{SA \text{ of Original}}\right)$	Volume Ratio $\left(\frac{V \text{ of New}}{V \text{ of Original}}\right)$
Original	2 by 5 by 3	62	30	______	______
A	4 by 10 by 6				
B	6 by 15 by 9				

2. **MAKE A CONJECTURE** Suppose you multiply each dimension of a prism by 2. What is the ratio of the surface area of the new prism to the surface area of the original prism? What is the ratio of the volumes?

3. If you multiply each dimension of a prism by 3, what is the ratio of the surface area of the new prism to the surface area of the original? What is the ratio of the volumes?

4. Suppose you multiply each dimension of a prism by a. Make a conjecture about the ratios of surface areas and volumes.

5. Repeat the activity using cylinders. To start, make a cylinder with radius 4 centimeters and height 5 centimeters. To compute surface area SA and volume V, use the formulas $SA = 2\pi r^2 + 2\pi rh$ and $V = \pi r^2 h$, where r is the radius and h is the height of the cylinder. Do the conjectures you made in Exercise 4 hold true for cylinders? Explain.

7-2 Dividing Monomials

Main Ideas

- Simplify expressions involving the quotient of monomials.
- Simplify expressions containing negative exponents.

New Vocabulary

zero exponent

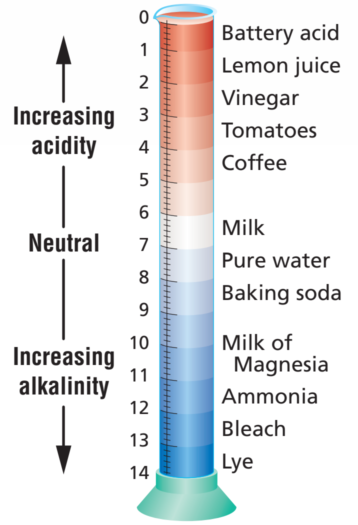
negative exponent

GET READY for the Lesson

To test whether a solution is a base or an acid, chemists use a pH test. This test measures the concentration c of hydrogen ions (in moles per liter) in the solution.

$$c = \left(\frac{1}{10}\right)^{\text{pH}}$$

The table gives examples of solutions with various pH levels. You can find the quotient of powers and use negative exponents to compare measures on the pH scale.



Source: U.S. Geological Survey

Quotients of Monomials Look for a pattern in the examples below.

$$\frac{4^5}{4^3} = \frac{\overbrace{\cancel{4} \cdot \cancel{4} \cdot \cancel{4} \cdot 4 \cdot 4}^{5 \text{ factors}}}{\underbrace{\cancel{4} \cdot \cancel{4} \cdot \cancel{4}}_{3 \text{ factors}}} = \underbrace{4 \cdot 4}_{5-3 \text{ or } 2 \text{ factors}} \text{ or } 4^2$$

$$\frac{3^6}{3^2} = \frac{\overbrace{\cancel{3} \cdot \cancel{3} \cdot 3 \cdot 3 \cdot 3 \cdot 3}^{6 \text{ factors}}}{\underbrace{\cancel{3} \cdot \cancel{3}}_{2 \text{ factors}}} = \underbrace{3 \cdot 3 \cdot 3 \cdot 3}_{6-2 \text{ or } 4 \text{ factors}} \text{ or } 3^4$$

KEY CONCEPT — Quotient of Powers

Words To divide two powers with the same base, subtract the exponents.

Symbols For all integers m and n and any nonzero number a, $\frac{a^m}{a^n} = a^{m-n}$.

Example $\frac{b^{15}}{b^7} = b^{15-7}$ or b^8

EXAMPLE Quotient of Powers

1 **Simplify $\frac{a^5b^8}{ab^3}$. Assume that no denominator is equal to zero.**

$\frac{a^5b^8}{ab^3} = \left(\frac{a^5}{a}\right)\left(\frac{b^8}{b^3}\right)$ Group powers that have the same base.

$= \left(a^{5-1}\right), \left(b^{8-3}\right)$ or a^4b^5 Quotient of Powers

CHECK Your Progress

1. Simplify $\frac{x^3y^4}{x^2y}$. Assume that no denominator is equal to zero.

Concepts in Motion
BrainPop®
algebra1.com

Look for a pattern in the example below.

$$\left(\frac{2}{5}\right)^3 = \underbrace{\left(\frac{2}{5}\right)\left(\frac{2}{5}\right)\left(\frac{2}{5}\right)}_{\text{3 factors}} = \frac{\overbrace{2 \cdot 2 \cdot 2}^{\text{3 factors}}}{\underbrace{5 \cdot 5 \cdot 5}_{\text{3 factors}}} \text{ or } \frac{2^3}{5^3}$$

KEY CONCEPT *Power of a Quotient*

Words To find the power of a quotient, find the power of the numerator and the power of the denominator.

Symbols For any integer m and any real numbers a and b, $b \neq 0$, $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$.

EXAMPLE Power of a Quotient

2 Simplify $\left(\frac{2p^2}{3}\right)^4$.

$\left(\frac{2p^2}{3}\right)^4 = \frac{(2p^2)^4}{3^4}$ Power of a Quotient

$= \frac{2^4(p^2)^4}{3^4}$ Power of a Product

$= \frac{16p^8}{81}$ Power of a Power

CHECK Your Progress Simplify each expression.

2A. $\left(\frac{3x^4}{4}\right)^3$

2B. $\left(\frac{5x^5y}{6}\right)^2$

Negative Exponents A graphing calculator can be used to investigate expressions with 0 as an exponent and negative exponents.

Study Tip

Graphing Calculator

To express a value as a fraction, press MATH ENTER ENTER.

GRAPHING CALCULATOR LAB

Zero Exponent and Negative Exponents

Use the [^] key to evaluate expressions with exponents.

1. Copy and complete the table.

Power	2^4	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}	2^{-4}
Value									

2. Describe the relationship between each pair of values.

a. 2^4 and 2^{-4} **b.** 2^3 and 2^{-3} **c.** 2^2 and 2^{-2} **d.** 2^1 and 2^{-1}

3. Make a conjecture as to the fractional value of 5^{-1}. Verify your conjecture using a calculator.

4. What is the value of 5^0?

5. What happens when you evaluate 0^0?

Study Tip

Alternative Method

Another way to look at the problem of simplifying $\frac{2^4}{2^4}$ is to recall that any nonzero number divided by itself is 1: $\frac{2^4}{2^4} = \frac{16}{16}$ or 1.

To understand why a calculator gives a value of 1 for 2^0, study the two methods used to simplify $\frac{2^4}{2^4}$.

Method 1

$\frac{2^4}{2^4} = 2^{4-4}$ Quotient of Powers

$= 2^0$ Subtract.

Method 2

$\frac{2^4}{2^4} = \frac{\overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}}}{\underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{2}}}$ Definition of powers

$= 1$ Simplify.

Since $\frac{2^4}{2^4}$ cannot have two different values, we can conclude that $2^0 = 1$.

KEY CONCEPT *Zero Exponent*

Words Any nonzero number raised to the zero power is 1.

Symbols For any nonzero number a, $a^0 = 1$.

Example $(-0.25)^0 = 1$

EXAMPLE **Zero Exponent**

3 **Simplify each expression. Assume that no denominator is equal to zero.**

a. $\left(-\frac{3x^5y}{8xy^7}\right)^0$

$\left(-\frac{3x^5y}{8xy^7}\right)^0 = 1$ $a^0 = 1$

b. $\frac{t^3s^0}{t}$

$\frac{t^3s^0}{t} = \frac{t^3(1)}{t}$ $a^0 = 1$

$= \frac{t^3}{t}$ Simplify.

$= t^2$ Quotient of Powers

CHECK Your Progress

3A. $\frac{x^0y^4}{y^2}$

3B. $\left(\frac{2x^3y^2z^5}{10xy^3z^4}\right)^0$

To investigate the meaning of a negative exponent, we can simplify expressions like $\frac{8^2}{8^5}$ in two ways.

Method 1

$\frac{8^2}{8^5} = 8^{2-5}$ Quotient of Powers

$= 8^{-3}$ Subtract.

Method 2

$\frac{8^2}{8^5} = \frac{\overset{1}{\cancel{8}} \cdot \overset{1}{\cancel{8}}}{\underset{1}{\cancel{8}} \cdot \underset{1}{\cancel{8}} \cdot 8 \cdot 8 \cdot 8}$ Definition of powers

$= \frac{1}{8^3}$ Simplify.

Since $\frac{8^2}{8^5}$ cannot have two different values, we can conclude that $8^{-3} = \frac{1}{8^3}$.

KEY CONCEPT — Negative Exponent

Words For any nonzero number a and any integer n, a^{-n} is the reciprocal of a^n. In addition, the reciprocal of a^{-n} is a^n.

Symbols For any nonzero number a and any integer n, $a^{-n} = \frac{1}{a^n}$ and $\frac{1}{a^{-n}} = a^n$.

Examples $5^{-2} = \frac{1}{5^2}$ or $\frac{1}{25}$ $\qquad \frac{1}{m^{-3}} = m^3$

Study Tip

Common Misconception

Do not confuse a negative number with a number raised to a negative power.

$3^{-1} = \frac{1}{3} \quad -3 \neq \frac{1}{3}$

An expression is simplified when it contains only positive exponents.

EXAMPLE Negative Exponents

4 **Simplify each expression. Assume that no denominator is equal to zero.**

a. $\frac{b^{-3}c^2}{d^{-5}}$

$$\frac{b^{-3}c^c}{d^{-5}} = \left(\frac{b^{-3}}{1}\right)\left(\frac{c^2}{1}\right)\left(\frac{1}{d^{-5}}\right) \quad \text{Write as a product of fractions.}$$

$$= \left(\frac{1}{b^3}\right)\left(\frac{c^2}{1}\right)\left(\frac{d^5}{1}\right) \quad a^{-n} = \frac{1}{a^n}$$

$$= \frac{c^2d^5}{b^3} \quad \text{Multiply fractions.}$$

b. $\frac{-3a^{-4}b^7}{21a^2b^7c^{-5}}$

$$\frac{-3a^{-4}b^7}{21a^2b^7c^{-5}} = \left(\frac{-3}{21}\right)\left(\frac{a^{-4}}{a^2}\right)\left(\frac{b^7}{b^7}\right)\left(\frac{1}{c^{-5}}\right) \quad \text{Group powers with the same base.}$$

$$= \frac{-1}{7}\left(a^{-4-2}\right)\left(b^{7-7}\right)\left(c^5\right) \quad \text{Quotient of Powers and Negative Exponent Properties}$$

$$= \frac{-1}{7}a^{-6}b^0c^5 \quad \text{Simplify.}$$

$$= \frac{-1}{7}\left(\frac{1}{a^6}\right)(1)c^5 \quad \text{Negative Exponent and Zero Exponent Properties}$$

$$= -\frac{c^5}{7a^6} \quad \text{Multiply fractions.}$$

c. $\frac{-3q^{-2}rs^4}{-12qr^{-3}s^{-5}}$

$$\frac{-3q^{-2}rs^4}{-12qr^{-3}s^{-5}} = \left(\frac{-3}{-12}\right)\left(\frac{q^{-2}}{q}\right)\left(\frac{r}{r^{-3}}\right)\left(\frac{s^4}{s^{-5}}\right) \quad \text{Group powers with the same base.}$$

$$= \frac{1}{4}q^{-3}r^4s^9 \quad \text{Simplify.}$$

$$= \frac{r^4s^9}{4q^3} \quad \text{Negative Exponent Property}$$

CHECK Your Progress

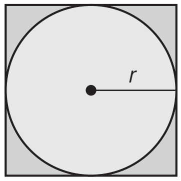
4A. $\frac{r^{-5}s^4}{t^{-3}}$ **4B.** $\frac{24x^{-2}y^4}{-6x^{-3}y^{-2}z^{-1}}$

Test-Taking Tip

Some problems can be solved using estimation. The area of the circle is less than the area of the square. Therefore, the ratio of the two areas must be less than 1. Use 3 as an approximate value for π to determine which of the choices is less than 1.

STANDARDIZED TEST EXAMPLE — Apply Properties of Exponents

5 **Write the ratio of the area of the circle to the area of the square in simplest form.**



A $\frac{\pi}{2}$ **C** $\frac{2\pi}{1}$

B $\frac{\pi}{4}$ **D** $\frac{\pi}{3}$

Read the Test Item

A ratio is a comparison of two quantities. It can be written in fraction form.

Solve the Test Item

- area of circle: πr^2
  length of square: diameter of circle or $2r$
  area of square: $(2r)^2$

- $\frac{\text{area of circle}}{\text{area of square}} = \frac{\pi r^2}{(2r)^2}$ Substitute.

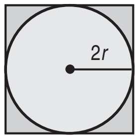
  $= \frac{\pi}{4} r^{2-2}$ Quotient of Powers

  $= \frac{\pi}{4} r^0$ or $\frac{\pi}{4}$ $r^0 = 1$

The answer is B.

CHECK Your Progress

5. Write the ratio of the area of the circle to the area of the square in simplest form.



F $\frac{\pi}{3}$ **G** $\frac{\pi}{2}$ **H** $\frac{\pi}{4}$ **J** $\frac{3\pi}{2}$

Online **Personal Tutor at** algebra1.com

CHECK Your Understanding

Simplify. Assume that no denominator is equal to zero.

Example 1 (p. 367)

1. $\frac{7^8}{7^2}$ **2.** $\frac{x^8y^{12}}{x^2y^7}$ **3.** $\frac{5pq^7}{10p^6q^3}$

Example 2 (p. 367)

4. $\left(\frac{2c^3d}{7z^2}\right)^3$ **5.** $\left(\frac{4a^2b}{2c^3}\right)^2$ **6.** $\left(\frac{3mn^3}{6n^2}\right)^2$

Example 3 (p. 369)

7. $y^0(y^5)(y^{-9})$ **8.** $\frac{(4m^{-3}n^5)^0}{mn}$ **9.** $\frac{(3x^2y^5)^0}{(21x^5y^2)^0}$

Example 4 (p. 370)

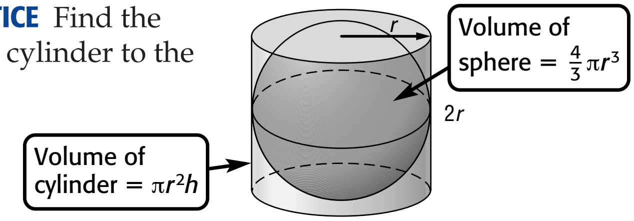
10. 13^{-2} **11.** $\frac{c^{-5}}{d^3g^{-8}}$ **12.** $\frac{(cd^{-2})^3}{(c^4d^9)^{-2}}$

Example 5
(p. 370)

13. STANDARDIZED TEST PRACTICE Find the ratio of the volume of the cylinder to the volume of the sphere.

A $\frac{1}{2}$ C $\frac{4}{3}$

B $\frac{3}{4}$ D $\frac{3}{2}$



Exercises

HOMEWORK HELP	
For Exercises	See Examples
14–17	1
18–19	2
20–21	3
22–28	4
29–30	5

Simplify. Assume that no denominator is equal to zero.

14. $\frac{4^{12}}{4^2}$

15. $\frac{3^{13}}{3^7}$

16. $\frac{p^7 n^3}{p^4 n^2}$

17. $\frac{y^3 z^9}{yz^2}$

18. $\left(\frac{5b^4 n}{2a^6}\right)^2$

19. $\left(\frac{3m^7}{4x^5 y^3}\right)^4$

20. $\left(\frac{r^{-2} t^5}{t^{-1}}\right)^0$

21. $\left(\frac{4c^{-2} d}{b^{-2} c^3 d^{-1}}\right)^0$

22. 6^{-2}

23. 5^{-3}

24. $\left(\frac{4}{5}\right)^{-2}$

25. $\left(\frac{3}{2}\right)^{-3}$

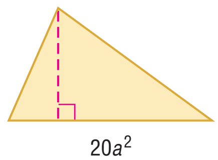
26. $n^2(p^{-4})(n^{-5})$

27. $\frac{28a^7 c^{-4}}{7a^3 b^0 c^{-8}}$

28. $x^3 y^0 x^{-7}$

29. The area of the rectangle is $24x^5y^3$ square units. Find the length of the rectangle.

30. The area of the triangle is $100a^3b$ square units. Find the height of the triangle.



Simplify. Assume that no denominator is equal to zero.

31. $\frac{-2a^3}{10a^8}$

32. $\frac{15b}{45b^5}$

33. $\frac{30h^{-2}k^{14}}{5hk^{-3}}$

34. $\frac{18x^3 y^4 z^7}{-2x^2 yz}$

35. $\frac{-19y^0 z^4}{-3z^{16}}$

36. $\frac{(5r^{-2})^{-2}}{(2r^3)^2}$

37. $\frac{p^{-4} q^{-3}}{(p^5 q^2)^{-1}}$

38. $\left(\frac{r^{-2} t^5}{t^{-1}}\right)^0$

39. $\left(\frac{5b^{-2} n^4}{n^2 z^{-3}}\right)^{-1}$

PROBABILITY **For Exercises 40 and 41, use the following information.**

If you toss a coin, the probability of getting heads is $\frac{1}{2}$. If you toss a coin 2 times, the probability of getting heads each time is $\frac{1}{2} \cdot \frac{1}{2}$ or $\left(\frac{1}{2}\right)^2$.

40. Write an expression to represent the probability of tossing a coin n times and getting n heads.

41. Express your answer to Exercise 40 as a power of 2.

Real-World Link

Timbre is the quality of the sound produced by a musical instrument. Sound quality is what distinguishes the sound of a note played on a flute from the sound of the same note played on a trumpet with the same frequency and intensity.

Source: www.school.discovery.com

SOUND **For Exercises 42–44, use the following information.**
The intensity of sound can be measured in watts per square meter. The table gives the watts per square meter for some common sounds.

