

Factoring

BIG Ideas

- Find the prime factorization of integers and monomials.
- Factor polynomials.
- Use the Zero Product Property to solve equations.

Key Vocabulary

factored form (p. 421)

perfect square trinomials (p. 454)

prime polynomial (p. 443)

Real-World Link

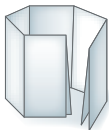
Dolphins Factoring is used to solve problems involving vertical motion. For example, factoring can be used to determine how long a dolphin that jumps out of the water is in the air.

FOLDABLES™

Study Organizer

Factoring Make this Foldable to help you organize your notes on factoring. Begin with a sheet of plain $8\frac{1}{2}$ " by 11" paper.

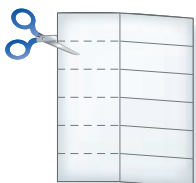
- 1 Fold** in thirds and then in half along the width.



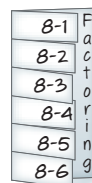
- 2 Open.** Fold lengthwise, leaving a $\frac{1}{2}$ " tab on the right.



- 3 Open.** Cut short side along folds to make tabs.



- 4 Label** each tab as shown.



GET READY for Chapter 8

Diagnose Readiness You have two options for checking Prerequisite Skills.

Option 2



Take the Online Readiness Quiz at algebra1.com.

Option 1

Take the Quick Check below. Refer to the Quick Review for help.

QUICK Check

Rewrite each expression using the Distributive Property. Then simplify.

(Lesson 1-5)

- $3(4 - x)$
- $a(a + 5)$
- $-7(n^2 - 3n + 1)$
- $6y(-3y - 5y + y^2 + y^3)$
- JOBS** In a typical week, Mr. Jackson averages 4 hours using e-mail, 10 hours of meeting in person, and 20 hours on the telephone. Write an expression that could be used to determine how many hours he will spend on these activities over the next month.

Find each product. (Lesson 7-6)

- $(x + 4)(x + 7)$
- $(3n - 4)(n + 5)$
- $(6a - 2b)(9a + b)$
- $(-x - 8y)(2x - 12y)$
- TABLE TENNIS** The dimensions of a homemade table tennis table are represented by a width of $2x + 3$ and a length of $x + 1$. Find an expression for the area of the table tennis table.

Find each product. (Lesson 7-7)

- $(y + 9)^2$
- $(3a - 2)^2$
- $(3m + 5n)^2$
- $(6r - 7s)^2$

QUICK Review

EXAMPLE 1

Rewrite $n\left(n - 3n^2 + 2 + \frac{4}{n}\right)$ using the Distributive Property. Then simplify.

$$\begin{aligned}n\left(n - 3n^2 + 2 + \frac{4}{n}\right) & \quad \text{Original expression} \\= (n)(n) + (n)(-3n^2) + (n)(2) + (n)\left(\frac{4}{n}\right) & \quad \text{Distribute } n \text{ to each term inside the parentheses.} \\= n^2 - 3n^3 + 2n + 4 & \quad \text{Multiply.} \\= -3n^3 + n^2 + 2n + 4 & \quad \text{Rewrite in descending order with respect to the exponents.}\end{aligned}$$

EXAMPLE 2

Find $(x + 2)(3x - 1)$.

$$\begin{aligned}(x + 2)(3x - 1) & \quad \text{Original expression} \\= (x)(3x) + (x)(-1) + (2)(3x) + (2)(-1) & \quad \text{FOIL Method} \\= 3x^2 - x + 6x - 2 & \quad \text{Multiply.} \\= 3x^2 + 5x - 2 & \quad \text{Combine like terms.}\end{aligned}$$

EXAMPLE 3

Find $(3 - g)^2$.

$$\begin{aligned}(3 - g)^2 &= (3 - g)(3 - g) & \quad \text{Laws of Exponents} \\&= 3^2 - 3g - 3g + g^2 & \quad \text{Multiply.} \\&= 3^2 - 6g + g^2 & \quad \text{Combine like terms.} \\&= 9 - 6g + g^2 & \quad \text{Simplify.}\end{aligned}$$

Monomials and Factoring



GET READY for the Lesson

In the search for extraterrestrial life, scientists listen to radio signals coming from faraway galaxies. How can they be sure that a particular radio signal was deliberately sent by intelligent beings instead of coming from some natural phenomenon? What if that signal began with a series of beeps in a pattern composed of the first 30 prime numbers (“beep-beep,” “beep-beep-beep,” and so on)?

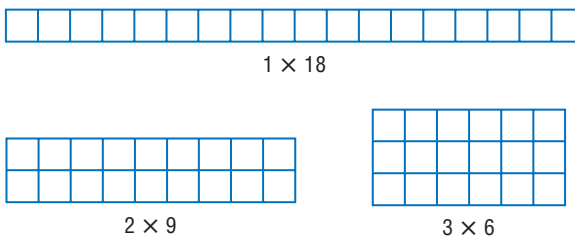
Main Ideas

- Find prime factorizations of monomials.
- Find the greatest common factors of monomials.

New Vocabulary

prime number
 composite number
 prime factorization
 factored form
 greatest common factor (GCF)

Prime Factorization Numbers that are multiplied are *factors* of the resulting product. Numbers that have whole number factors can be represented geometrically. Consider all of the possible rectangles with whole number dimensions that have areas of 18 square units.



The number 18 has six factors: 1, 2, 3, 6, 9, and 18.

Study Tip

Prime Numbers

Before deciding that a number is prime, try dividing it by all of the prime numbers that are less than the square root of that number.

KEY CONCEPT

Prime and Composite Numbers

Words

A whole number, greater than 1, for which the only factors are 1 and itself, is called a **prime number**.

A whole number, greater than 1, that has more than two factors is called a **composite number**.

Examples

2, 3, 5, 7, 11, 13, 17, 19

4, 6, 8, 9, 10, 12, 14, 15

0 and 1 are neither prime nor composite.

A whole number expressed as the product of prime factors is called the **prime factorization** of the number. Two methods of factoring 90 are shown.

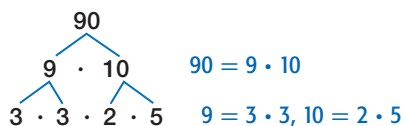
Method 1 Find the least prime factors.

$$90 = 2 \cdot 45 \quad \text{The least prime factor of 90 is 2.}$$

$$= 2 \cdot 3 \cdot 15 \quad \text{The least prime factor of 45 is 3.}$$

$$= 2 \cdot 3 \cdot 3 \cdot 5 \quad \text{The least prime factor of 15 is 3.}$$

Method 2 Use a factor tree.



All of the factors in the last step are prime. Thus, the prime factorization of 90 is $2 \cdot 3 \cdot 3 \cdot 5$ or $2 \cdot 3^2 \cdot 5$.

Usually the factors are ordered from the least prime factor to the greatest.

Factoring a monomial is similar to factoring a whole number. A monomial is in **factored form** when it is expressed as the product of prime numbers and variables, and no variable has an exponent greater than 1.

EXAMPLE Prime Factorization of a Monomial

1 Factor $-12a^2b^3$ completely.

$$\begin{aligned}
 -12a^2b^3 &= -1 \cdot 12a^2b^3 && \text{Express } -12 \text{ as } -1 \cdot 12 \\
 &= -1 \cdot 2 \cdot 6 \cdot a \cdot a \cdot b \cdot b \cdot b && 12 = 2 \cdot 6, a^2 = a \cdot a, \text{ and } b^3 = b \cdot b \cdot b \\
 &= -1 \cdot 2 \cdot 2 \cdot 3 \cdot a \cdot a \cdot b \cdot b \cdot b && 6 = 2 \cdot 3
 \end{aligned}$$

Thus, $-12a^2b^3$ in factored form is $-1 \cdot 2 \cdot 2 \cdot 3 \cdot a \cdot a \cdot b \cdot b \cdot b$.

CHECK Your Progress

Factor each monomial completely.

1A. $38rs^2t$

1B. $-66pq^2$

Cross-Curricular Project



Math online Finding the GCF of distances will help you make a scale model of the solar system. Visit algebra1.com to continue work on your project.

Greatest Common Factor Two or more numbers may have some common prime factors. Consider the prime factorization of 48 and 60.

$$\begin{array}{l}
 48 = \textcircled{2} \cdot \textcircled{2} \cdot 2 \cdot 2 \cdot \textcircled{3} \\
 60 = \textcircled{2} \cdot \textcircled{2} \cdot \textcircled{3} \cdot 5
 \end{array}$$

Factor each number.
Circle the common prime factors.