Watts per Square Meter	Common Sounds
10^2	jet plane (30 m away)
10^1	pain level
10^0	amplified music (2 m away)
10^{-2}	noisy kitchen
10^{-3}	heavy traffic
10^{-6}	normal conversation
10^{-7}	average home
10^{-9}	soft whisper
10^{-12}	barely audible

42. How many times more intense is the sound from heavy traffic than the sound from normal conversation?

43. What sound is 10,000 times as loud as a noisy kitchen?

44. How does the intensity of a whisper compare to that of normal conversation?

LIGHT **For Exercises 45 and 46, use the table at the right.**

45. Express the range of the wavelengths of visible light using positive exponents. Then evaluate each expression.

46. Express the range of the wavelengths of X rays using positive exponents. Then evaluate each expression.

Spectrum of Electromagnetic Radiation

Region	Wavelength (om)
Radio	greater than 10
Microwave	10^1 to 10^{-2}
Infrared	10^{-2} to 10^{-5}
Visible	10^{-5} to 10^{-4}
Ultraviolet	10^{-4} to 10^{-7}
X rays	10^{-7} to 10^{-9}
Gamma Rays	less than 10^{-9}

EXTRA PRACTICE
See pages 731, 750.

Math Online
Self-Check Quiz at algebra1.com

47. COMPUTERS In 1993, the processing speed of a desktop computer was about 10^8 instructions per second. By 2004, it had increased to 10^{10} instructions per second. How many times faster is the newer computer?

H.O.T. Problems

48. OPEN ENDED Name two monomials whose product is $54x^2y^3$.

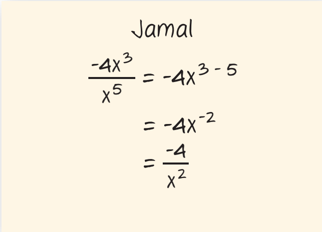
49. ALTERNATIVE METHODS Describe a method of simplifying $\frac{a^3b^5}{ab^2}$ using negative exponents instead of the Quotient of Powers Property.

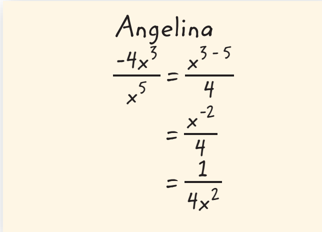
CHALLENGE **Simplify. Assume that no denominator equals zero.**

50. $a^n(a^3)$ **51.** $(5^{4x-3})(5^{2x+1})$ **52.** $\frac{c^{x+7}}{c^{x-4}}$

53. REASONING Write a convincing argument to show why $3^0 = 1$ using the following pattern: $3^5 = 243$, $3^4 = 81$, $3^3 = 27$, $3^2 = 9$.

54. FIND THE ERROR Jamal and Angelina are simplifying $\frac{-4x^3}{x^5}$. Who is correct? Explain your reasoning.





55. *Writing in Math* Use the information about pH levels on page 366 to explain how you can use the properties of exponents to compare measures on the pH scale. Demonstrate an example comparing two pH levels using the properties of exponents.

STANDARDIZED TEST PRACTICE

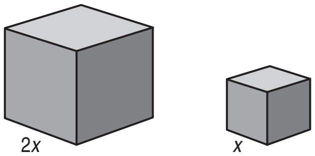
56. How many times greater is the volume of the larger cube than the volume of the smaller cube?

A 2

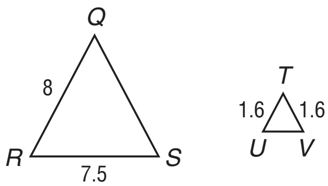
B 4

C 8

D 16



57. REVIEW $\triangle QRS$ is similar to $\triangle TUV$. What is the length of $\overline{UV}$?



F 11.0 **G** 2.3 **H** 1.7 **J** 1.5

Spiral Review

Simplify. (Lesson 7-1)

58. $(m^3n)(mn^2)$ **59.** $(3x^4y^3)(4x^4y)$ **60.** $(a^3x^2)^4$

61. $(3cd^5)^2$ **62.** $[(2^3)^2]^2$ **63.** $(-3ab)^3(2b^3)^2$

NUTRITION For Exercises 64 and 65, use the following information.
Between the ages of 11 and 18, you should get at least 1200 milligrams of calcium each day. One ounce of mozzarella cheese has 147 milligrams of calcium, and one ounce of Swiss cheese has 219 milligrams. Suppose you want to eat no more than 8 ounces of cheese. (Lesson 6-8)

64. Draw a graph showing the possible amounts of each type of cheese you can eat and still get your daily requirement of calcium. Let x be the amount of mozzarella cheese and y be the amount of Swiss cheese.

65. List three possible solutions.

GET READY for the Next Lesson

PREREQUISITE SKILL Evaluate each expression when $a = 5$, $b = -2$, and $c = 3$. (Lesson 1-2)

66. $5b^2$ **67.** $b^3 + 3ac$ **68.** $-2b^4 - 5b^3 - b$

READING MATH

Mathematical Prefixes and Everyday Prefixes

You may have noticed that many prefixes used in mathematics are also used in everyday language. You can use the everyday meaning of these prefixes to better understand their mathematical meaning. The table shows four mathematical prefixes along with their meaning and an example of an everyday word using that prefix.

Prefix	Everyday Meaning	Example
mono-	**1.** one; single; alone	**monologue** A continuous series of jokes or comic stories delivered by one comedian.
bi-	**1.** two **2.** both **3.** both sides, parts, or directions	**bicycle** A vehicle with two wheels behind one another.
tri-	**1.** three **2.** occurring at intervals of three **3.** occurring three times during	**trilogy** A group of three dramatic or literary works related in subject or theme.
poly-	**1.** more than one; many; much	**polygon** A closed plane figure bounded by three or more line segments.

Source: *The American Heritage Dictionary of the English Language*

You can use your everyday understanding of prefixes to help you understand mathematical terms that use those prefixes.

Reading to Learn

1. Give an example of a geometry term that uses one of these prefixes. Then define that term.

2. **MAKE A CONJECTURE** Given your knowledge of the meaning of the word monomial, make a conjecture as to the meaning of each of the following mathematical terms.

a. binomial **b.** trinomial **c.** polynomial

3. Research the following prefixes and their meanings.

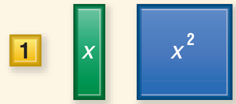
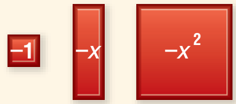
a. semi- **b.** hexa- **c.** octa-

d. penta- **e.** tri- **f.** quad-

EXPLORE
7-3

Algebra Lab
Polynomials

Algebra tiles can be used to model polynomials. A polynomial is a monomial or the sum of monomials. The diagram at the right shows the models.

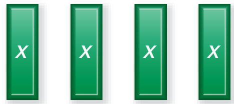
Polynomial Models	
Polynomials are modeled using three types of tiles.	
Each tile has an opposite.	

ACTIVITY

Use algebra tiles to model each polynomial.

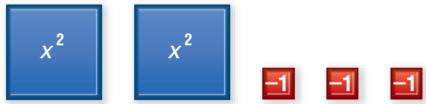
- $4x$

  To model this polynomial, you will need 4 green x-tiles.



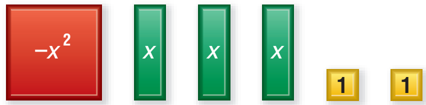
- $2x^2 - 3$

  To model this polynomial, you will need 2 blue x^2-tiles and 3 red -1-tiles.



- $-x^2 + 3x + 2$

  To model this polynomial, you will need 1 red $-x^2$-tile, 3 green x-tiles, and 2 yellow 1-tiles.



Model and Analyze

Use algebra tiles to model each polynomial. Then draw a diagram of your model.

1. $-2x^2$

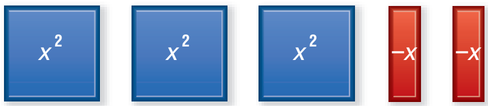
2. $5x - 4$

3. $3x^2 - x$

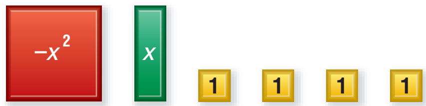
4. $x^2 + 4x + 3$

Write an algebraic expression for each model.

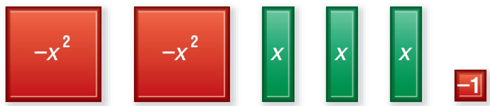
5.



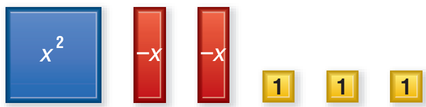
6.



7.



8.



9. **MAKE A CONJECTURE** Write a sentence or two explaining why algebra tiles are sometimes called *area tiles*.

7-3 Polynomials

Main Ideas

- Find the degree of a polynomial.
- Arrange the terms of a polynomial in ascending or descending order.

New Vocabulary

polynomial
binomial
trinomial
degree of a monomial
degree of a polynomial

GET READY for the Lesson

The number of hours H spent per person per year playing video games from 2000 through 2005 is shown in the table. These data can be modeled by the equation

$$H = \frac{1}{4}(t^4 - 9t^3 + 24t^2 + 19t + 280),$$

where t is the number of years since 2000. The expression $t^4 - 9t^3 + 24t^2 + 19t + 280$ is an example of a polynomial.

Video Game Usage

Year	Hours per Year
2000	70
2001	79
2002	90
2003	97
2004	103
2005	115

Source: U.S. Census Bureau

Degree of a Polynomial A **polynomial** is a monomial or a sum of monomials. Some polynomials have special names. A **binomial** is the sum of *two* monomials, and a **trinomial** is the sum of *three* monomials.

Monomial	Binomial	Trinomial
7	$3 + 4y$	$x + y + z$
$4ab^3c^2$	$7pqr + pq^2$	$3v^2 - 2w + ab^3$

EXAMPLE Identify Polynomials

1 **State whether each expression is a polynomial. If it is a polynomial, identify it as a *monomial*, *binomial*, or *trinomial*.**

	Expression	Polynomial?	Monomial, Binomial, or Trinomial?
a.	$2x - 3yz$	Yes, $2x - 3yz = 2x + (-3yz)$, the sum of two monomials.	binomial
b.	$8n^3 + 5n^{-2}$	No. $5n^{-2} = \frac{5}{n^2}$, which is not a monomial.	none of these
c.	-8	Yes, -8 is a real number.	monomial
d.	$4a^2 + 5a + a + 9$	Yes, the expression simplifies to $4a^2 + 6a + 9$, so it is the sum of three monomials.	trinomial

CHECK Your Progress

1A. x

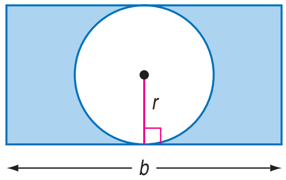
1B. $-3y^2 - 2y + 4y - 1$

1C. $5rs + 7tuv$

1D. $10x^{-4} - 8x^3$

EXAMPLE Write a Polynomial

2 **GEOMETRY** **Write a polynomial to represent the area of the shaded region.**



Words The area of the shaded region is the area of the rectangle minus the area of the circle.

Variables
area of shaded region = A
width of rectangle = $2r$
rectangle area = $b(2r)$
circle area = πr^2

Area of shaded region = rectangle area − circle area.

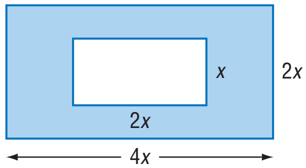
Equation

$$A = b(2r) - \pi r^2$$
$$A = 2br - \pi r^2$$

The polynomial representing the area of the shaded region is $2br - \pi r^2$.

CHECK Your Progress

2. Write a polynomial to represent the area of the shaded region.



Online Personal Tutor at algebra1.com

Concepts in Motion
Interactive Lab
algebra1.com

The **degree of a monomial** is the sum of the exponents of all its variables.

Monomial	Degree
$8y^4$	4
$3a$	1
$-2xy^2z^3$	1 + 2 + 3 or 6
7	0

The **degree of a polynomial** is the greatest degree of any term in the polynomial. To find the degree of a polynomial, you must find the degree of each term.

Reading Math

Degrees of 1 and 0

- Since $a = a^1$, the monomial $3a$ can be rewritten as $3a^1$. Thus $3a$ has degree 1.
- Since $x^0 = 1$, the monomial 7 can be rewritten as $7x^0$. Thus 7 has degree 0.

EXAMPLE Degree of a Polynomial

3 **Find the degree of each polynomial.**

	Polynomial	Terms	Degree of Each Term	Degree of Polynomial
a.	$5mn^2$	$5mn^2$	3	3
b.	$-4x^2y^2 + 3x^2 + 5$	$-4x^2y^2, 3x^2, 5$	4, 2, 0	4
c.	$3a + 7ab - 2a^2b + 16$	$3a, 7ab, -2a^2b, 16$	1, 2, 3, 0	3

CHECK Your Progress

3A. $7xy^5z$ **3B.** $12m^3n^2 - 8mn^2 + 3$ **3C.** $2rs - 3rs^2 - 7r^2s^2 - 13$

Write Polynomials in Order The terms of a polynomial are usually arranged so that the powers of one variable are in *ascending* (increasing) order or *descending* (decreasing) order.

Math Online Extra Examples at algebra1.com

EXAMPLE 4 Arrange Polynomials in Ascending Order

Arrange the terms of each polynomial so that the powers of x are in ascending order.

a. $7x^2 + 2x^4 - 11$

$7x^2 + 2x^4 - 11 = 7x^2 + 2x^4 - 11x^0$ $\quad x^0 = 1$

$= -11 + 7x^2 + 2x^4$ $\quad$ Compare powers of x: $0 < 2 < 4$.

b. $2xy^3 + y^2 + 5x^3 - 3x^2y$

$2xy^3 + y^2 + 5x^3 - 3x^2y$

$= 2x^1y^3 + y^2 + 5x^3 - 3x^2y^1$ $\quad x = x^1$

$= y^2 + 2xy^3 - 3x^2y + 5x^3$ $\quad$ Compare powers of x: $0 < 1 < 2 < 3$.

CHECK Your Progress

4A. $3x^2y^4 + 2x^4y^2 - 4x^3y + x^5 - y^2$ **4B.** $7x^3 - 4xy^4 + 3x^2y^3 - 11x^6y$

EXAMPLE 5 Arrange Polynomials in Descending Order

Arrange the terms of each polynomial so that the powers of x are in descending order.

a. $6x^2 + 5 - 8x - 2x^3$

$6x^2 + 5 - 8x - 2x^3 = 6x^2 + 5x^0 - 8x^1 - 2x^3$ $\quad x^0 = 1$ and $x = x^1$

$= -2x^3 + 6x^2 - 8x + 5$ $\quad 3 > 2 > 1 > 0$

b. $3a^3x^2 - a^4 + 4ax^5 + 9a^2x$

$3a^3x^2 - a^4 + 4ax^5 + 9a^2x$

$= 3a^3x^2 - a^4x^0 + 4a^1x^5 + 9a^2x^1$ $\quad a = a^1$, $x^0 = 1$, and $x = x^1$

$= 4ax^5 + 3a^3x^2 + 9a^2x - a^4$ $\quad 5 > 2 > 1 > 0$.

CHECK Your Progress

5A. $4x^2 + 2x^3y + 5 - x$ **5B.** $x + 2x^7y - 5x^4y^8 - x^2y^2 + 3$

CHECK Your Understanding

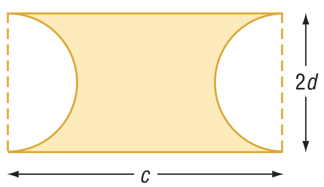
Example 1 (p. 376)

State whether each expression is a polynomial. If the expression is a polynomial, identify it as a *monomial*, a *binomial*, or a *trinomial*.

1. $5x - 3xy + 2x$ **2.** $\frac{2z}{5}$ **3.** $9a^2 + 7a - 5$

Example 2 (p. 377)

4. GEOMETRY Write a polynomial to represent the area of the shaded region.



Example 3 (p. 377)

Find the degree of each polynomial.

5. 1 **6.** $3x + 2$ **7.** $2x^2y^3 + 6x^4$

Example 4 (p. 378)

Arrange the terms of each polynomial so that the powers of *x* are in ascending order.

8. $6x^3 - 12 + 5x$ **9.** $-7a^2x^3 + 4x^2 - 2ax^5 + 2a$

Example 5 (p. 378)

Arrange the terms of each polynomial so that the powers of *x* are in descending order.

10. $2c^5 + 9cx^2 + 3x$ **11.** $y^3 + x^3 + 3x^2y + 3xy^2$

Exercises

HOMEWORK HELP

For Exercises	See Examples
12–17	1
18–21	2
22–33	3
34–41	4
42–49	5

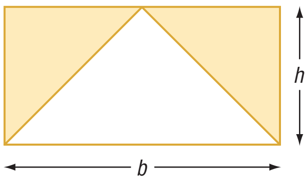
State whether each expression is a polynomial. If the expression is a polynomial, identify it as a *monomial*, a *binomial*, or a *trinomial*.

12. 14 **13.** $\frac{6m^2}{p} + p^3$ **14.** $7b - 3.2c + 8b$

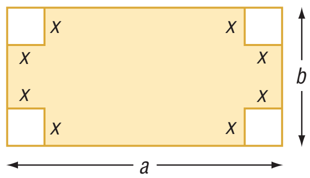
15. $\frac{1}{3}x^2 + x - 2$ **16.** $6gh^2 - 4g^2h + g$ **17.** $-4 + 2a + \frac{5}{a^2}$

GEOMETRY **Write a polynomial to represent the area of each shaded region.**

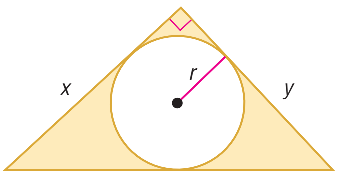
18.



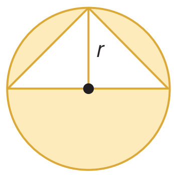
19.



20.



21.



Find the degree of each polynomial.

22. $5x^3$ **23.** $9y$ **24.** $4ab$

25. -13 **26.** $c^4 + 7c^2$ **27.** $6n^3 - n^2p^2$

28. $15 - 8ag$ **29.** $3a^2b^3c^4 - 18a^5c$ **30.** $2x^3 - 4y + 7xy$

31. $3z^5 - 2x^2y^3z - 4x^2z$ **32.** $7 + d^5 - b^2c^2d^3 + b^6$ **33.** $11r^2t^4 - 2s^4t^5 + 24$

Arrange the terms of each polynomial so that the powers of *x* are in ascending order.