The common prime factors of 48 and 60 are 2, 2, and 3.

The product of the common prime factors, $2 \cdot 2 \cdot 3$ or 12, is called the greatest common factor of 48 and 60. The **greatest common factor (GCF)** is the greatest number that is a factor of both original numbers. The GCF of two or more monomials can be found in a similar way.

KEY CONCEPT

Greatest Common Factor (GCF)

- The GCF of two or more monomials is the product of their common factors when each monomial is written in factored form.
- If two or more integers or monomials have a GCF of 1, then the integers or monomials are said to be *relatively prime*.

Study Tip

Alternative Method

You can also find the greatest common factor by listing the factors of each number and finding which of the common factors is the greatest.

Consider Example 2.

15: ①, 3, 5, 15

16: ①, 2, 4, 8, 16

The only common factor, and therefore the greatest common factor, is 1.

EXAMPLE Finding GCF

- 2 GEOMETRY** The areas of two rectangles are 15 square inches and 16 square inches, respectively. The length and width of both figures are whole numbers. If the rectangles have the same width, what is the greatest possible value for their widths?

Find the GCF of 15 and 16.

$$15 = 3 \cdot 5 \quad \text{Factor each number.}$$

$$16 = 2 \cdot 2 \cdot 2 \cdot 2 \quad \text{There are no common prime factors.}$$

The GCF of 15 and 16 is 1, so 15 and 16 are relatively prime. The width of the rectangles is 1 inch.

CHECK Your Progress

2. What is the greatest possible value for the widths if the rectangles described above have areas of 84 square inches and 70 square inches, respectively?

EXAMPLE GCF of a Set of Monomials

- 3** Find the GCF of $36x^2y$ and $54xy^2z$.

$$36x^2y = \textcircled{2} \cdot \textcircled{2} \cdot \textcircled{3} \cdot \textcircled{3} \cdot \textcircled{x} \cdot x \cdot \textcircled{y} \quad \text{Factor each number.}$$

$$54xy^2z = \textcircled{2} \cdot \textcircled{3} \cdot \textcircled{3} \cdot 3 \cdot \textcircled{x} \cdot \textcircled{y} \cdot y \cdot z \quad \text{Circle the common prime factors.}$$

The GCF of $36x^2y$ and $54xy^2z$ is $2 \cdot 3 \cdot 3 \cdot x \cdot y$ or $18xy$.

CHECK Your Progress

Find the GCF of each set of monomials.

3A. $17d^3, 5d^2$

3B. $22p^2q, 32pr^2t$

 **Online Personal Tutor at algebra1.com**

CHECK Your Understanding

Example 1
(p. 421)

Factor each monomial completely.

1. $4p^2$

2. $39b^3c^2$

3. $-100x^3yz^2$

4. **GARDENING** Corey is planting 120 jalapeno pepper plants in a rectangular arrangement in his garden. In what ways can he arrange them so that he has the same number of plants in each row, at least 4 rows of plants, and at least 6 plants in each row?

Examples 2, 3
(p. 422)

Find the GCF of each set of monomials.

5. 10, 15

6. $54, 63$

7. $18xy, 36y^2$

8. $25n, 21m$

9. $12qr, 8r^2, 16rs$

10. $42a^2b, 6a^2, 18a^3$

Exercises

HOMEWORK HELP	
For Exercises	See Examples
11–18	1
19–27	2, 3

Factor each monomial completely.

11. $66d^4$

12. $85x^2y^2$

13. $-49a^3b^2$

14. $50gh$

15. $160pq^2$

16. $-243n^3m$

17. **GEOMETRY** A rectangle has an area of 96 square millimeters and its length and width are both whole numbers. What are the minimum and maximum values for the perimeter of the rectangle? Explain your reasoning.

18. **MARCHING BANDS** The number of members in two high school marching bands is shown in the table. During the halftime show, the bands plan to march into the stadium from opposite ends using formations with the same number of rows. If the bands match up in the center of the field, what is the maximum number of rows, and how many band members will be in each row?

High School	Number of Band Members
Logan	75
Northeast	90

Find the GCF of each set of monomials.

19. 27, 72

20. 32, 48

21. 18, 35

22. $15a, 28b^2$

23. $24d^2, 30c^2d$

24. $20gh, 36g^2h^2$

25. $15r^2s, 35s^2, 70rs$

26. $28a^2b^2, 63a^3b^2, 91b^3$

27. $14m^2n^2, 18mn, 2m^2n^3$

28. **NUMBER THEORY** *Twin primes* are two consecutive odd numbers that are prime. The first pair of twin primes is 3 and 5. List the next five pairs of twin primes.

29. **GEOMETRY** The area of a triangle is 20 square centimeters. What are possible whole-number dimensions for the base and height of the triangle?

30. **RESEARCH** Use the Internet or another source to investigate *Mersenne primes*. Describe what they are, and then list three Mersenne primes.

31. **REASONING** Determine whether the following statement is *true* or *false*. If false, provide a counterexample. *All prime numbers are odd.*

32. **CHALLENGE** Suppose 6 is a factor of ab , where a and b are natural numbers. Make a valid argument to explain why each assertion is *true* or provide a counterexample to show that an assertion is *false*.

- 6 must be a factor of a or of b .
- 3 must be a factor of a or of b .
- 3 must be a factor of a and of b .

33. **OPEN ENDED** Name two monomials whose GCF is $5x^2$. Justify your choices.

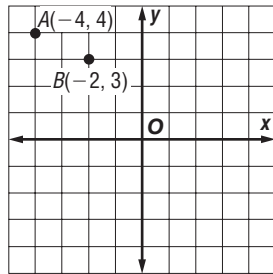
34. **Writing in Math** Use the information about signals on page 420 to explain how prime numbers are related to the search for extraterrestrial life. Include a list of the first 30 prime numbers and an explanation of how you found them.

EXTRA PRACTICE
See pages 733, 751.

Math online
Self-Check Quiz at algebra1.com

H.O.T. Problems

35. If a line passes through A and B , approximately where will the line cross the x -axis?



- A between -1 and 0
 B between 1 and 2
 C between 2.5 and 3.5
 D between 3.5 and 4.5

36. **REVIEW** A shoe store organizes its sale shoes by size. The chart below shows how many pairs of shoes in different styles are on each size rack.

Style	Number of Pairs of Shoes
athletic shoes	15
loafers	8
sandals	22
boots	5

If Bethany chooses a pair without looking, what is the probability that she will choose a pair of boots?

- F $\frac{5}{8}$
 G $\frac{3}{5}$
 H $\frac{4}{25}$
 J $\frac{1}{10}$

Spiral Review

Find each product. (Lessons 7-6 and 7-7)

37. $(2x - 1)^2$

38. $(3a + 5)(3a - 5)$

39. $(7p^2 + 4)(7p^2 + 4)$

40. $(6r + 7)(2r - 5)$

41. $(10h + k)(2h + 5k)$

42. $(b + 4)(b^2 + 3b - 18)$

43. **VIDEOS** Professional closed-captioning services cost \$10 per video minute plus a fee of \$50. A company budgeted \$500 for closed-captioning for an instructional video. Define a variable. Then write and solve an inequality to find the number of video minutes for which they can have closed-captioning and stay within their budget. (Lesson 6-3)

Find the value of r so that the line that passes through the given points has the given slope. (Lesson 4-1)

44. $(1, 2), (-2, r), m = 3$

45. $(-5, 9), (r, 6), m = -\frac{3}{5}$

46. **RETAIL SALES** A department store buys clothing at wholesale prices and then marks the clothing up 25% to sell at retail price to customers. If the retail price of a jacket is \$79, what was the wholesale price? (Lesson 2-7)

GET READY for the Next Lesson

PREREQUISITE SKILL Use the Distributive Property to rewrite each expression. (Lesson 1-5)

47. $5(2x + 8)$

48. $a(3a + 1)$

49. $2g(3g - 4)$

50. $-4y(3y - 6)$

51. $7b + 7c$

52. $2x + 3x$

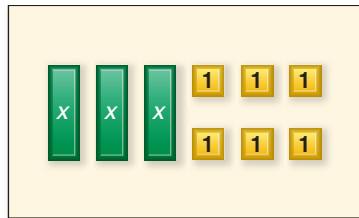
Algebra Lab

Factoring Using the Distributive Property

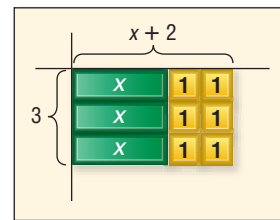
Sometimes you know the product of binomials and are asked to find the factors. This is called factoring. You can use algebra tiles to factor binomials.