34. $2x + 3x^2 - 1$ **35.** $9x^3 + 7 - 3x^5$

36. $c^2x^3 - c^3x^2 + 8c$ **37.** $x^3 + 4a + 5a^2x^6$

38. $4 + 3ax^5 + 2ax^2 - 5a^7$ **39.** $10x^3y^2 - 3x^9y + 5y^4 + 2x^2$

40. $3xy^2 - 4x^3 + x^2y + 6y$ **41.** $-8a^5x + 2ax^4 - 5 - a^2x^2$

EXTRA PRACTICE

See pages 731, 750.

Math Online

Self-Check Quiz at algebra1.com

Real-World Link

From 1980 to 1999, the number of triplet and higher births rose approximately 532% (from 1377 to 7321 births). This steep climb in multiple births coincides with the increased use of fertility drugs.

Source: National Center for Health and Statistics

Arrange the terms of each polynomial so that the powers of x are in descending order.

42. $5 + x^5 + 3x^3$

43. $2x - 1 + 6x^2$

44. $4a^3x^2 - 5a + 2a^2x^3$

45. $b^2 + x^2 - 2xb$

46. $c^2 + cx^3 - 5c^3x^2 + 11x$

47. $9x^2 + 3 + 4ax^3 - 2a^2x$

48. $8x - 9x^2y + 7y^2 - 2x^4$

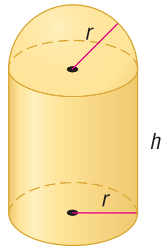
49. $4x^3y + 3xy^4 - x^2y^3 + y^4$

50. **MONEY** Write a polynomial to represent the value of q quarters, d dimes and n nickels.

51. **MULTIPLE BIRTHS** The rate of quadruplet births Q in the United States in recent years can be modeled by $Q = -0.5t^3 + 11.7t^2 - 21.5t + 218.6$, where t represents the number of years since 1992. For what values of t does this model no longer give realistic data? Explain your reasoning.

PACKAGING **For Exercises 52 and 53, use the following information.**

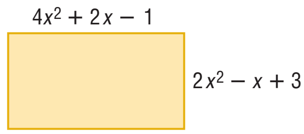
A convenience store sells milkshakes in cups with semispherical lids. The volume of a cylinder is the product of π, the square of the radius r, and the height h. The volume of a sphere is the product of $\frac{4}{3}$, π, and the cube of the radius.



52. Write a polynomial that represents the volume of the container.

53. If the height of the container is 6 inches and the radius is 2 inches, find the volume of the container.

54. Write two polynomials that represent the perimeter and area of the rectangle shown at right.



H.O.T. Problems

55. **OPEN ENDED** Give an example of a monomial of degree zero.

56. **REASONING** Explain why a polynomial cannot contain a variable term with a negative power.

57. **CHALLENGE** Tell whether the following statement is *true* or *false*. Explain your reasoning.

The degree of a binomial can never be zero.

58. **REASONING** Determine whether each statement is *true* or *false*. If false, give a counterexample.

a. All binomials are polynomials.

b. All polynomials are monomials.

c. All monomials are polynomials.

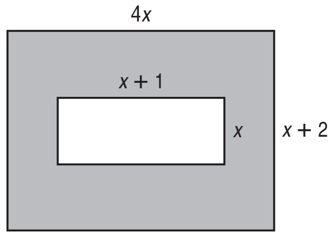
59. ***Writing in Math*** Use the information about video game usage on page 376 to explain how polynomials can be useful in modeling data. Include a discussion of the accuracy of the equation by evaluating the polynomial for $t = \{0, 1, 2, 3, 4, 5\}$ and an example of how and why someone might use this equation.

STANDARDIZED TEST PRACTICE

60. Which expression could be used to represent the area of the shaded region of the rectangle, reduced to simplest terms?

A $x^2 + 3x$ **C** $3x^2 - 7x$

B $4x^2 + 8x$ **D** $4x^2 + 7x$



61. REVIEW Lawanda rolled a six-sided game cube 30 times and recorded her results in the table below. Each side of the cube is a different color. Which color has the same experimental probability and theoretical probability?

F Purple

G Red

H White

J Orange

Color	Rolls
Red	4
Blue	8
White	4
Orange	3
Green	6
Purple	5

Spiral Review

Simplify. Assume that no denominator is equal to zero. (Lesson 7-2)

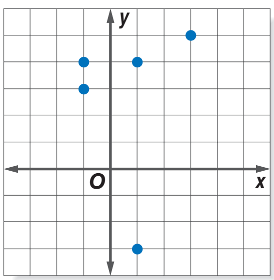
62. $a^0b^{-2}c^{-1}$ **63.** $\frac{-5n^5}{n^8}$ **64.** $\left(\frac{4x^3y^2}{3z}\right)^2$ **65.** $\frac{(-y)^5m^8}{y^3m^{-7}}$

Determine whether each expression is a monomial. Write *yes* or *no*. (Lesson 7-1)

66. $3a + 4b$ **67.** $\frac{6}{n}$ **68.** $\frac{v^2}{3}$

Determine whether each relation is a function. (Lesson 3-6)

69.



70.

x	y
−2	−2
0	1
3	4
5	−2

71. MAPS The scale of a road map is 1.5 inches = 100 miles. The distance between New Hartford, Connecticut, and Westerly, Rhode Island, by highway on the map is about 1.0 inch. What is the distance between these two cities? (Lesson 2-6)

Find each square root. Round to the nearest hundredth if necessary. (Lesson 1-8)

72. $\pm\sqrt{121}$ **73.** $\sqrt{3.24}$ **74.** $-\sqrt{52}$

GET READY for the Next Lesson

PREREQUISITE SKILL Simplify each expression, if possible. If not possible, write *in simplest form*. (Lesson 1-5)

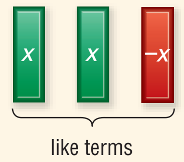
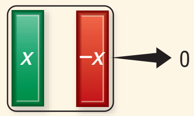
75. $3n + 5n$ **76.** $9a^2 + 3a - 2a^2$

77. $-3a + 5b + 4a - 7b$ **78.** $4x + 3y - 6 + 7x + 8 - 10y$

EXPLORE 7-4

Algebra Lab
Adding and Subtracting Polynomials

Monomials such as $5x$ and $-3x$ are called *like terms* because they have the same variable to the same power. When you use algebra tiles, you can recognize like terms because these tiles have the same size and shape as each other.

Polynomial Models	
Like terms are represented by tiles that have the same shape and size.	
A *zero pair* may be formed by pairing one tile with its opposite. You can remove or add zero pairs without changing the value of the polynomial.	

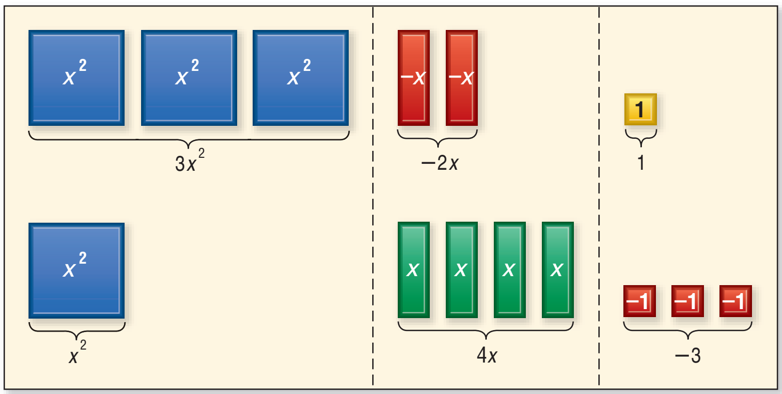
ACTIVITY 1 **Use algebra tiles to find $(3x^2 - 2x + 1) + (x^2 + 4x - 3)$.**

Concepts in MOtion
BrainPOP®
algebra1.com

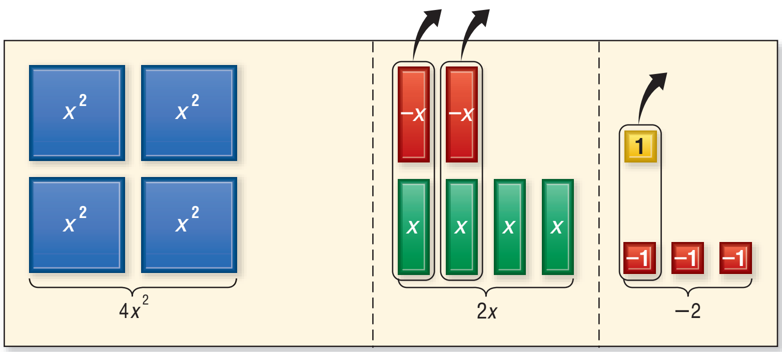
Step 1 Model each polynomial.

$3x^2 - 2x + 1 \longrightarrow$

$x^2 + 4x - 3 \longrightarrow$



Step 2 Combine like terms and remove zero pairs.

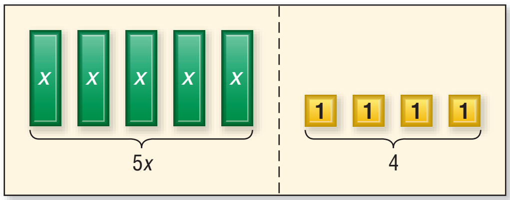


Step 3 Write the polynomial for the tiles that remain.

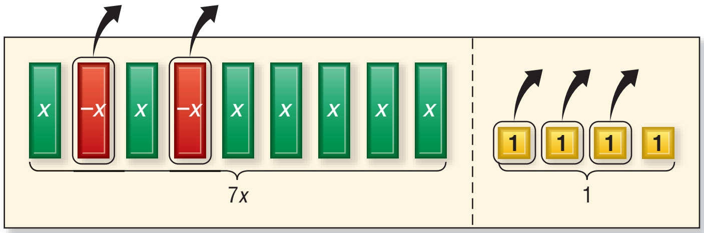
$(3x^2 - 2x + 1) + (x^2 + 4x - 3) = 4x^2 + 2x - 2$

ACTIVITY 2 Use algebra tiles to find $(5x + 4) - (-2x + 3)$.

Step 1 Model the polynomial $5x + 4$.



Step 2 To subtract $-2x + 3$, you must remove 2 $-x$-tiles and 3 1-tiles. You can remove the 1-tiles, but there are no $-x$-tiles. Add 2 zero pairs of x-tiles. Then remove the 2 $-x$-tiles.



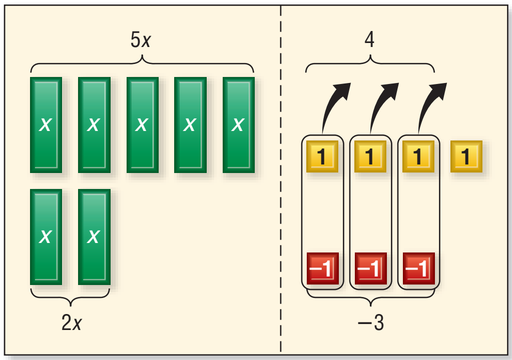
Step 3 The remaining tiles model $7x + 1$.

Recall that you can subtract a number by adding its additive inverse or opposite. Similarly, you can subtract a polynomial by adding its opposite.

ACTIVITY 3 Use algebra tiles and the additive inverse, or opposite, to find $(5x + 4) - (-2x + 3)$.

Step 1 To find the difference of $5x + 4$ and $-2x + 3$, add $5x + 4$ and the opposite of $-2x + 3$. The opposite of $-2x + 3$ is $2x - 3$.

Step 2 Write the polynomial for the tiles that remain.
$(5x + 4) - (-2x + 3) = 7x + 1$
Notice that this is the same answer as in Activity 2.



Concepts in Motion
Animation algebra1.com

Model and Analyze

Use algebra tiles to find each sum or difference.

1. $(5x^2 + 3x - 4) + (2x^2 - 4x + 1)$
2. $(2x^2 + 5) + (3x^2 + 2x + 6)$
3. $(-4x^2 + x) + (5x - 2)$
4. $(3x^2 + 4x + 2) - (x^2 - 5x - 5)$
5. $(-x^2 + 7x) - (-x^2 + 3x)$
6. $(8x + 4) - (6x^2 + x - 3)$
7. Find $(2x^2 - 3x + 1) - (2x + 3)$ using each method from Activities 2 and 3. Illustrate and explain how zero pairs are used in each case.

7-4 Adding and Subtracting Polynomials

Main Ideas

- Add polynomials.
- Subtract polynomials.

GET READY for the Lesson

From 2000 to 2003, the amount of sales (in millions of dollars) of rap/hip-hop music R and country music C in the United States can be modeled by the following equations, where t is the number of years since 2000.

$$R = -132.32t^3 + 624.74t^2 - 773.61t + 1847.67$$
$$C = -3.42t^3 + 8.6t^2 - 94.95t + 1532.56$$

The total music sales T of rap/hip-hop and country music is $R + C$.

Add Polynomials To add polynomials, you can group like terms horizontally or write them in column form, aligning like terms. Adding polynomials involves adding like terms.

EXAMPLE 1 **Add Polynomials**

Find $(3x^2 - 4x + 8) + (2x - 7x^2 - 5)$.

Method 1 Horizontal

$(3x^2 - 4x + 8) + (2x - 7x^2 - 5)$

$= [3x^2 + (-7x^2)] + (-4x + 2x) + [8 + (-5)]$ Group like terms.

$= -4x^2 - 2x + 3$ Add like terms.

Method 2 Vertical

$$\begin{array}{r} 3x^2 - 4x + 8 \\ (+)\ -7x^2 + 2x - 5 \\ \hline -4x^2 - 2x + 3 \end{array}$$

Notice that terms are in descending order with like terms aligned.

Study Tip

Adding Columns

When adding like terms in column form, remember that you are adding integers. Rewrite each monomial to eliminate subtractions. For example, you could rewrite $3x^2 - 4x + 8$ as $3x^2 + (-4x) + 8$.

CHECK Your Progress

1. Find $(5x^2 - 3x + 4) + (6x - 3x^2 - 3)$.

Subtract Polynomials Recall that you can subtract a real number by adding its opposite or additive inverse. Similarly, you can subtract a polynomial by adding its additive inverse.

To find the additive inverse of a polynomial, replace each term with its additive inverse.

Polynomial	Additive Inverse
$-5m + 3n$	$5m - 3n$
$2y^2 - 6y + 11$	$-2y^2 + 6y - 11$
$7a + 9b - 4$	$-7a - 9b + 4$

EXAMPLE 2 Subtract Polynomials

Find $(3n^2 + 13n^3 + 5n) - (7n + 4n^3)$.

Method 1 Horizontal

Subtract $7n + 4n^3$ by adding its additive inverse.

$$
\begin{aligned}
&(3n^2 + 13n^3 + 5n) - (7n + 4n^3) \\
&= (3n^2 + 13n^3 + 5n) + (-7n - 4n^3) && \text{The additive inverse of } 7n + 4n^3 \text{ is } -7n - 4n^3. \\
&= 3n^2 + [13n^3 + (-4n^3)] + [5n + (-7n)] && \text{Group like terms.} \\
&= 3n^2 + 9n^3 - 2n && \text{Combine like terms.}
\end{aligned}
$$

Method 2 Vertical

Align like terms in columns and subtract by adding the additive inverse.

$$
\begin{array}{rr}
 & 3n^2 + 13n^3 + 5n \\
(-) & 4n^3 + 7n \\ \hline
\end{array}
\quad \text{Add the opposite.} \quad
\begin{array}{rr}
 & 3n^2 + 13n^3 + 5n \\
(+) & -4n^3 - 7n \\ \hline
 & 3n^2 + 9n^3 - 2n
\end{array}
$$

Thus, $(3n^2 + 13n^3 + 5n) - (7n + 4n^3) = 3n^2 + 9n^3 - 2n$ or, arranged in descending order, $9n^3 + 3n^2 - 2n$.

Check Your Progress

2. Find $(4x^3 - 3x^2 + 6x - 4) - (-2x^3 + x^2 - 2)$.

When polynomials are used to model real-world data, their sums and differences can have real-world meaning, too.

Real-World Career

Teacher

The educational requirements for a teaching license vary by state. In the 2004–2005 school year, the average public K–12 teacher salary was $47,808.

Math Online

For more information, go to algebra1.com.

Real-World EXAMPLE 3

EDUCATION The total number of public school teachers T consists of two groups, elementary E and secondary S. From 1992 through 2003, the number (in thousands) of secondary teachers and total teachers could be modeled by the following equations, where n is the number of years since 1992.

$S = 29n + 949 \quad T = 58n + 2401$

a. Find an equation that models the number of elementary teachers E for this time period.

Subtract the polynomial for S from the polynomial for T.

$$
\begin{array}{l}
\text{Total} \\
-\ \text{Secondary} \\ \hline
\text{Elementary}
\end{array}
\quad
\begin{array}{rr}
 & 58n + 2401 \\
(-) & 29n + 949 \\ \hline
\end{array}
\quad \text{Add the opposite.} \quad
\begin{array}{rr}
 & 58n + 2401 \\
(+) & -29n - 949 \\ \hline
 & 29n + 1452
\end{array}
$$

An equation is $E = 29n + 1452$.

b. Use the equation to predict the number of elementary teachers in 2015.

The year 2015 is 2015 − 1992 or 23 years after the year 1992.

If this trend continues, the number of elementary teachers in 2015 would be 29(23) + 1452 or about 2,119,000.

Study Tip

Inverse of a Polynomial

When finding the additive inverse of a polynomial, remember to find the additive inverse of *every* term.

CHECK Your Progress

3. **WIRELESS DEVICES** An electronics store sells cell phones and pagers. The equations below represent the monthly sales m of cell phones C and pagers P. Write an equation that represents the total monthly sales T of wireless devices. Use the equation to predict the number of wireless devices sold in 10 months.

$C = 7m + 137 \quad P = 4m + 78$

Online Personal Tutor at algebra1.com

CHECK Your Understanding

Examples 1, 2 (pp. 384, 385)

Find each sum or difference.

1. $(4p^2 + 5p) + (-2p^2 + p)$
2. $(5y^2 - 3y + 8) + (4y^2 - 9)$
3. $(8cd - 3d + 4c) + (-6 + 2cd)$
4. $(-8xy + 3x^2 - 5y) + (4x^2 - 2y + 6xy)$
5. $(6a^2 + 7a - 9) - (-5a^2 + a - 10)$
6. $(g^3 - 2g^2 + 5g + 6) - (g^2 + 2g)$
7. $(3ax^2 - 5x - 3a) - (6a - 8a^2x + 4x)$
8. $(4rst - 8r^2s + s^2) - (6rs^2 + 5rst - 2s^2)$

Example 3 (p. 385)

POPULATION For Exercises 9 and 10, use the following information.
From 1980 through 2003, the female population F and the male population M of the United States (in thousands) are modeled by the following equations, where n is the number of years since 1980.

$F = 1{,}379n + 115{,}513 \qquad M = 1{,}450n + 108{,}882$

9. Find an equation that models the total population T of the United States in thousands for this time period.
10. If this trend continues, what will the population of the U. S. be in 2010?

Exercises

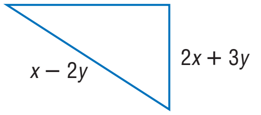
HOMEWORK HELP	
For Exercises	See Examples
11–16	1
17–22	2
23–24	3

Find each sum or difference.

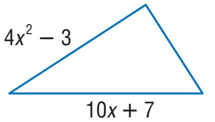
11. $(6n^2 - 4) + (-2n^2 + 9)$
12. $(9z - 3z^2) + (4z - 7z^2)$
13. $(3 + a^2 + 2a) + (a^2 - 8a + 5)$
14. $(-3n^2 - 8 + 2n) + (5n + 13 + n^2)$
15. $(x + 5) + (2y + 4x - 2)$
16. $(2b^3 - 4b + b^2) + (-9b^2 + 3b^3)$
17. $(11 + 4d^2) - (3 - 6d^2)$
18. $(4g^3 - 5g) - (2g^3 + 4g)$
19. $(-4y^3 - y + 10) - (4y^3 + 3y^2 - 7)$
20. $(4x + 5xy + 3y) - (3y + 6x + 8xy)$
21. $(3x^2 + 8x + 4) - (5x^2 - 4)$
22. $(5ab^2 + 3ab) - (2ab^2 + 4 - 8ab)$

GEOMETRY The measures of two sides of a triangle are given. If P is the perimeter, find the measure of the third side.

23. $P = 7x + 3y$



24. $P = 10x^2 - 5x + 16$



Find each sum or difference.

25. $(3a + 2b - 7c) + (6b - 4a + 9c) + (-7c - 3a - 2b)$

26. $(5x^2 - 3) + (x^2 - x + 11) + (2x^2 - 5x + 7)$

27. $(3y^2 - 8) + (5y + 9) - (y^2 + 6y - 4)$

28. $(9x^3 + 3x - 13) - (6x^2 - 5x) + (2x^3 - x^2 - 8x + 4)$

Real-World Link

In 2002, attendance at movie theaters was at its highest point in 40 years with 1.63 billion tickets sold for a record $9.52 billion in gross income.

Source: The National Association of Theatre Owners

MOVIES **For Exercises 29 and 30, use the following information.**
From 1995 to 2004, the number of indoor movie screens I and total movie screens T in the U.S. could be modeled by the following equations, where n is the number of years since 1995.

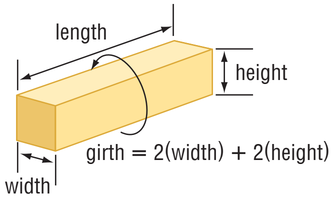
$$I = -194.8n^2 + 2{,}658n + 26{,}933 \qquad T = -193n^2 + 2{,}616n + 27{,}793$$

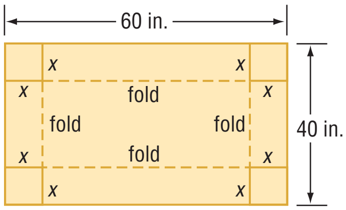
29. Find an equation that models the number of outdoor movie screens D.

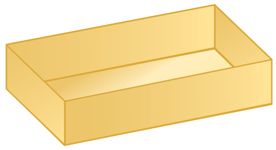
30. If this trend continues, how many outdoor screens will there be in 2010?

POSTAL SERVICE **For Exercises 31–33, use the following information.**
The U.S. Postal Service restricts the sizes of boxes shipped by parcel post. The sum of the length and the girth of the box must not exceed 108 inches.

Suppose you want to make an open box using a 60-by-40-inch piece of cardboard by cutting squares out of each corner and folding up the flaps. The lid will be made from another piece of cardboard. You do not know how big the squares should be, so for now call the length of the side of each square x.







31. Write polynomials to represent the length, width, and girth of the box formed.

32. Write and solve an inequality to find the least possible value of x you could use in designing this box so it meets postal regulations.

33. What is the greatest integral value of x you could use to design this box if it does not have to meet regulations?

EXTRA PRACTICE

See pages 731, 750.

Math Online

Self-Check Quiz at algebra1.com

H.O.T. Problems

34. REASONING Explain why $5xy^2$ and $3x^2y$ are *not* like terms.

35. OPEN ENDED Write two polynomials with a difference of $2x^2 + x + 3$.

36. FIND THE ERROR Esteban and Kendra are finding $(5a - 6b) - (2a + 5b)$. Who is correct? Explain your reasoning.

Esteban	Kendra
$(5a - 6b) - (2a + 5b)$	$(5a - 6b) - (2a + 5b)$
$= (-5a + 6b) + (-2a - 5b)$	$= (5a - 6b) + (-2a - 5b)$
$= -7a + b$	$= 3a - 11b$

Robert D. Macklin

CHALLENGE **For Exercises 37–39, suppose x is an integer.**

37. Write an expression for the next integer greater than x.

38. Show that the sum of two consecutive integers, x and the next integer after x, is always odd. (*Hint:* A number is considered even if it is divisible by 2.)

39. What is the least number of consecutive integers that must be added together to always arrive at an even integer?

40. ***Writing in Math*** Use the information about music sales on page 384 to explain how you can use polynomials to model sales. Include an equation that models total music sales, and an example of how and why someone might use this equation in your answer.

STANDARDIZED TEST PRACTICE

41. The perimeter of the rectangle shown below is $16a + 2b$. Which expression represents the width of the rectangle?

A $3a + 2b$

B $10a + 2b$

C $2a - 3b$

D $6a + 4b$

42. **REVIEW** The scale factor of two similar polygons is 4:5. The perimeter of the larger polygon is 200 inches. What is the perimeter of the smaller polygon?

F 250 inches

G 160 inches

H 80 inches

J 40 inches

Spiral Review

Find the degree of each polynomial. (Lesson 7-3)

43. $15t^3y^2$

44. 24

45. $m^2 + n^3$

46. $4x^2y^3z - 5x^3z$

Simplify. Assume no denominator is equal to zero. (Lesson 7-2)

47. $\dfrac{49a^4b^7c^2}{7ab^4c^3}$

48. $\dfrac{-4n^3p^{-5}}{n^{-2}}$

49. $\dfrac{(8n^7)^2}{(3n^2)^{-3}}$

KEYBOARDING **For Exercises 50–53, use the table that shows keyboarding speeds of 12 students in words per minute (wpm) and weeks of experience.** (Lesson 4-7)

Experience (weeks)	4	7	8	1	6	3	5	2	9	6	7	10
Keyboarding Speed (wpm)	33	45	46	20	40	30	38	22	52	44	42	55

50. Make a scatter plot of these data. Then draw a line of fit.

51. Find the equation of the line.

52. Use the equation to predict the speed of a student after a 12-week course.

53. Why is this equation not used to predict the speed for any number of weeks of experience?

GET READY for the Next Lesson

PREREQUISITE SKILL. **Simplify.** (Lesson 1-5)

54. $6(3x - 8)$

55. $-2(b + 9)$

56. $-7(-5p + 4q)$

57. $9(3a + 5b - c)$

58. $8(x^2 + 3x - 4)$

59. $-3(2a^2 - 5a + 7)$

CHAPTER 7

Mid-Chapter Quiz

Lessons 7-1 through 7-4

Simplify. (Lesson 7-1)

1. $n^3(n^4)(n)$
2. $4ad(3a^3d)$
3. $(-2w^3z^4)^3(-4wz^3)^2$

4. **MULTIPLE CHOICE** Ruby says that $(xy)^2 = x^2 + 2xy + y^2$ for every value of x and y, but Ebony disagrees. What does $(xy)^2$ really equal? (Lesson 7-1)

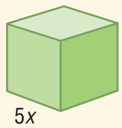
   A $2x^2y$

   B $2xy$

   C xy^2

   D x^2y^2

5. **MULTIPLE CHOICE** Which expression represents the volume of the cube? (Lesson 7-1)



   F $15x^3$

   G $25x^2$

   H $25x^3$

   J $125x^3$

Simplify. Assume that no denominator is equal to zero. (Lesson 7-2)

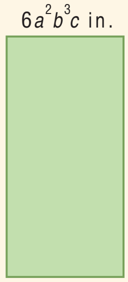
6. $\dfrac{25p^{10}}{15p^3}$

7. $\left(\dfrac{6k^3}{7np^4}\right)^2$

8. $\dfrac{4x^0y^2}{(3y^{-3}z^5)^{-2}}$

9. $\dfrac{(m^2np^3)^{-3}}{(m^5n^3p^6)^{-4}}$

10. **GEOMETRY** The area of the rectangle is $66a^3b^5c^7$ square inches. Find the length of the rectangle. (Lesson 7-2)



11. **MULTIPLE CHOICE** The wavelength of a microwave is 10^{-2} centimeters, and the wavelength of an X ray is 10^{-8} centimeters. How many times greater is the length of a microwave than an X ray? (Lesson 7-3)

    A 10^{10}

    B 10^6

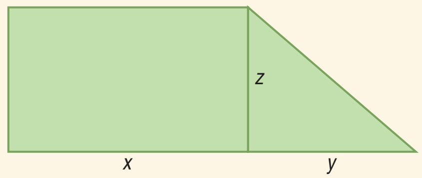
    C 10^{-6}

    D 10^{-10}

Find the degree of each polynomial. (Lesson 7-3)

12. $5x^4$
13. $-9n^3p^4$
14. $7a^2 - 2ab^2$
15. $-6 - 8x^2y^2 + 5y^3$

GEOMETRY For Exercises 16 and 17, use the figure below. (Lesson 7-3)



16. Write a polynomial that represents the area of the figure.

17. If $x = 7$ feet, $y = 3$ feet, and $z = 2$ feet, find the area of the figure.

Arrange the terms of each polynomial so that the powers of x are in ascending order. (Lesson 7-4)

18. $4x^2 + 9x - 12 + 5x^3$
19. $2xy^4 + x^3y^5 + 5x^5y - 13x^2$

20. **MULTIPLE CHOICE** If three consecutive integers are x, $x + 1$, and $x + 2$, what is the sum of these three integers? (Lesson 7-4)

    F $2x + 3$

    G $3x + 3$

    H $x(x + 1)(x + 2)$

    J $x^3 + 3x^2 + 2x$

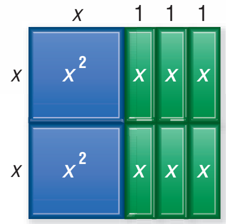
7-5 Multiplying a Polynomial by a Monomial

Main Ideas

- Find the product of a monomial and a polynomial.
- Solve equations involving polynomials.

GET READY for the Lesson

The algebra tiles shown are grouped together to form a rectangle with a width of $2x$ and a length of $x + 3$. Notice that the rectangle consists of 2 blue x^2-tiles and 6 green x-tiles. The area of the rectangle is the sum of these algebra tiles or $2x^2 + 6x$.



Product of Monomial and Polynomial The Distributive Property can be used to multiply a polynomial by a monomial.

EXAMPLE Multiply a Polynomial by a Monomial

1 **Find $-2x^2(3x^2 - 7x + 10)$.**

Method 1 Horizontal

$-2x^2(3x^2 - 7x + 10)$

$= -2x^2(3x^2) - (-2x^2)(7x) + (-2x^2)(10)$ Distributive Property

$= -6x^4 - (-14x^3) + (-20x^2)$ Multiply.

$= -6x^4 + 14x^3 - 20x^2$ Simplify.

Method 2 Vertical

$$\begin{array}{lr} & 3x^2 - 7x + 10 \\ (\times) & -2x^2 \\ \hline & -6x^4 + 14x^3 - 20x^2 \end{array}$$

Distributive Property

Multiply.

CHECK Your Progress

1. Find $5a^2(-4a^2 + 2a - 7)$.

Review Vocabulary

Distributive Property: For any numbers a, b, and c, $a(b + c) = ab + ac$ and $a(b - c) = ab - ac$. (Lesson 1–5)

EXAMPLE Simplify Expressions

2 **Simplify $4(3d^2 + 5d) - d(d^2 - 7d + 12)$.**

$4(3d^2 + 5d) - d(d^2 - 7d + 12)$

$= 4(3d^2) + 4(5d) + (-d)(d^2) - (-d)(7d) + (-d)(12)$ Distributive Property

$= 12d^2 + 20d + (-d^3) - (-7d^2) + (-12d)$ Product of Powers

$= 12d^2 + 20d - d^3 + 7d^2 - 12d$ Simplify.

$= -d^3 + (12d^2 + 7d^2) + (20d - 12d)$ Commutative and Associative Properties

$= -d^3 + 19d^2 + 8d$ Combine like terms.

CHECK Your Progress

2. Simplify $3(5x^2 + 2x - 4) - x(7x^2 + 2x - 3)$.

Real-World Link

About 98% of long-distance companies service their calls using the network of one of three companies. Since the quality of phone service is basically the same, a company's rates are the primary factor in choosing a long-distance provider.

Source: Chamberland Enterprises

Real-World EXAMPLE

3 **PHONE SERVICE** **Greg pays a fee of \$20 a month for local calls. Long-distance rates are 6¢ per minute for in-state calls and 5¢ per minute for out-of-state calls. Suppose Greg makes 300 minutes of long-distance phone calls in January and *m* of those minutes are for in-state calls.**

a. Find an expression for Greg's phone bill for January.

Words	Bill	=	service fee	+	in-state minutes	·	6¢ per minute	+	out-of-state minutes	·	5¢ per minute.
Variables	If m = number of minutes of in-state calls, then $300 - m$ = number of minutes of out-of-state calls. Let B = phone bill for the month of January.										
Equation	B	=	20	+	m	·	0.06	+	$(300 - m)$	·	0.05

$B = 20 + m \cdot 0.06 + (300 - m) \cdot 0.05$ — Write the equation.

$= 20 + 0.06m + 300(0.05) - m(0.05)$ — Distributive Property

$= 20 + 0.06m + 15 - 0.05m$ — Simplify.

$= 35 + 0.01m$ — Simplify.

Greg's bill for January is $35 + 0.01m$, for m minutes of in-state calls.

b. Evaluate the expression to find the cost if Greg had 37 minutes of in-state calls in January.

$35 + 0.01m = 35 + 0.01(37)$ — $m = 37$

$= 35 + 0.37$ — Multiply.

$= 35.37$ — Add.

Greg's bill was \$35.37.

CHECK Your Progress

3. A parking garage charges \$30 per month plus \$0.50 per daytime hour and \$0.25 per hour during nights and weekends. Suppose Juana parks in the garage for 47 hours in January and *h* of those are night and weekend hours. Find an expression for her January bill. Then find the cost if Juana had 12 hours of night and weekend hours.

Online Personal Tutor at algebra1.com

Solve Equations with Polynomial Expressions Many equations contain polynomials that must be added, subtracted, or multiplied.

EXAMPLE Polynomials on Both Sides

4 **Solve $y(y - 12) + y(y + 2) + 25 = 2y(y + 5) - 15$.**

$y(y - 12) + y(y + 2) + 25 = 2y(y + 5) - 15$ — Original equation

$y^2 - 12y + y^2 + 2y + 25 = 2y^2 + 10y - 15$ — Distributive Property

$2y^2 - 10y + 25 = 2y^2 + 10y - 15$ — Combine like terms.

$-10y + 25 = 10y - 15$ — Subtract $2y^2$ from each side.

$-20y + 25 = -15$ — Subtract $10y$ from each side.

$-20y = -40$ — Subtract 25 from each side.

$y = 2$ — Divide each side by −20.

CHECK $y(y - 12) + y(y + 2) + 25 = 2y(y + 5) - 15$ Original equation

$2(2 - 12) + 2(2 + 2) + 25 \stackrel{?}{=} 2(2)(2 + 5) - 15$ $y = 2$

$2(-10) + 2(4) + 25 \stackrel{?}{=} 4(7) - 15$ Simplify.

$-20 + 8 + 25 \stackrel{?}{=} 28 - 15$ Multiply.

$13 = 13$ ✓ Add and subtract.

CHECK Your Progress

4. Solve $2x(x + 4) + 7 = (x + 8) + 2x(x + 1) + 12$.

CHECK Your Understanding

Example 1 (p. 390)

Find each product.

1. $-3y(5y + 2)$

2. $9b^2(2b^3 - 3b^2 + b - 8)$

3. $2x(4a^4 - 3ax + 6x^2)$

4. $-4xy(5x^2 - 12xy + 7y^2)$

Example 2 (p. 390)

Simplify.

5. $t(5t - 9) - 2t$

6. $x(3x + 4) + 2(7x - 3)$

7. $5n(4n^3 + 6n^2 - 2n + 3) - 4(n^2 + 7n)$

8. $4y^2(y^2 - 2y + 5) + 3y(2y^2 - 2)$

Example 3 (p. 391)

SAVINGS For Exercises 9–11, use the following information.
Matthew's grandmother left him $10,000 for college. Matthew puts some of the money into a savings account earning 3% interest per year. With the rest, he buys a certificate of deposit (CD) earning 5% per year.

9. If Matthew puts x dollars into the savings account, write an expression to represent the amount of the CD.

10. Write an equation for the total amount of money T Matthew will have saved for college after one year.

11. If Matthew puts $3000 in savings, how much money will he have in one year?

Example 4 (p. 391)

Solve each equation.