ACTIVITY 1 Use algebra tiles to factor $3x + 6$.

Step 1 Model the polynomial $3x + 6$.



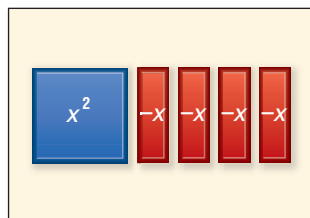
Step 2 Arrange the tiles into a rectangle. The total area of the rectangle represents the product, and its length and width represent the factors.



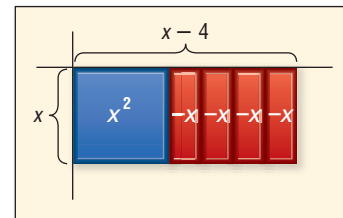
The rectangle has a width of 3 and a length of $x + 2$. So, $3x + 6 = 3(x + 2)$.

ACTIVITY 2 Use algebra tiles to factor $x^2 - 4x$.

Step 1 Model the polynomial $x^2 - 4x$.



Step 2 Arrange the tiles into a rectangle.



The rectangle has a width of x and a length of $x - 4$. So, $x^2 - 4x = x(x - 4)$.

ANALYZE THE RESULTS

Use algebra tiles to factor each binomial.

1. $2x + 10$

2. $6x - 8$

3. $5x^2 + 2x$

4. $9 - 3x$

Tell whether each binomial can be factored. Justify your answer with a drawing.

5. $4x - 10$

6. $3x - 7$

7. $x^2 + 2x$

8. $x^2 + 4$

9. **MAKE A CONJECTURE** Explain how you can use algebra tiles to determine whether a binomial can be factored. Include an example of one binomial that can be factored and one that cannot.

Factoring Using the Distributive Property

Main Ideas

- Factor polynomials by using the Distributive Property.
- Solve quadratic equations of the form $ax^2 + bx = 0$.

New Vocabulary

factoring
factoring by grouping
Zero Products Property
roots

GET READY for the Lesson

Roger Clemens, pitcher for the Houston Astros, has had fastballs clocked at 98 miles per hour or about 151 feet per second. If he threw a ball directly upward with the same velocity, the height h of the ball in feet above the point at which he released it could be modeled by the formula $h = 151t - 16t^2$, where t is the time in seconds. You can use factoring and the Zero Product Property to determine how long the ball would remain in the air before returning to his glove.



Factor by Using the Distributive Property In Chapter 7, you used the Distributive Property to multiply a polynomial by a monomial.

$$\begin{aligned} 2a(6a + 8) &= 2a(6a) + 2a(8) \\ &= 12a^2 + 16a \end{aligned}$$

You can reverse this process to express a polynomial as the product of a monomial factor and a polynomial factor.

$$\begin{aligned} 12a^2 + 16a &= 2a(6a) + 2a(8) \\ &= 2a(6a + 8) \end{aligned}$$

Thus, a *factored form* of $12a^2 + 16a$ is $2a(6a + 8)$. **Factoring** a polynomial means to find its *completely* factored form.

EXAMPLE Use the Distributive Property

1 Use the Distributive Property to factor each polynomial.

a. $12a^2 + 16a$

First, find the GCF of $12a^2$ and $16a$.

$$12a^2 = \underbrace{2 \cdot 2 \cdot 3}_{\text{prime factors}} \cdot \underbrace{a \cdot a}_{\text{prime factors}} \quad \text{Factor each monomial.}$$

$$16a = \underbrace{2 \cdot 2 \cdot 2 \cdot 2}_{\text{prime factors}} \cdot \underbrace{a}_{\text{prime factor}} \quad \text{Circle the common prime factors.}$$

$$\text{GCF: } 2 \cdot 2 \cdot a \text{ or } 4a$$

Write each term as the product of the GCF and its remaining factors. Then use the Distributive Property to factor out the GCF.

$$\begin{aligned} 12a^2 + 16a &= 4a(3 \cdot a) + 4a(2 \cdot 2) && \text{Rewrite each term using the GCF.} \\ &= 4a(3a) + 4a(4) && \text{Simplify remaining factors.} \\ &= 4a(3a + 4) && \text{Distributive Property} \end{aligned}$$

Thus, the completely factored form of $12a^2 + 16a$ is $4a(3a + 4)$.

b. $18cd^2 + 12c^2d + 9cd$

$18cd^2 = 2 \cdot \textcircled{3} \cdot 3 \cdot \textcircled{c} \cdot \textcircled{d} \cdot d$ Factor each monomial.