12. $-2(w + 1) + w = 7 - 4w$

13. $3(y - 2) + 2y = 4y + 14$

14. $a(a + 3) + a(a - 6) + 35 = a(a - 5) + a(a + 7)$

15. $n(n - 4) + n(n + 8) = n(n - 13) + n(n + 1) + 16$

Exercises

HOMEWORK HELP

For Exercises	See Examples
16–25	1
26–31	2
32–35	3
36–43	4

Find each product.

16. $r(5r + r^2)$

17. $w(2w^3 - 9w^2)$

18. $-4x(8 + 3x)$

19. $5y(-2y^2 - 7y)$

20. $7ag(g^3 + 2ag)$

21. $-3np(n^2 - 2p)$

22. $-2b^2(3b^2 - 4b + 9)$

23. $6x^3(5 + 3x - 11x^2)$

24. $8x^2y(5x + 2y^2 - 3)$

25. $-cd^2(3d + 2c^2d - 4c)$

Simplify.

26. $d(-2d + 4) + 15d$

27. $-x(4x^2 - 2x) - 5x^3$

28. $3w(6w - 4) + 2(w^2 - 3w + 5)$

29. $5n(2n^3 + n^2 + 8) + n(4 - n)$

30. $10(4m^3 - 3m + 2) - 2m(-3m^2 - 7m + 1)$

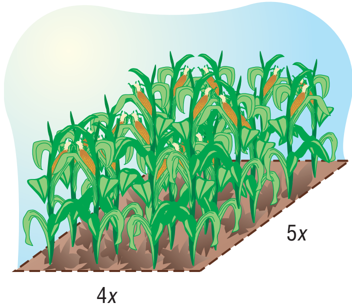
31. $4y(y^2 - 8y + 6) - 3(2y^3 - 5y^2 + 2)$

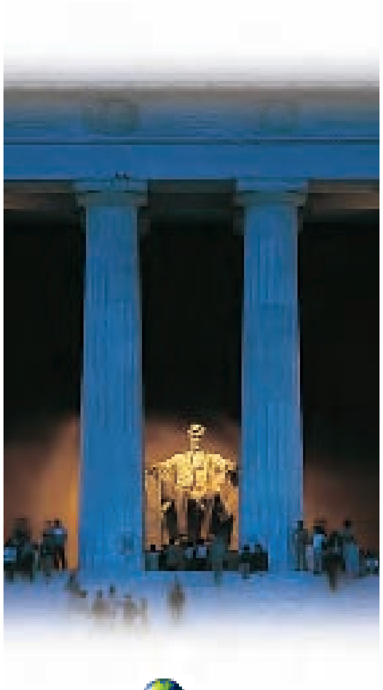
SAVINGS **For Exercises 32 and 33, use the following information.**
Marta has \$6000 to invest. She puts x dollars of this money into a savings account that earns 2% interest per year. With the rest, she buys a certificate of deposit that earns 4% per year.

32. Write an equation for the amount of money T Marta will have in one year.

33. Suppose at the end of one year, Marta has a total of \$6210. How much money did Marta invest in each account?

34. **FARMING** A farmer plants corn in a field with a length to width ratio of 5:4. Next year, he plans to increase the field's area by increasing its length by 12 feet. Write an expression for this new area.





Real-World Link
Inside the Lincoln Memorial is a 19-foot marble statue of the United States' 16th president. The statue is flanked on either side by the inscriptions of Lincoln's Second Inaugural Address and Gettysburg Address.

Source: washington.org

35. **CLASS TRIP** Mr. Wong's American History class will take taxis from their hotel in Washington, D.C., to the Lincoln Memorial. The fare is \$2.75 for the first mile and \$1.25 for each additional mile. If the distance is m miles and t taxis are needed, write an expression for the cost to transport the group.

Solve each equation.

36. $2(4x - 7) = 5(-2x - 9) - 5$

37. $4(3p + 9) - 5 = -3(12p - 5)$

38. $d(d - 1) + 4d = d(d - 8)$

39. $c(c + 3) - c(c - 4) = 9c - 16$

40. $a(3a - 2) + 2a(a + 4) = a(a + 2) + 4a(a - 3) + 48$

41. $3(4w - 2) + 6(w + 4) - 3 = 4w - 7(w + 2) + 5(3w + 7)$

Expand and simplify.

42. $4(x + 2) - 6$

43. $3x - 2(x + 1)$

Find each product.

44. $-\frac{3}{4}hk^2(20k^2 + 5h - 8)$

45. $\frac{2}{3}a^2b(6a^3 - 4ab + 9b^2)$

46. $-5a^3b(2b + 5ab - b^2 + a^3)$

47. $4p^2q^2(2p^2 - q^2 + 9p^3 + 3q)$

Simplify.

48. $-3c^2(2c + 7) + 4c(3c^2 - c + 5) + 2(c^2 - 4)$

49. $4x^2(x + 2) + 3x(5x^2 + 2x - 6) - 5(3x^2 - 4x)$

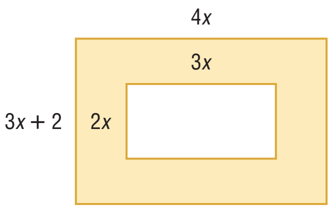
Solve each equation.

50. $2n(n + 4) + 18 = n(n + 5) + n(n - 2) - 7$

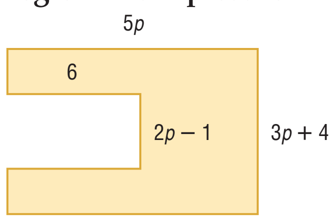
51. $3g(g - 4) - 2g(g - 7) = g(g + 6) - 28$

GEOMETRY **Find the area of each shaded region in simplest form.**

52.



53.



Real-World Link

Approximately one third of young people in grades 7–12 suggested that "working for the good of my community and country" and "helping others or volunteering" were important future goals.

Source: Primeday/Roper National Youth Opinion Survey

VOLUNTEERING **For Exercises 54 and 55, use the following information.**
Loretta is making baskets of apples and oranges for homeless shelters. She wants to place a total of 10 pieces of fruit in each basket. Apples cost 25¢ each, and oranges cost 20¢ each.

54. If a represents the number of apples Loretta uses, write a polynomial model in simplest form for the total amount of money T Loretta will spend.

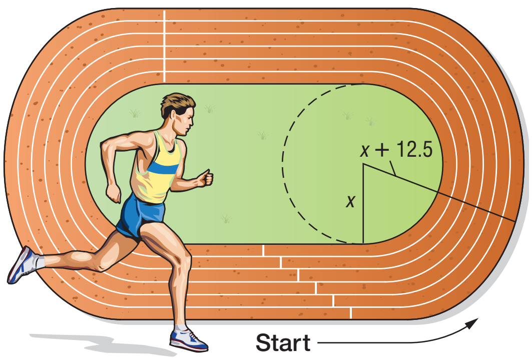
55. If Loretta uses 4 apples in each basket, find the total cost for fruit.

SALES **For Exercises 56 and 57, use the following information.**
A store advertises that all sports equipment is 30% off the retail price. In addition, the store asks customers to select and pop a balloon to receive a coupon for an additional n percent off one of their purchases.

56. Write an expression for the cost of a pair of inline skates with retail price p.

57. Use this expression to calculate the cost, not including sales tax, of a \$200 pair of inline skates for an additional 10% off.

58. **SPORTS** You may have noticed that when runners race around a curved track, their starting points are staggered. This is so each contestant runs the same distance to the finish line.



If the radius of the inside lane is x and each lane is 2.5 feet wide, how far apart should the officials start the runners in the inside lane and the outside (6th) lane? (*Hint*: Circumference = $2\pi r$, where r is the radius of the circle)

EXTRA PRACTICE
See pages 732, 750.
Math Online
Self-Check Quiz at algebra1.com

NUMBER THEORY **For Exercises 59 and 60, let x be an odd integer.**

59. Write an expression for the next odd integer.

60. Find the product of x and the next odd integer.

H.O.T. Problems

61. **OPEN ENDED** Write a monomial and a trinomial involving one variable. Then find their product.

CHALLENGE **For Exercises 62–64, use the following information.**
An even number can be represented by $2x$, where x is any integer.

62. Show that the product of two even integers is always even.

63. Write a representation for an odd integer.

64. Show that the product of an even and an odd integer is always even.

65. ***Writing in Math*** Use the information about the area of a rectangle on page 390 to explain how the product of a monomial and a polynomial relate to finding the area of a rectangle. Include the product of $2x$ and $x + 3$ derived algebraically in your answer.

STANDARDIZED TEST PRACTICE

66. A plumber charges $70 for the first thirty minutes of each house call plus $4 for each additional minute that she works. The plumber charges Ke-Min $122 for her time. What amount of time, in minutes, did the plumber work?

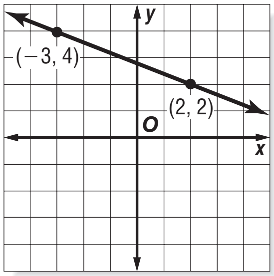
A 43

B 48

C 58

D 64

67. REVIEW What is the slope of this line?



F $-\frac{5}{2}$ **G** -2 **H** $-\frac{1}{2}$ **J** $-\frac{2}{5}$

Spiral Review

Find each sum or difference. (Lesson 7-4)

68. $(4x^2 + 5x) + (-7x^2 + x)$

69. $(3y^2 + 5y - 6) - (7y^2 - 9)$

70. $(5b - 7ab + 8a) - (5ab - 4a)$

71. $(6p^3 + 3p^2 - 7) + (p^3 - 6p^2 - 2p)$

State whether each expression is a polynomial. If the expression is a polynomial, identify it as a *monomial,* a *binomial,* or a *trinomial.* (Lesson 7-3)

72. $4x^2 - 10ab + 6$

73. $4c + ab - c$

74. $\frac{7}{y} + y^2$

75. $\frac{n^2}{3}$

Define a variable, write an inequality, and solve each problem. Then check your solution. (Lesson 6-3)

76. Six increased by ten times a number is less than nine times the number.

77. Nine times a number increased by four is no less than seven decreased by thirteen times the number.

Write an equation of the line that passes through each pair of points. (Lesson 4-4)

78. $(-3, -8), (1, 4)$

79. $(-4, 5), (2, -7)$

80. $(3, -1), (-3, 2)$

Solve each equation. (Lesson 2-5)

81. $2(x + 3) + 3 = 4x - 5$

82. $3(y - 3) - 6 = 9y - 15$

83. $2(3a + 6) - 3 = 6a + 12$

84. BASKETBALL Tremaine scored 54 three-point field goals, 84 two-point field goals, and 106 free throws in 23 games. How many points did he score on average per game? (Lesson 2-4)

GET READY for the Next Lesson

PREREQUISITE SKILL **Simplify.** (Lesson 7-1)

85. $(a)(a)$

86. $2x(3x^2)$

87. $-3y^2(8y^2)$

88. $4y(3y) - 4y(6)$

89. $-5n(2n^2) - (-5n)(8n) + (-5n)(4)$

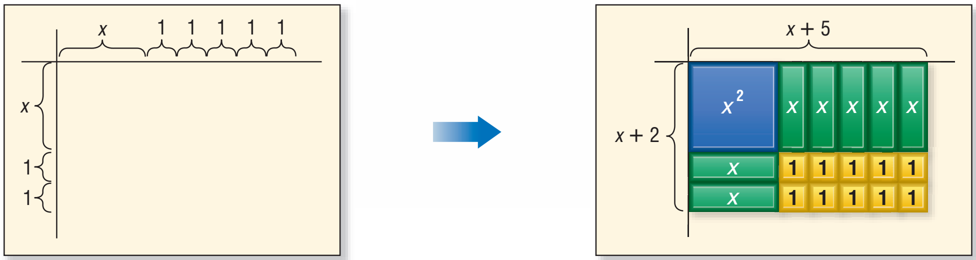
90. $3p^2(6p^2) - 3p^2(8p) + 3p^2(12)$

EXPLORE 7-6

Algebra Lab
Multiplying Polynomials

ACTIVITY 1 **Use algebra tiles to find $(x + 2)(x + 5)$.**

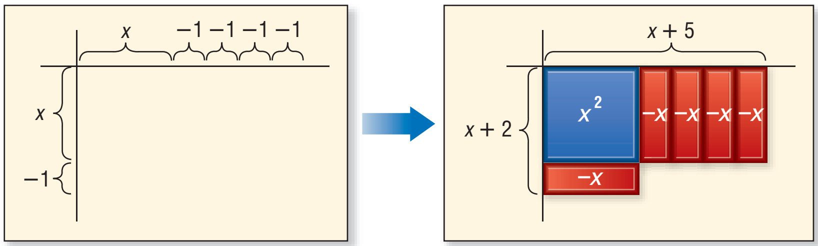
The rectangle will have a width of $x + 2$ and a length of $x + 5$. Use algebra tiles to mark off the dimensions on a product mat. Then complete the rectangle with algebra tiles.



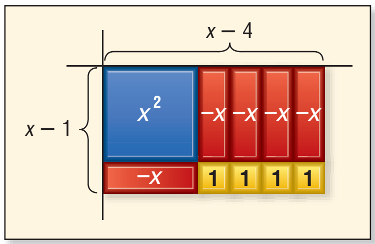
The rectangle consists of 1 blue x^2-tile, 7 green x-tiles, and 10 yellow 1-tiles. The area of the rectangle is $x^2 + 7x + 10$. Therefore, $(x + 2)(x + 5) = x^2 + 7x + 10$.

ACTIVITY 2 **Use algebra tiles to find $(x - 1)(x - 4)$.**

Step 1 The rectangle will have a width of $x - 1$ and a length of $x - 4$. Use algebra tiles to mark off the dimensions on a product mat. Then begin to make the rectangle with algebra tiles.



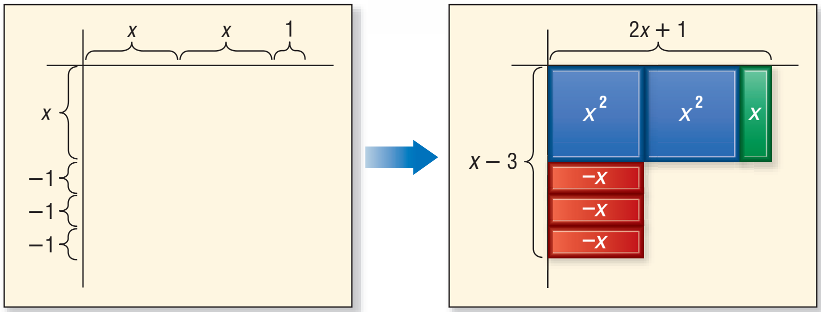
Step 2 Determine whether to use 4 yellow 1-tiles or 4 red −1-tiles to complete the rectangle. Remember that the numbers at the top and side give the dimensions of the tile needed. The area of each tile is the product of −1 and −1 or 1. This is represented by a yellow 1-tile. Fill in the space with 4 yellow 1-tiles to complete the rectangle.



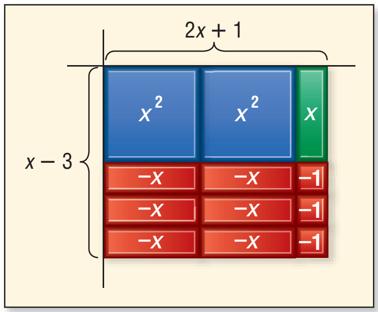
The rectangle consists of 1 blue x^2-tile, 5 red $-x$-tiles, and 4 yellow 1-tiles. The area of the rectangle is $x^2 - 5x + 4$. Therefore, $(x - 1)(x - 4) = x^2 - 5x + 4$.

ACTIVITY 3 **Use algebra tiles to find $(x - 3)(2x + 1)$.**

Step 1 The rectangle will have a width of $x - 3$ and a length of $2x + 1$. Mark off the dimensions on a product mat. Then make the rectangle.

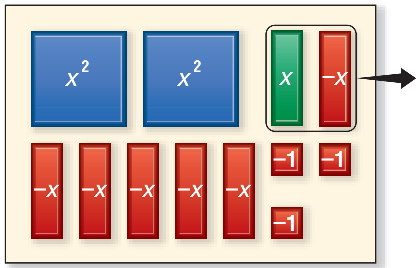


Step 2 Determine what color x-tiles and what color 1-tiles to use to complete the rectangle. The area of each $-x$-tile is the product of x and -1. This is represented by a red $-x$-tile. The area of the -1-tile is represented by the product of 1 and -1 or -1. This is represented by a red -1-tile. Complete the rectangle with 3 red $-x$-tiles and 3 red -1-tiles.



Step 3 Rearrange the tiles to simplify the diagram. Notice that a zero pair is formed by one positive and one negative x-tile.

There are 2 blue x^2-tiles, 5 red $-x$-tiles, and 3 red -1-tiles left. $(x - 3)(2x + 1) = 2x^2 - 5x - 3$.



Concepts in Motion
Animation
algebra1.com

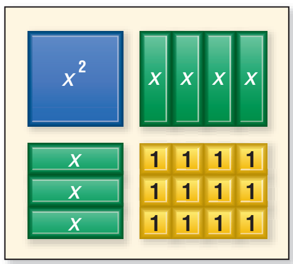
Model

Use algebra tiles to find each product.

1. $(x + 2)(x + 3)$
2. $(x - 1)(x - 3)$
3. $(x + 1)(x - 2)$
4. $(x + 1)(2x + 1)$
5. $(x - 2)(2x - 3)$
6. $(x + 3)(2x - 4)$

Analyze the Results

7. You can also use the Distributive Property to find the product of two binomials. The figure at the right shows the model for $(x + 3)(x + 4)$ separated into four parts. Write a sentence or two explaining how this model shows the use of the Distributive Property.



7-6 Multiplying Polynomials

Main Ideas

- Multiply two binomials by using the FOIL method.
- Multiply two polynomials by using the Distributive Property.

New Vocabulary

FOIL method

GET READY for the Lesson

To compute 24 × 36, we multiply each digit in 24 by each digit in 36, paying close attention to the place value of each digit.