$12c^2d = 2 \cdot 2 \cdot \textcircled{3} \cdot \textcircled{c} \cdot c \cdot \textcircled{d}$ Circle the common prime factors.

$9cd = \textcircled{3} \cdot 3 \cdot \textcircled{c} \cdot \textcircled{d}$

GCF: $3 \cdot c \cdot d$ or $3cd$

$$\begin{aligned} 18cd^2 + 12c^2d + 9cd &= 3cd(6d) + 3cd(4c) + 3cd(3) && \text{Rewrite each term using the GCF.} \\ &= 3cd(6d + 4c + 3) && \text{Distributive Property} \end{aligned}$$

CHECK Your Progress

1A. $16a + 4b$

1B. $3p^2q - 9pq^2 + 36pq$

Using the Distributive Property to factor polynomials having four or more terms is called **factoring by grouping** because pairs of terms are grouped together and factored. The Distributive Property is then applied a second time to factor a common binomial factor.

Study Tip

Factoring by Grouping

Sometimes you can group terms in more than one way when factoring a polynomial. For example, the polynomial in Example 2 could have been factored in the following way.

$$\begin{aligned} 4ab + 8b + 3a + 6 &= (4ab + 3a) + (8b + 6) \\ &= a(4b + 3) + 2(4b + 3) \\ &= (4b + 3)(a + 2) \end{aligned}$$

Notice that this result is the same as in Example 2.

EXAMPLE Use Grouping

2 Factor $4ab + 8b + 3a + 6$.

$4ab + 8b + 3a + 6$

$= (4ab + 8b) + (3a + 6)$ Group terms with common factors.

$= 4b(a + 2) + 3(a + 2)$ Factor the GCF from each grouping.

$= (a + 2)(4b + 3)$ Distributive Property

CHECK Your Progress Factor each polynomial.

2A. $6x^2 - 15x - 8x + 20$

2B. $rs + 5s - r - 5$

Recognizing binomials that are additive inverses is often helpful when factoring by grouping. For example, $7 - y$ and $y - 7$ are additive inverses. By rewriting $7 - y$ as $-1(y - 7)$, factoring by grouping is possible in the following example.

EXAMPLE Use the Additive Inverse Property

3 Factor $35x - 5xy + 3y - 21$.

$35x - 5xy + 3y - 21 = (35x - 5xy) + (3y - 21)$ Group terms with common factors.

$= 5x(7 - y) + 3(y - 7)$ Factor the GCF from each grouping.

$= 5x(-1)(y - 7) + 3(y - 7)$ $7 - y = -1(y - 7)$

$= -5x(y - 7) + 3(y - 7)$ $5x(-1) = -5x$

$= (y - 7)(-5x + 3)$ Distributive Property

CHECK Your Progress Factor each polynomial.

3A. $c - 2cd + 8d - 4$

3B. $3p - 2p^2 - 18p + 27$



CONCEPT SUMMARY

Factoring by Grouping

Words A polynomial can be factored by grouping if *all* of the following situations exist.

- There are four or more terms.
- Terms with common factors can be grouped together.
- The two common binomial factors are identical or are additive inverses of each other.

Symbols $ax + bx + ay + by = x(a + b) + y(a + b)$
 $= (a + b)(x + y)$

Solve Equations by Factoring Some equations can be solved by factoring. Consider the following products.

$$6(0) = 0 \quad 0(-3) = 0 \quad (5 - 5)(0) = 0 \quad -2(-3 + 3) = 0$$

Notice that in each case, *at least one* of the factors is zero. These examples illustrate the **Zero Product Property**.

Study Tip

Zero Product Property

If the product of two factors is equal to a *nonzero* value, then you *cannot* use the Zero Product Property. You must first multiply all the factors, and then put all the terms on one side of the equation, with zero on the other. Then you must factor the new expression and use the Zero Product Property.

KEY CONCEPT

Zero Product Property

Word If the product of two factors is 0, then at least one of the factors must be 0.

Symbols For any real numbers a and b , if $ab = 0$, then either $a = 0$, $b = 0$, or both a and b equal zero.