Step 1	Step 2	Step 3
Multiply by the ones.	Multiply by the tens.	Add like place values.
24 × 36 144	24 × 36 144 720	24 × 36 144 + 720 864
$6 \times 24 = 6(20 + 4)$ $= 120 + 24$ or 144	$30 \times 24 = 30(20 + 4)$ $= 600 + 120$ or 720	

You can multiply two binomials in a similar way.

Multiply Binomials To multiply two binomials, apply the Distributive Property twice as you do when multiplying two-digit numbers.

EXAMPLE The Distributive Property

1 **Find $(x + 3)(x + 2)$.**

Method 1 Vertical

Multiply by 2.	Multiply by x.	Combine like terms.
$x + 3$ $(\times)\ x + 2$ $2x + 6$	$x + 3$ $(\times)\ x + 2$ $2x + 6$ $x^2 + 3x$	$x + 3$ $(\times)\ x + 2$ $2x + 6$ $x^2 + 3x$ $x^2 + 5x + 6$
$2(x + 3) = 2x + 6$	$x(x + 3) = x^2 + 3x$	

Method 2 Horizontal

$(x + 3)(x + 2) = x(x + 2) + 3(x + 2)$ Distributive Property

$= x(x) + x(2) + 3(x) + 3(2)$ Distributive Property

$= x^2 + 2x + 3x + 6$ Multiply.

$= x^2 + 5x + 6$ Combine like terms.

CHECK Your Progress

Find each product.

1A. $(m + 4)(m + 5)$

1B. $(y - 2)(y + 8)$

There is a shortcut version of the Distributive Property called the **FOIL method.** You can use the FOIL method to multiply two binomials.

KEY CONCEPT — *FOIL Method*

Words To multiply two binomials, find the sum of the products of

F the *First* terms,

O the *Outer* terms,

I the *Inner* terms, and

L the *Last* terms.

Example

	Product of First Terms		Product of Outer Terms		Product of Inner Terms		Product of Last Terms
$(x + 3)(x - 2) =$	$(x)(x)$	+	$(-2)(x)$	+	$(3)(x)$	+	$(3)(-2)$

$= x^2 - 2x + 3x - 6$

$= x^2 + x - 6$

Concepts in Motion
Animation
algebra1.com

EXAMPLE FOIL Method

2 **Find each product.**

a. $(x - 5)(x + 7)$

$(x - 5)(x + 7) = (x)(x) + (x)(7) + (-5)(x) + (-5)(7)$	F O I L	FOIL method
$= x^2 + 7x - 5x - 35$		Multiply.
$= x^2 + 2x - 35$		Combine like terms.

b. $(2y + 3)(6y - 7)$

$(2y + 3)(6y - 7)$

$= (2y)(6y) + (2y)(-7) + (3)(6y) + (3)(-7)$ (F O I L)	FOIL method
$= 12y^2 - 14y + 18y - 21$	Multiply.
$= 12y^2 + 4y - 21$	Combine like terms.

Study Tip

Checking Your Work

You can check your products in Examples 2a and 2b by reworking each problem using the Distributive Property.

CHECK Your Progress

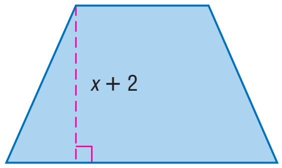
2A. $(x + 3)(x - 4)$

2B. $(4a - 5)(3a + 2)$

The FOIL method can be used to find an expression that represents the area of geometric shapes when the lengths of the sides are given as binomials.

EXAMPLE FOIL Method

3 **GEOMETRY** **The area A of a trapezoid is one half the height h times the sum of the bases, b_1 and b_2. Write an expression for the area of the trapezoid.**



Explore Identify the height and bases.

$h = x + 2$

$b_1 = 3x - 7$

$b_2 = 2x + 1$

Plan Now write and apply the formula.

Area	equals	one-half	height	times	sum of bases.
A	$=$	$\frac{1}{2}$	h	$\cdot$	$(b_1 + b_2)$

Solve

$A = \frac{1}{2}h(b_1 + b_2)$	Original formula
$= \frac{1}{2}(x + 2)[(3x - 7) + (2x + 1)]$	Substitution
$= \frac{1}{2}(x + 2)(5x - 6)$	Add polynomials in the brackets.
$= \frac{1}{2}[x(5x) + x(-6) + 2(5x) + 2(-6)]$	FOIL Method
$= \frac{1}{2}(5x^2 - 6x + 10x - 12)$	Multiply.
$= \frac{1}{2}(5x^2 + 4x - 12)$	Combine like terms.
$= \frac{5}{2}x^2 + 2x - 6$	Distributive Property

Check The area of the trapezoid is $\frac{5}{2}x^2 + 2x - 6$ square units.

CHECK Your Progress

3. Write an expression for the area of a triangle with a base of $2x + 3$ and a height of $3x - 1$.

Online **Personal Tutor at** algebra1.com

Study Tip

Common Misconception

A common mistake when multiplying polynomials horizontally is to combine terms that are not alike. For this reason, you may prefer to multiply polynomials in column form, aligning like terms.

Multiply Polynomials The Distributive Property can be used to multiply any two polynomials.

EXAMPLE The Distributive Property

4 **Find each product.**

a. $(4x + 9)(2x^2 - 5x + 3)$

$(4x + 9)(2x^2 - 5x + 3)$

$= 4x(2x^2 - 5x + 3) + 9(2x^2 - 5x + 3)$	Distributive Property
$= 8x^3 - 20x^2 + 12x + 18x^2 - 45x + 27$	Distributive Property
$= 8x^3 - 2x^2 - 33x + 27$	Combine like terms.

b. $(y^2 - 2y + 5)(6y^2 - 3y + 1)$

$(y^2 - 2y + 5)(6y^2 - 3y + 1)$

$= y^2(6y^2 - 3y + 1) - 2y(6y^2 - 3y + 1) + 5(6y^2 - 3y + 1)$

$= 6y^4 - 3y^3 + y^2 - 12y^3 + 6y^2 - 2y + 30y^2 - 15y + 5$

$= 6y^4 - 15y^3 + 37y^2 - 17y + 5$

CHECK Your Progress

4A. $(3x - 5)(2x^2 + 7x - 8)$ **4B.** $(m^2 + 2m - 3)(4m^2 - 7m + 5)$

CHECK Your Understanding

Examples 1–2 (pp. 398–399)

Find each product.

1. $(y + 4)(y + 3)$

2. $(x - 2)(x + 6)$

3. $(a - 8)(a + 5)$

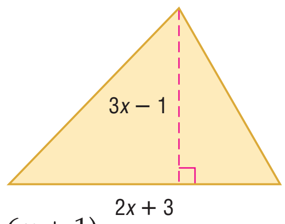
4. $(4h + 5)(h + 7)$

5. $(9p - 1)(3p - 2)$

6. $(2g + 7)(5g - 8)$

Example 3 (p. 400)

7. GEOMETRY The area A of a triangle is half the product of the base b times the height h. Write a polynomial expression that represents the area of the triangle at the right.



Example 4 (p. 400)

Find each product.

8. $(3k - 5)(2k^2 + 4k - 3)$

9. $(4x^2 - 2)(2x^2 + 6x + 1)$

10. $(y^2 - 5y + 3)(4y^2 + 2y - 6)$

11. $(3m^2 + 2m - 7)(5m^2 + m + 9)$

Exercises

HOMEWORK HELP

For Exercises	See Examples
12–29	1, 2
30–33	3
34–41	4

Find each product.

12. $(b + 8)(b + 2)$

13. $(n + 6)(n + 7)$

14. $(x - 4)(x - 9)$

15. $(a - 3)(a - 5)$

16. $(y + 4)(y - 8)$

17. $(p + 2)(p - 10)$

18. $(2w - 5)(w + 7)$

19. $(k + 12)(3k - 2)$

20. $(8d + 3)(5d + 2)$

21. $(4g + 3)(9g + 6)$

22. $(7x - 4)(5x - 1)$

23. $(6a - 5)(3a - 8)$

24. $(2n + 3)(2n + 3)$

25. $(5m - 6)(5m - 6)$

26. $(10r - 4)(10r + 4)$

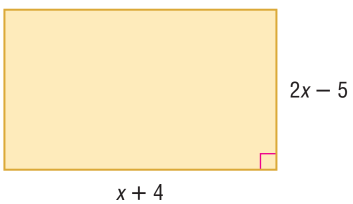
27. $(7t + 5)(7t - 5)$

28. $(8x + 2y)(5x - 4y)$

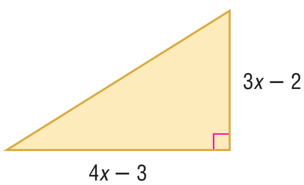
29. $(11a - 6b)(2a + 3b)$

GEOMETRY **Write an expression to represent the area of each figure.**

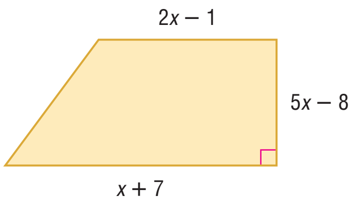
30.



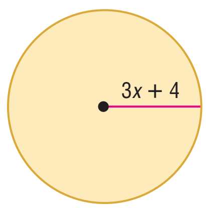
31.



32.



33.



Find each product.

34. $(p + 4)(p^2 + 2p - 7)$

35. $(a - 3)(a^2 - 8a + 5)$

36. $(2x - 5)(3x^2 - 4x + 1)$

37. $(3k + 4)(7k^2 + 2k - 9)$

38. $(n^2 - 3n + 2)(n^2 + 5n - 4)$

39. $(y^2 + 7y - 1)(y^2 - 6y + 5)$

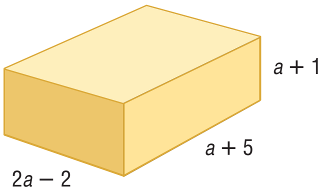
Simplify.

40. $(m + 2)[(m^2 + 3m - 6) + (m^2 - 2m + 4)]$

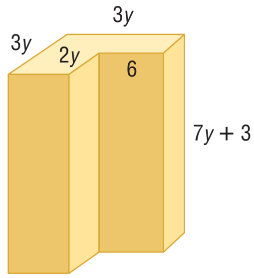
41. $[(t^2 + 3t - 8) - (t^2 - 2t + 6)](t - 4)$

GEOMETRY The volume V of a prism equals the area of the base B times the height h. Write an expression to represent the volume of each prism.

42.



43.

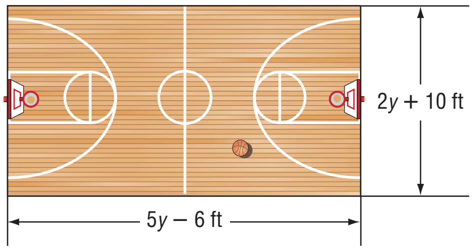


Real-World Link

More than 200 million people a year pay to see basketball games. That is more admissions than for any other American sport.

Source: *Compton's Encyclopedia*

44. BASKETBALL The dimensions of a professional basketball court are represented by a width of $5y - 6$ feet and a length of $2y + 10$ feet. Find an expression for the area of the court.



OFFICE SPACE For Exercises 45–47, use the following information.

LaTanya's modular office is square. Her office in the company's new building will be 2 feet shorter in one direction and 4 feet longer in the other.

45. Write expressions for the dimensions of LaTanya's new office.

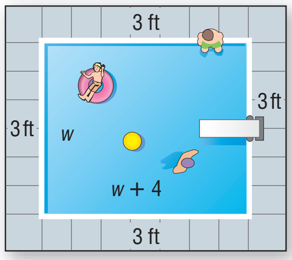
46. Write a polynomial expression for the area of her new office.

47. Suppose her office is presently 9 feet by 9 feet. Will her new office be bigger or smaller than her old office and by how much? Explain.

EXTRA PRACTICE
See pages 732, 750.

Math Online
Self-Check Quiz at algebra1.com

48. POOL CONSTRUCTION A homeowner is installing a swimming pool in his backyard. He wants its length to be 4 feet longer than its width. Then he wants to surround it with a concrete walkway 3 feet wide. If he can only afford 300 square feet of concrete for the walkway, what should the dimensions of the pool be?



H.O.T. Problems

49. REASONING Compare and contrast the procedure used to multiply a trinomial by a binomial using the vertical method with the procedure used to multiply a three-digit number by a two-digit number.

50. ALGEBRA TILES Draw a diagram to show how you would use algebra tiles to find the product of $2x - 1$ and $x + 3$.

51. CHALLENGE Determine whether the following statement is *sometimes*, *always*, or *never* true. Explain your reasoning.

The product of a binomial and a trinomial is a polynomial with four terms.

52. **OPEN ENDED** Write a binomial and a trinomial involving a single variable. Then find their product.

53. ***Writing in Math*** Using the information about multiplying binomials on page 398 explain how multiplying binomials is similar to multiplying two-digit numbers. Include a demonstration of a horizontal method for multiplying 24×36 in your answer.

STANDARDIZED TEST PRACTICE

54. A rectangle's width is represented by x and its length by y. Which expression best represents the area of the rectangle if the length and width are doubled?

A $2xy$

B $2(xy)^2$

C $4xy$

D $(xy)^2$

55. **REVIEW** Tania's age is 4 years less than twice her little brother Billy's age. If Tania is 12, which equation can be used to determine Billy's age?

F $x = 12$

G $12 = 4 - 2x$

H $12(2) - 4 = x$

J $12 = 2x - 4$

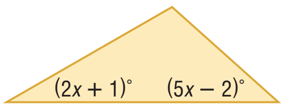
Spiral Review

Simplify. (Lesson 7-5)

56. $3x(2x - 4) + 6(5x^2 + 2x - 7)$

57. $4a(5a^2 + 2a - 7) - 3(2a^2 - 6a - 9)$

58. **GEOMETRY** The sum of the degree measures of the angles of a triangle is 180. (Lesson 7-4)



a. Write an expression to represent the measure of the third angle of the triangle.

b. If $x = 15$, find the measures of the three angles of the triangle.

If $f(x) = 2x - 5$ and $g(x) = x^2 + 3x$, find each value. (Lesson 3-6)

59. $f(-4)$

60. $g(-2) + 7$

61. $f(a + 3)$

Solve each equation or formula for the variable specified. (Lesson 2-8)

62. $a = \frac{v}{t}$ for t

63. $ax - by = 2cz$ for y

64. $4x + 3y = 7$ for y

Solve each equation. (Lesson 2-4)

65. $\frac{d - 2}{3} = 7$

66. $3n + 6 = -15$

67. $35 + 20h = 100$

GET READY for the Next Lesson

PREREQUISITE SKILL **Simplify.** (Lesson 7–1)

68. $(6a)^2$

69. $(7x)^2$

70. $(9b)^2$

71. $(4y^2)^2$

72. $(2v^3)^2$

73. $(3g^4)^2$

7-7 Special Products

Main Ideas

- Find squares of sums and differences.
- Find the product of a sum and a difference.

New Vocabulary

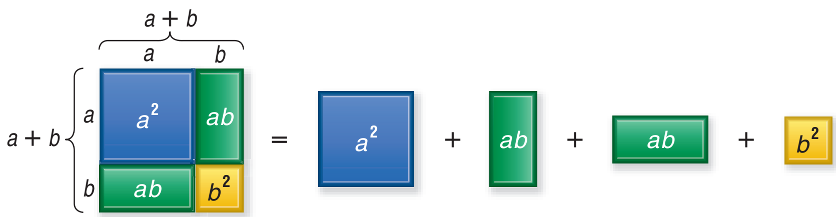
difference of two squares

GET READY for the Lesson

In the previous lesson, you learned how to multiply two binomials using the FOIL method. You may have noticed that the *Outer* and *Inner* terms often combine to produce a trinomial product. This is not always the case, however. Notice that the product of $x + 3$ and $x - 3$ is a binomial.

$(x + 5)(x - 3)$	$(x + 3)(x - 3)$
F O I L	F O I L
$= x^2 - 3x + 5x - 15$	$= x^2 - 3x + 3x - 9$
$= x^2 + 2x - 15$	$= x^2 + 0x - 9$
	$= x^2 - 9$

Squares of Sums and Differences While you can always use the FOIL method to find the product of two binomials, some pairs of binomials have products that follow a specific pattern. One such pattern is the *square of a sum*, $(a + b)^2$ or $(a + b)(a + b)$.



$$(a + b)^2 = a^2 + ab + ab + b^2$$
$$= a^2 + 2ab + b^2$$

KEY CONCEPT — Square of a Sum

Words The square of $a + b$ is the square of a plus twice the product of a and b plus the square of b.

Symbols $(a + b)^2 = (a + b)(a + b) = a^2 + 2ab + b^2$

Example $(x + 7)^2 = x^2 + 2(x)(7) + 7^2 = x^2 + 14x + 49$

EXAMPLE Square of a Sum

1 **Find $(4y + 5)^2$.**

$(a + b)^2 = a^2 + 2ab + b^2$

$(4y + 5)^2 = (4y)^2 + 2(4y)(5) + 5^2$ $a = 4y$ and $b = 5$

$= 16y^2 + 40y + 25$ Check by using FOIL.

Concepts in Motion
Animation
algebra1.com

Extra Examples at algebra1.com

CHECK Your Progress Find each product.

1A. $(8c + 3d)^2$ **1B.** $(3x + 4y)^2$

Study Tip

$(a + b)^2$
In the pattern for $(a + b)^2$, a and b can be numbers, variables, or expressions with numbers and variables.

To find the pattern for the *square of a difference*, $(a - b)^2$, write $a - b$ as $a + (-b)$ and square it using the square of a sum pattern.

$$(a - b)^2 = [a + (-b)]^2$$
$$= a^2 + 2(a)(-b) + (-b)^2 \quad \text{Square of a Sum}$$
$$= a^2 - 2ab + b^2 \quad \text{Simplify. Note that } (-b)^2 = (-b)(-b) \text{ or } b^2.$$

KEY CONCEPT *Square of a Difference*

Words The square of $a - b$ is the square of a minus twice the product of a and b plus the square of b.

Symbols $(a - b)^2 = (a - b)(a - b) = a^2 - 2ab + b^2$

Example $(x - 4)^2 = x^2 - 2(x)(4) + 4^2 = x^2 - 8x + 16$

EXAMPLE Square of a Difference

2 Find $(5m^3 - 2n)^2$.

$$(a - b)^2 = a^2 - 2ab + b^2$$
$$(5m^3 - 2n)^2 = (5m^3)^2 - 2(5m^3)(2n) + (2n)^2 \quad a = 5m^3 \text{ and } b = 2n$$
$$= 25m^6 - 20m^3n + 4n^2 \quad \text{Simplify.}$$

CHECK Your Progress Find each product.

2A. $(6p - 1)^2$ **2B.** $(a - 2b)^2$

Real-World Career
Geneticist

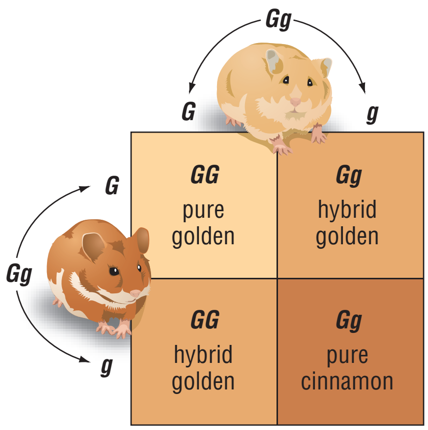
Laboratory geneticists work in medicine to find cures for disease, in agriculture to breed new crops and livestock, and in police work to identify criminals.

Math Online
For more information, go to algebra1.com

Real-World EXAMPLE

3 **GENETICS** **The Punnett square shows the possible gene combinations between two hamsters. Each hamster passes on one *dominant* gene *G* for golden coloring and one *recessive* gene *g* for cinnamon coloring.**

Show how combinations can be modeled by the square of a binomial. Then determine what percent of the offspring will be pure golden, hybrid golden, and pure cinnamon.



Each parent has half the genes necessary for golden coloring and half the genes necessary for cinnamon coloring. The makeup of each parent can be modeled by $0.5G + 0.5g$. Their offspring can be modeled by the product of $0.5G + 0.5g$ and $0.5G + 0.5g$ or $(0.5G + 0.5g)^2$.

Use this product to determine possible colors of the offspring.

$$(a + b)^2 = a^2 + 2ab + b^2 \qquad \text{Square of a Sum}$$
$$(0.5G + 0.5g)^2 = (0.5G)^2 + 2(0.5G)(0.5g) + (0.5g)^2 \qquad a = 0.5G \text{ and } b = 0.5g$$
$$= 0.25G^2 + 0.5Gg + 0.25g^2 \qquad \text{Simplify.}$$
$$= 0.25GG + 0.5Gg + 0.25gg \qquad G^2 = GG \text{ and } g^2 = gg$$

Thus, 25% of the offspring are *GG* or pure golden, 50% are *Gg* or hybrid golden, and 25% are *gg* or pure cinnamon.

Check Your Progress

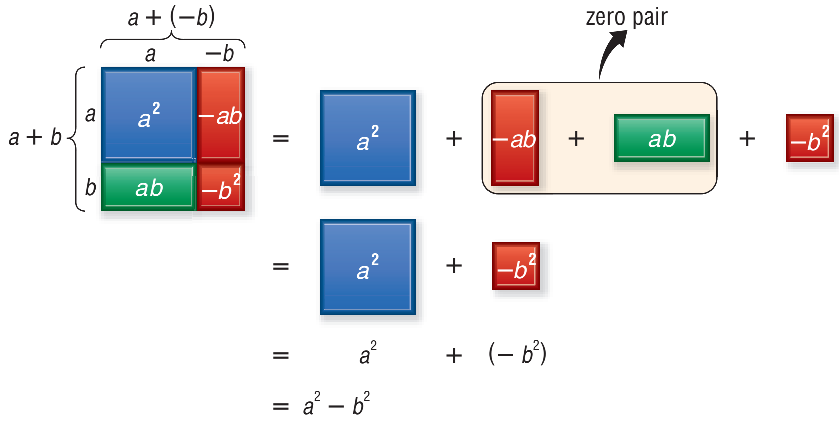
3. Andrew has a garden that is x feet long and x feet wide. He decides that he wants to add 3 feet to the length and the width in order to grow more vegetables. Show how the new area of the garden can be modeled by the square of a binomial.

Online **Personal Tutor at** algebra1.com

Concepts in Motion
Animation
algebra1.com

Product of a Sum and a Difference

You can use the diagram below to find the pattern for the product of the sum and difference of the *same two terms*, $(a + b)(a - b)$. Recall that $a - b$ can be rewritten as $a + (-b)$.



The resulting product, $a^2 - b^2$, is called the **difference of two squares.**

KEY CONCEPT	***Product of a Sum and a Difference***
Words	The product of $a + b$ and $a - b$ is the square of a minus the square of b.
Symbols	$(a + b)(a - b) = (a - b)(a + b) = a^2 - b^2$
Example	$(x + 9)(x - 9) = x^2 - 9^2 = x^2 - 81$

EXAMPLE 4 Product of a Sum and a Difference

Find $(11v - 8w^2)(11v + 8w^2)$.

$$(a - b)(a + b) = a^2 - b^2$$
$$(11v - 8w^2)(11v + 8w^2) = (11v)^2 - (8w^2)^2 \qquad a = 11v \text{ and } b = 8w^2$$
$$= 121v^2 - 64w^4 \qquad \text{Simplify.}$$

Check Your Progress **Find each product.**

4A. $(3n + 2)(3n - 2)$ **4B.** $(4c - 7d)(4c + 7d)$

CHECK Your Understanding

Examples 1–2
(pp. 404–405)

Find each product.

1. $(a + 6)^2$
2. $(2a + 7b)^2$
3. $(3x + 9y)^2$
4. $(4n - 3)(4n - 3)$
5. $(x^2 - 6y)^2$
6. $(9 - p)^2$

Example 3
(pp. 405–406)

GENETICS For Exercises 7 and 8, use the following information.
Dalila has brown eyes and Bob has blue eyes. Brown genes B are dominant over blue genes b. A person with genes BB or Bb has brown eyes. Someone with genes bb has blue eyes. Suppose Dalila's genes for eye color are Bb.

7. Write an expression for the possible eye coloring of Dalila and Bob's children.
8. What is the probability that a child of Dalila and Bob would have blue eyes?

Example 4
(p. 406)

Find each product.

9. $(8x - 5)(8x + 5)$
10. $(3a + 7b)(3a - 7b)$
11. $(4y^2 + 3z)(4y^2 - 3z)$

Exercises

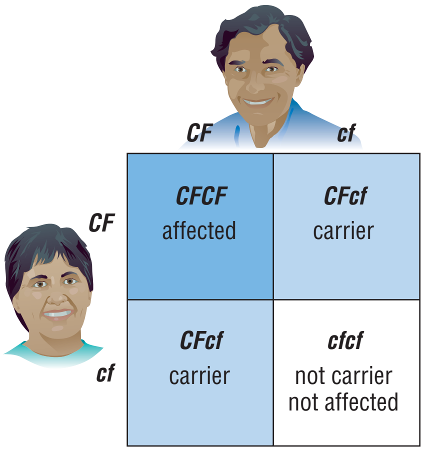
HOMEWORK HELP

For Exercises	See Examples
12–14	1
15–17	2
18–19	3
20–22	4

Find each product.

12. $(k + 8)(k + 8)$
13. $(y + 4)^2$
14. $(2g + 5)^2$
15. $(a - 5)(a - 5)$
16. $(n - 12)^2$
17. $(7 - 4y)^2$

GENETICS For Exercises 18 and 19, use the following information and the Punnett square.
Cystic fibrosis is inherited from parents only if both parents have the abnormal CF gene. Children of two parents with the CF gene will either be affected with the disease, a carrier but not affected, or not have the gene.



18. Write an expression for the genetic makeup of children of two parents that are carriers of cystic fibrosis.
19. What is the probability that a child will not be affected and not be a carrier?

Find each product.

20. $(b + 7)(b - 7)$
21. $(c - 2)(c + 2)$
22. $(11r + 8)(11r - 8)$

Find each product.

23. $(9x + 3)^2$
24. $(4 - 6h)^2$
25. $(12p - 3)(12p + 3)$
26. $(a + 5b)^2$
27. $(m + 7n)^2$
28. $(2x - 9y)^2$
29. $(3n - 10p)^2$
30. $(5w + 14)(5w - 14)$
31. $(4d - 13)(4d + 13)$
32. $(x^3 + 4y)^2$
33. $(3a^2 - b^2)^2$
34. $(8a^2 - 9b^3)(8a^2 + 9b^3)$
35. $(5x^4 - y)(5x^4 + y)$
36. $\left(\frac{2}{3}x - 6\right)^2$
37. $\left(\frac{4}{5}x + 10\right)^2$
38. $(2n + 1)(2n - 1)(n + 5)$
39. $(p + 3)(p - 4)(p - 3)(p + 4)$

EXTRA PRACTICE
See pages 732, 750.
Math Online
Self-Check Quiz at algebra1.com

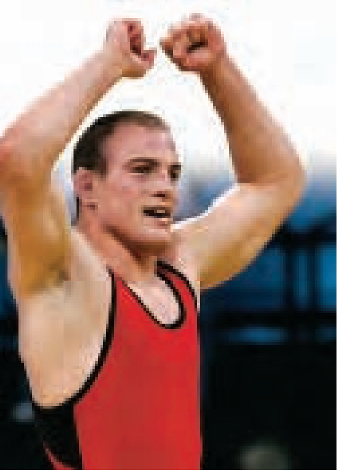
MAGIC TRICK **For Exercises 40–43, use the following information.**
Madison says that she can perform a magic trick with numbers. She asks you to pick an integer, any integer. Square that integer. Then, add twice your original number. Next add 1. Take the square root of the result. Finally, subtract your original number. Then Madison exclaims with authority, "Your answer is 1!"

40. Pick an integer and follow Madison's directions. Is your result 1?

41. Let a represent the integer you chose. Then, find a polynomial representation for the first three steps of Madison's directions.

42. The polynomial you wrote in Exercise 41 is the square of what binomial sum?

43. Take the square root of the perfect square you wrote in Exercise 42, then subtract a, your original integer. What is the result?



Real-World Link
Cael Sanderson of Iowa State University is the only wrestler in NCAA Division 1 history to be undefeated for four years. He compiled a 159–0 record from 1999–2002.

Source: www.teamsanderson.cc

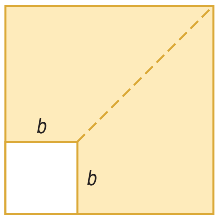
WRESTLING **For Exercises 44–46, use the following information.**
A high school wrestling mat must be a square with 38-foot sides and contain two circles as shown. Suppose the inner circle has a radius of s feet, and the outer circle's radius is nine feet longer than the inner circle.

44. Write an expression for the area of the larger circle.

45. Write an expression for the area of the square outside the circle.

46. Use the expression to find the area if $s = 1$.

47. **GEOMETRY** The area of the shaded region models the difference of two squares, $a^2 - b^2$. Show that the area of the shaded region is also equal to $(a - b)(a + b)$. (*Hint*: Divide the shaded region into two trapezoids as shown.)



H.O.T. Problems

48. **REASONING** Compare and contrast the pattern for the square of a sum with the pattern for the square of a difference.

49. **ALGEBRA TILES** Draw a diagram to show how you would use algebra tiles to model the product of $x - 3$ and $x - 3$, or $(x - 3)^2$.

50. **OPEN ENDED** Write two binomials whose product is a difference of squares. Then multiply to verify your answer.

51. **CHALLENGE** Does a pattern exist for the cube of a sum, $(a + b)^3$?

a. Investigate this question by finding the product of $(a + b)(a + b)(a + b)$.

b. Use the pattern you discovered in part a to find $(x + 2)^3$.

c. Draw a diagram of a geometric model for the cube of a sum.

52. *Writing in Math* Using the information about the product of two binomials on page 404 distinguish when the product of two binomials is also a binomial. Include an example of two binomials whose product is a binomial and an example of two binomials whose product is not a binomial in your answer.

STANDARDIZED TEST PRACTICE

53. The base of a triangle is represented by $x - 4$, and the height is represented by $x + 4$. Which of the following represents the area of the triangle?

A $x^2 - 16$

B $\frac{1}{2}x^2 + 4x - 8$

C $x^2 + 8x - 16$

D $\frac{1}{2}x^2 - 8$

54. REVIEW The sum of a number and 8 is −19. Which equation shows this relationship?

F $8n = -19$

G $n + 8 = -19$

H $n - 8 = 19$

J $n - 19 = 8$

Spiral Review

Find each product. (Lesson 7-6)

55. $(x + 2)(x + 7)$

56. $(c - 9)(c + 3)$

57. $(4y - 1)(5y - 6)$

58. $(3n - 5)(8n + 5)$

59. $(x - 2)(3x^2 - 5x + 4)$

60. $(2k + 5)(2k^2 - 8k + 7)$

Solve. (Lesson 7-5)

61. $6(x + 2) + 4 = 5(3x - 4)$

62. $-3(3a - 8) + 2a = 4(2a + 1)$

63. $p(p + 2) + 3p = p(p - 3)$

64. $y(y - 4) + 2y = y(y + 12) - 7$

Use elimination to solve each system of equations. (Lesson 5-3, 5-4)

65. $\frac{3}{4}x + \frac{1}{5}y = 5$

$\frac{3}{4}x - \frac{1}{5}y = -5$

66. $2x - y = 10$

$5x + 3y = 3$

67. $2x = 4 - 3y$

$3y - x = -11$

Write the slope-intercept form of an equation that passes through the given point and is perpendicular to the graph of each equation. (Lesson 4-6)

68. $5x + 5y = 35, (-3, 2)$

69. $2x - 5y = 3, (-2, 7)$

70. $5x + y = 2, (0, 6)$

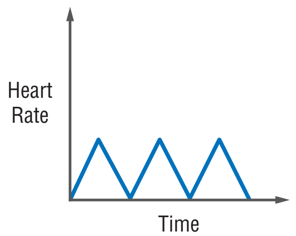
Find the *n*th term of each arithmetic sequence described. (Lesson 3-7)

71. $a_1 = 3, d = 4, n = 18$

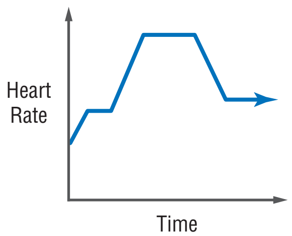
72. $-5, 1, 7, 13, \ldots$ for $n = 12$

73. PHYSICAL FITNESS Mitchell likes to exercise regularly. He likes to warm up by walking two miles. Then he runs five miles. Finally, he cools down by walking another mile. Identify the graph that best represents Mitchell's heart rate as a function of time. (Lesson 1-9)

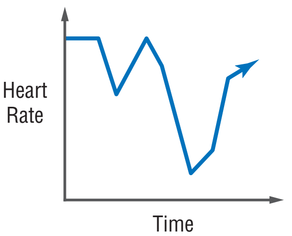
a.



b.



c.

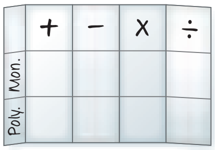


Chapter 7 Study Guide and Review

Download Vocabulary Review from algebra1.com

FOLDABLES Study Organizer: GET READY to Study

Be sure the following Key Concepts are noted in your Foldable.



Key Concepts

Multiplying Monomials (Lesson 7-1)

- To multiply two powers that have the same base, add exponents.
- To find the power of a power, multiply exponents.
- The power of a product is the product of the powers.

Dividing Monomials (Lesson 7-2)

- To divide two powers that have the same base, subtract the exponents.
- To find the power of a quotient, find the power of the numerator and the power of the denominator.
- Any nonzero number raised to the zero power is 1.
- For any nonzero number a and any integer n,

$a^{-n} = \frac{1}{a^n}$ and $\frac{1}{a^{-n}} = a^n$.

Polynomials (Lesson 7-3)

- The degree of a monomial is the sum of the exponents of all its variables.
- The degree of a polynomial is the greatest degree of any term. To find the degree of a polynomial, you must find the degree of each term.

Operations with Polynomials (Lessons 7-4 to 7-7)

- The Distributive Property can be used to multiply a polynomial by a monomial.
- Square of a Sum: $(a + b)^2 = a^2 + 2ab + b^2$
- Square of a Difference: $(a - b)^2 = a^2 - 2ab + b^2$
- Product of a Sum and a Difference: $(a + b)(a - b) = (a - b)(a + b) = a^2 - b^2$

Key Vocabulary

binomial (p. 376)
constant (p. 358)
degree of a monomial (p. 377)
degree of a polynomial (p. 377)
difference of two squares (p. 406)
FOIL method (p. 399)
monomial (p. 358)
negative exponent (p. 369)
polynomial (p. 376)
Power of a Power (p. 359)
Power of a Product (p. 360)
Power of a Quotient (p. 367)
Product of Powers (p. 359)
Quotient of Powers (p. 366)
trinomial (p. 376)
zero exponent (p. 368)

Vocabulary Check

Choose a term from the vocabulary list that best matches each example.

1. $4^{-3} = \frac{1}{4^3}$

2. $(n^3)^5 = n^{15}$

3. $\frac{4x^2y}{8xy^3} = \frac{x}{2y^2}$

4. $4x^2$

5. $x^2 - 3x + 1$

6. $2^0 = 1$

7. $x^4 - 3x^3 + 3x^2 - 1$

8. $x^2 + 2$

9. $(a^3b)(2ab^2) = 2a^4b^3$

10. $(x + 3)(x - 4) = x^2 - 4x + 3x - 12$

Vocabulary Review at algebra1.com

Lesson-by-Lesson Review

7–1 Multiplying Monomials (pp. 358–364)

Simplify.

11. $y^3 \cdot y^3 \cdot y$

12. $(3ab)(-4a^2b^3)$

13. $(-4a^2x)(-5a^3x^4)$

14. $(4a^2b)^3$

15. $(-3xy)^2(4x)^3$

16. $(-2c^2d)^4(-3c^2)^3$

17. $-\frac{1}{2}(m^2n^4)^2$

18. $(5a^2)^3 + 7(a^6)$

19. **GEOMETRY** A cone has a radius of $4x^3$ and a height of $3b^2$. Use the formula $V = \frac{1}{3}(\pi r^2 \ell)$ to find the volume of the cone.