The solutions of an equation are called the **roots** of the equation.

EXAMPLE Solve an Equation

4 Solve each equation. Check the solutions.

a. $(d - 5)(3d + 4) = 0$

If $(d - 5)(3d + 4) = 0$, then according to the Zero Product Property either $d - 5 = 0$ or $3d + 4 = 0$.

$$(d - 5)(3d + 4) = 0$$

Original equation

$$d - 5 = 0 \quad \text{or} \quad 3d + 4 = 0$$

Set each factor equal to zero.

$$d = 5$$

$$3d = -4$$

Solve each equation.

$$d = -\frac{4}{3}$$

The roots are 5 and $-\frac{4}{3}$.

CHECK Substitute 5 and $-\frac{4}{3}$ for d in the original equation.

$$(d - 5)(3d + 4) = 0$$

$$(d - 5)(3d + 4) = 0$$

$$(5 - 5)[3(5) + 4] \stackrel{?}{=} 0$$

$$\left[\left(-\frac{4}{3} \right) - 5 \right] \left[3 \left(-\frac{4}{3} \right) + 4 \right] \stackrel{?}{=} 0$$

$$(0)(19) \stackrel{?}{=} 0$$

$$\left(-\frac{19}{3} \right) (0) \stackrel{?}{=} 0$$

$$0 = 0 \quad \checkmark$$

$$0 = 0 \quad \checkmark$$

Study Tip

Common Misconception

You may be tempted to try to solve the equation in Example 4b by dividing each side of the equation by x .

Remember, however, that x is an *unknown* quantity. If you divide by x , you may actually be dividing by zero, which is undefined.

b. $x^2 = 7x$

Write the equation so that it is of the form $ab = 0$.

$$x^2 = 7x$$

Original equation

$$x^2 - 7x = 0$$

Subtract $7x$ from each side.

$$x(x - 7) = 0$$

Factor using the GCF of x^2 and $-7x$, which is x .

$$x = 0 \quad \text{or} \quad x - 7 = 0$$

Zero Product Property

$$x = 7$$

Solve each equation.

The roots are 0 and 7.

Check by substituting 0 and 7 for x in the original equation.

CHECK Your Progress

4A. $3n(n + 2) = 0$

4B. $7d^2 - 35d = 0$

4C. $x^2 = -10x$

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CHECK Your Understanding

Examples 1–3
(pp. 426–427)

Factor each polynomial.

1. $9x^2 + 36x$

2. $4r^2 + 8rs + 28r$

3. $5y^2 - 15y + 4y - 12$

4. $5c - 10c^2 + 2d - 4cd$

Example 4
(pp. 428–429)

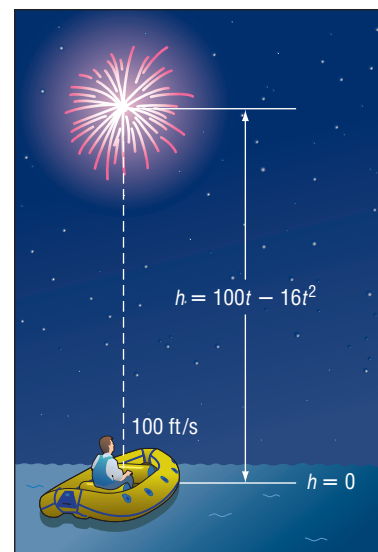
Solve each equation. Check the solutions.

5. $h(h + 5) = 0$

6. $(n - 4)(n + 2) = 0$

7. $5m = 3m^2$

8. **PHYSICAL SCIENCE** A flare is launched from a life raft. The height h of the flare in feet above the sea is modeled by the formula $h = 100t - 16t^2$, where t is the time in seconds after the flare is launched. Let $h = 0$ and solve $0 = 100t - 16t^2$ for t . How many seconds will it take for the flare to return to the sea? Explain your reasoning.



Exercises

HOMEWORK HELP

For Exercises	See Examples
9–14	1
15–20	2, 3
21–30	4

Factor each polynomial.