Example 1 **Simplify $(2ab^2)(3a^2b^3)$.**

$(2ab^2)(3a^2b^3)$

$= (2 \cdot 3)(a \cdot a^2)(b^2 \cdot b^3)$ Commutative Property

$= 6a^3b^5$ Product of Powers

Example 2 **Simplify $(2x^2y^3)^3$.**

$(2x^2y^3)^3 = 2^3(x^2)^3(y^3)^3$ Power of a Product

$= 8x^6y^9$ Power of a Power

7–2 Dividing Monomials (pp. 366–373)

Simplify. Assume that no denominator is equal to zero.

20. $\frac{(3y)^0}{6a}$

21. $\left(\frac{3bc^2}{4d}\right)^3$

22. $x^{-2}y^0z^3$

23. $\frac{27b^{-2}}{14b^{-3}}$

24. $\frac{(3a^3bc^2)^2}{18a^2b^3c^4}$

25. $\frac{-16a^3b^2x^4y}{-48a^4bxy^3}$

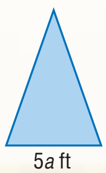
26. $\frac{(-a)^5b^8}{a^5b^2}$

27. $\frac{(4a^{-1})^{-2}}{(2a^4)^2}$

28. $\left(\frac{5xy^{-2}}{35x^{-2}y^6}\right)^0$

29. $\frac{12}{3}\left(\frac{m}{n^3}\right)\left(\frac{n^4}{m^3}\right)$

30. **GEOMETRY** The area of a triangle is $50a^2b$ square feet. The base of the triangle is $5a$ feet. What is the height of the triangle?



Example 3 **Simplify $\frac{2x^6y}{8x^2y^2}$. Assume that no denominator is equal to zero.**

$\frac{2x^6y}{8x^2y^2} = \left(\frac{2}{8}\right)\left(\frac{x^6}{x^2}\right)\left(\frac{y}{y^2}\right)$ Group the powers with the same base.

$= \left(\frac{1}{4}\right)(x^{6-2})(y^{1-2})$ Quotient of Powers

$= \frac{x^4}{4y}$ Simplify.

Example 4 **Simplify $\frac{m^{-4}n^3p^0}{mn^{-2}}$. Assume that no denominator is zero.**

$\frac{m^{-4}n^3p^0}{mn^{-2}} = \left(\frac{m^{-4}}{m}\right)\left(\frac{n^3}{n^{-2}}\right)(p^0)$ Group the powers with the same base.

$= (m^{-4-1})(n^{3+2})$ Quotient of Powers and Zero Exponent

$= \frac{n^5}{m^5}$ Simplify.

Study Guide and Review

7-3 Polynomials (pp. 376–381)

Find the degree of each polynomial.

31. $n - 2p^2$

32. $29n^2 + 17n^2t^2$

33. $4xy + 9x^3z^2 + 17rs^3$

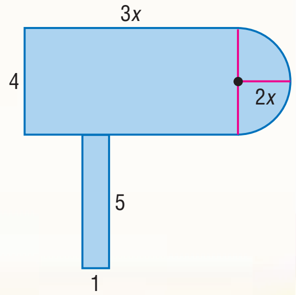
34. $-6x^5y - 2y^4 + 4 - 8y^2$

Arrange the terms of each polynomial so that the powers of x are in descending order.

35. $3x^4 - x + x^2 - 5$

36. $-2x^2y^3 - 27 - 4x^4 + xy + 5x^3y^2$

37. **CONSTRUCTION** Ben is building a brick patio with pavers using the drawing below. Write a polynomial to represent the area of the patio.



Example 5 Find the degree of $2xy^3 + x^2y$.

Polynomial:	$2xy^3 + x^2y$
Terms:	$2xy^3, x^2y$
Degree of Each Term:	4, 3
Degree of Polynomial:	4

Example 6 Arrange the terms of $4x^2 + 9x^3 - 2 - x$ so that the powers of x are in descending order.

$4x^2 + 9x^3 - 2 - x$

$= 4x^2 + 9x^3 - 2x^0 - x^1$ $x^0 = 1$ and $x = x^1$

$= 9x^3 + 4x^2 - x - 2$ $3 > 2 > 1 > 0$

7-4 Adding and Subtracting Polynomials (pp. 384–388)

Find each sum or difference.

38. $(2x^2 - 5x + 7) - (3x^3 + x^2 + 2)$

39. $(x^2 - 6xy + 7y^2) + (3x^2 + xy - y^2)$

40. $(7z^2 + 4) - (3z^2 + 2z - 6)$

41. $(13m^4 - 7m - 10) + (8m^4 - 3m + 9)$

42. $(11m^2n^2 + 4mn - 6) + (5m^2n^2 + 6mn + 17)$

43. $(-5p^2 + 3p + 49) - (2p^2 + 5p + 24)$

44. **GARDENING** Kyle is planting flowers around the perimeter of his rectangular garden. If the perimeter of his garden is $110x$ and one side measures $25x$, find the length of the other side.

Example 7 Find $(7r^2 + 9r) - (12r^2 - 4)$.

$(7r^2 + 9r) - (12r^2 - 4)$

$= 7r^2 + 9r + (-12r^2 + 4)$ The additive inverse of $12r^2 - 4$ is $-12r^2 + 4$.

$= (7r^2 - 12r^2) + 9r + 4$ Group like terms.

$= -5r^2 + 9r + 4$ Add like terms.

Mixed Problem Solving
For mixed problem-solving practice, see page 750.

7–5 Multiplying a Polynomial by a Monomial (pp. 390–395)

Simplify.

45. $b(4b - 1) + 10b$

46. $x(3x - 5) + 7(x^2 - 2x + 9)$

47. $8y(11y^2 - 2y + 13) - 9(3y^3 - 7y + 2)$

48. $2x(x - y^2 + 5) - 5y^2(3x - 2)$

Solve each equation.

49. $m(2m - 5) + m = 2m(m - 6) + 16$

50. $2(3w + w^2) - 6 = 2w(w - 4) + 10$

51. **SHOPPING** Nicole bought x shirts for \$15.00 each, y pants for \$25.72 each, and z belts for \$12.53 each. Sales tax on these items was 7%. Write an expression to find the total cost of Nicole's purchases.

Example 8 **Simplify $x^2(x + 2) + 3(x^3 + 4x^2)$.**

$x^2(x + 2) + 3(x^3 + 4x^2)$

$= x^2(x) + x^2(2) + 3(x^3) + 3(4x^2)$

$= x^3 + 2x^2 + 3x^3 + 12x^2$ Multiply.

$= 4x^3 + 14x^2$ Combine like terms.

Example 9 **Solve $x(x - 10) + x(x + 2) + 3 = 2x(x + 1) - 7$.**

$x(x - 10) + x(x + 2) + 3 = 2x(x + 1) - 7$

$x^2 - 10x + x^2 + 2x + 3 = 2x^2 + 2x - 7$

$2x^2 - 8x + 3 = 2x^2 + 2x - 7$

$-8x + 3 = 2x - 7$

$-10x = -10$

$x = 1$

7–6 Multiplying Polynomials (pp. 398–403)

Find each product.

52. $(r - 3)(r + 7)$

53. $(4a - 3)(a + 4)$

54. $(5r - 7s)(4r + 3s)$

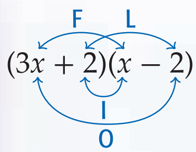
55. $(3x + 0.25)(6x - 0.5)$

56. $(2k + 1)(k^2 + 7k - 9)$

57. $(4p - 3)(3p^2 - p + 2)$

58. **MANUFACTURING** A company is designing a box in the shape of a rectangular prism for dry pasta. The length is 2 inches more than twice the width and the height is 3 inches more than the length. Write an expression for the volume of the box.

Example 10 **Find $(3x + 2)(x - 2)$.**



F O I L

$= (3x)(x) + (3x)(-2) + (2)(x) + (2)(-2)$

$= 3x^2 - 6x + 2x - 4$ Multiply

$= 3x^2 - 4x - 4$ Combine like terms.

Example 11 **Find $(2y - 5)(4y^2 + 3y - 7)$.**

$(2y - 5)(4y^2 + 3y - 7)$

$= 2y(4y^2 + 3y - 7) - 5(4y^2 + 3y - 7)$

$= 8y^3 + 6y^2 - 14y - 20y^2 - 15y + 35$

$= 8y^3 - 14y^2 - 29y + 35$

Study Guide and Review

7–7 Special Products (pp. 404–409)

Find each product.

59. $(x - 6)(x + 6)$

60. $(4x + 7)^2$

61. $(8x - 5)^2$

62. $(5x - 3y)(5x + 3y)$

63. $(6a - 5b)^2$

64. $(3m + 4n)^2$

65. **GENETICS** Emily and Santos are both able to roll their tongues. Tongue rolling genes *R* are dominant over nonrolling genes *r*. A person with genes *RR* or *Rr* is able to roll their tongue. Someone with genes *rr* cannot roll their tongue. Suppose Emily's and Santos's genes for tongue rolling are *Rr*. Write an expression for the possible tongue-rolling abilities of Santos's and Emily's children. What is the probability that a child of Emily and Santos could not roll their tongue?

Example 12 Find $(r - 5)^2$.

$(a - b)^2 = a^2 - 2ab + b^2$ Square of a Difference

$(r - 5)^2 = r^2 - 2(r)(5) + 5^2$ $a = r$ and $b = 5$

$= r^2 - 10r + 25$ Simplify.

Example 13 Find $(2c + 9)(2c - 9)$.

$(a + b)(a - b) = a^2 - b^2$

$(2c + 9)(2c - 9) = (2c)^2 - 9^2$ $a = 2c$ and $b = 9$

$= 4c^2 - 81$ Simplify.

CHAPTER 7 Practice Test

Simplify. Assume that no denominator is equal to zero.

1. $(a^2b^4)(a^3b^5)$

2. $(-12abc)(4a^2b^4)$

3. $\left(\frac{3}{5}m\right)^2$

4. $(-3a)^4(a^5b)^2$

5. $(-5a^2)(-6b^3)^2$

6. $\frac{mn^4}{m^3n^2}$

7. $\frac{9a^2bc^2}{63a^4bc}$

8. $\frac{48a^2bc^5}{(3ab^3c^2)^2}$

Find the degree of each polynomial. Then arrange the terms so that the powers of *y* are in descending order.

9. $2y^2 + 8y^4 + 9y$

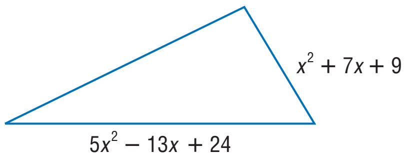
10. $5xy - 7 + 2y^4 - x^2y^3$

Find each sum or difference.

11. $(5a + 3a^2 - 7a^3) + (2a - 8a^2 + 4)$

12. $(x^3 - 3x^2y + 4xy^2 + y^3)$
$- (7x^3 + x^2y - 9xy^2 + y^3)$

13. GEOMETRY The measures of two sides of a triangle are given. If the perimeter is represented by $11x^2 - 29x + 10$, find the measure of the third side.



14. MULTIPLE CHOICE What is the area of the square with sides that measure $x - 6$?

A $4x - 24$

B $x^2 - 12x + 36$

C $x^2 + 12x + 36$

D $x^2 - 36$

Simplify.

15. $(h - 5)^2$

16. $(4x - y)(4x + y)$

17. $3x^2y^3(2x - xy^2)$

18. $(2a^2b + b^2)^2$

19. $(4m + 3n)(2m - 5n)$

20. $(2c + 5)(3c^2 - 4c + 2)$

Solve each equation.

21. $2x(x - 3) = 2(x^2 - 7) + 2$

22. $3a(a^2 + 5) - 11 = a(3a^2 + 4)$

23. MULTIPLE CHOICE If $x^2 + 2xy + y^2 = 8$, find $3(x + y)^2$.

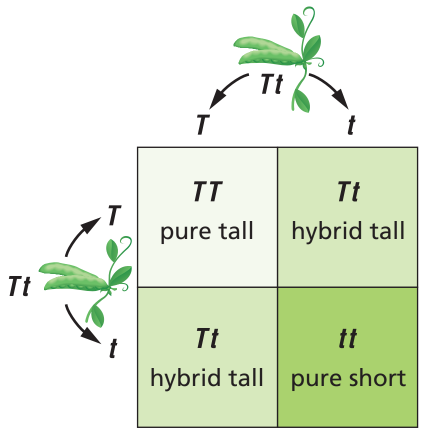
F 2

G 4

H 12

J 24

GENETICS The Punnett square shows the possible gene combinations of a cross between two pea plants. Each plant passes on one dominant gene *T* for tallness and one recessive gene *t* for shortness.



24. Show how the possible combinations can be modeled by the square of a binomial.

25. What is the probability that the offspring will be pure tall (*TT*), hybrid tall (*Tt*), and pure short (*tt*)?

CHAPTER 7

Standardized Test Practice

Cumulative, Chapters 1–7

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. Which expression describes the area in square units of a rectangle that has a width of $2a^2b$ and a length of $4a^3b^4$?

A $8a^6b^4$ C $6a^6b^4$

B $8a^5b^5$ D $6a^5b^5$

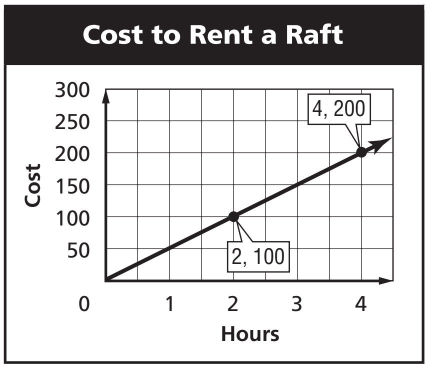
TEST-TAKING TIP

QUESTION 1 On multiple-choice questions, try to compute the answer first. Then compare your answer to the given answer choices. If you don't find your answer among the choices, check your calculations.

2. GRIDDABLE What is the value of r in the equation below?

$$r = \frac{5(2) - 3}{2(7 - 6)}$$

3. Which statement is true for the graph below?



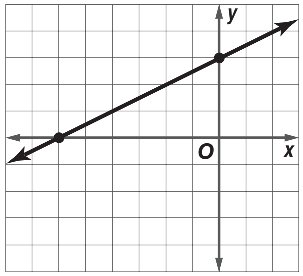
F It will cost Chelsea \$200 to rent a raft for 4 hours.

G It will cost Maria \$150 to rent a raft for 2 hours.

H It will cost John \$100 to rent a raft for 1 hour.

J It will cost Marcus \$100 to rent a raft for 3 hours.

4. The graph of the linear equation $y = \frac{1}{2}x + 3$ is shown below.

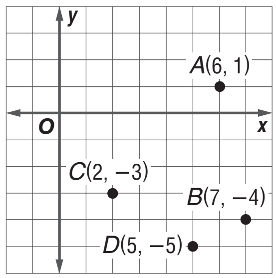


Which point is not in the solution set of $y > \frac{1}{2}x + 3$?

A (1, 5) C (−3, 4)

B (−6, 1) D (−2, 1)

5. Which point on the grid below satisfies the conditions $x < 5$ and $y > -6$?



F Point A

G Point B

H Point C

J Point D

6. Maya used 18 square feet of material to make a blanket. The blanket is in the shape of a square. She would like to put a binding around the outside of the blanket. How can she find the length of a side?

A Find the square root of the area.

B Multiply the area by 4.

C Divide the area by 4.

D Divide the are by 2.

Preparing for Standardized Tests
For test-taking strategies and more practice, see pages 756–773.

7. Describe the effect on the area of a circle when the radius is tripled.

F The area is reduced by $\frac{1}{3}$.

G The area remains constant.

H The area is tripled.

J The area is increased nine times.

8. **GRIDDABLE** Bradley needs to stain his deck. The deck measures 14 by 16 feet. If the stain costs \$1.25 per square foot, including tax, how much will it cost to stain his deck?

9. The table shows the sum of the interior angle measures in certain convex polygons.

Convex Polygon	Number of Sides	Sum of Angle Measures
triangle	3	180°
quadrilateral	4	360°
pentagon	5	540°
hexagon	6	720°

Based on the table, what is the sum of the angle measures of an octagon?

A 720°

B 900°

C 1000°

D 1080°

10. Carlos builds circular parachutes. The area of one parachute is 100 square feet. If he triples the radius of the parachute to build a new parachute, what will the area of the new parachute be?

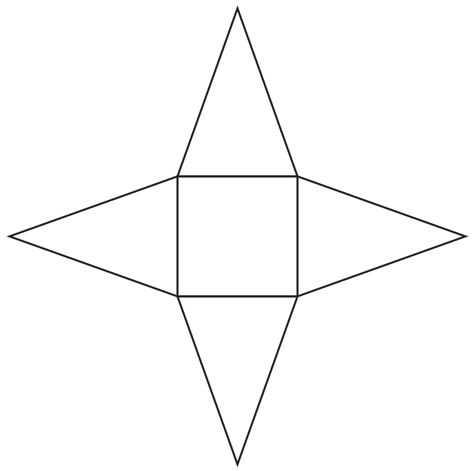
F 100 ft^2

G 300 ft^2

H 600 ft^2

J 900 ft^2

11. The net of a square pyramid is shown below.



Measure the dimensions of the pyramid to the nearest $\frac{1}{8}$ inch. Find the surface area of the pyramid to the nearest square inch.

A 3 in^2 **C** 6 in^2

B 4 in^2 **D** 8 in^2

Pre-AP

Record your answers on a sheet of paper. Show your work.

12. Two cars leave at the same time and both drive to Nashville. The cars' distance from Knoxville, in miles, can be represented by the two equations below, where t represents time in hours.

Car A: $A = 65t + 10$; Car B: $B = 55t + 20$

a. Which car is faster? Explain.

b. How far did Car B travel after 2 hours?

c. Find an expression that models the distance between the two cars.

d. How far apart are the cars after $3\frac{1}{2}$ hours?

NEED EXTRA HELP?												
If You Missed Question...	1	2	3	4	5	6	7	8	9	10	11	12
Go to Lesson or Page...	7-1	1-3	3-3	6-6	6-7	7-2	7-2	7-2	7-1	7-2	7-1	7-4