9. $5x + 30y$

10. $a^5b - a$

11. $14gh - 18h$

12. $8bc^2 + 24bc$

13. $15x^2y^2 + 25xy + x$

14. $12ax^3 + 20bx^2 + 32cx$

15. $x^2 + 2x + 3x + 6$

16. $12y^2 + 9y + 8y + 6$

17. $18x^2 - 30x - 3x + 5$

18. $2my + 7x + 7m + 2xy$

19. $8ax - 6x - 12a + 9$

20. $10x^2 - 14xy - 15x + 21y$

Solve each equation. Check the solutions.

21. $x(x - 24) = 0$

22. $a(a + 16) = 0$

23. $(q + 4)(3q - 15) = 0$

24. $(3y + 9)(y - 7) = 0$

25. $(2b - 3)(3b - 8) = 0$

26. $(4n + 5)(3n - 7) = 0$

27. $3z^2 + 12z = 0$

28. $2x^2 = 5x$

29. **BASEBALL** Malik popped a ball straight up with an initial upward velocity of 45 feet per second. The height h , in feet, of the ball above the ground is modeled by the equation $h = 2 + 48t - 16t^2$. How long was the ball in the air if the catcher catches the ball when it is 2 feet above the ground? Is your answer reasonable in the context of this situation?

30. **MARINE BIOLOGY** In a pool at an aquarium, a dolphin jumps out of the water traveling at 20 feet per second. Its height h , in feet, above the water after t seconds is given by the formula $h = 20t - 16t^2$. Solve the equation for $h = 0$ and interpret the solution.

Factor each polynomial.

31. $12x^2y^2z + 40xy^3z^2$

32. $18a^2bc^2 - 48abc^3$

GEOMETRY Find an expression for the area of a square with the given perimeter.

33. $P = (12x + 20y)$ in.

34. $P = (36a - 16b)$ cm

35. **GEOMETRY** The expression $\frac{1}{2}n^2 - \frac{3}{2}n$ can be used to find the number of diagonals in a polygon that has n sides. Write the expression in factored form and find the number of diagonals in a decagon (10-sided polygon).

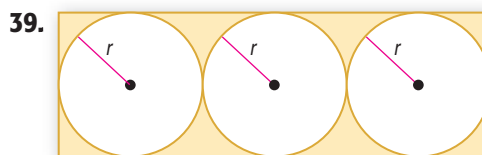
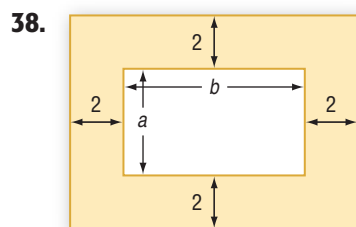
SOFTBALL For Exercises 36 and 37, use the following information.

Alisha is scheduling the games for a softball league. To find the number of games she needs to schedule, she uses the equation $g = \frac{1}{2}n^2 - \frac{1}{2}n$, where g represents the number of games needed for each team to play each other exactly once and n represents the number of teams.

36. Write this equation in factored form.

37. How many games are needed for 7 teams to play each other exactly 3 times?

GEOMETRY Write an expression in factored form for the area of each shaded region.



40. **REASONING** Represent $4x^2 + 12x$ as a product of factors in three different ways. Then decide which of the three is the completely factored form. Explain your reasoning.



Real-World Career

Marine Biologist

A marine biologist uses math to study and analyze factors that affect organisms living in and near the ocean.



For more information, go to algebra1.com.

EXTRA PRACTICE
See pages 733, 751.
Math online
Self-Check Quiz at algebra1.com

H.O.T. Problems

41. **OPEN ENDED** Write an equation that can be solved by using the Zero Product Property. Describe how to solve the equation and then find the roots.
42. **REASONING** Explain why $(x - 2)(x + 4) = 0$ cannot be solved by dividing each side by $x - 2$.
43. **CHALLENGE** Factor $a^{x+y} + a^x b^y - a^y b^x - b^{x+y}$. Describe your steps.
44. **Writing in Math** Use the information about Roger Clemens on page 426 to explain how you can determine how long a baseball will remain in the air. Explain how to use factoring and the Zero Product Property to solve the problem. Then interpret each solution in the context of the problem.

STANDARDIZED TEST PRACTICE

45. Which of the following shows $16x^2 - 4x$ factored completely?
- A $4x(x)$
 B $4x(4x - 1)$
 C $x(4x - 1)$
 D $x(x - 4)$

46. **REVIEW** The frequency table shows the results of a survey in which students were asked to name the colors of their bicycles. Which measure of data describes the most popular color for a bicycle?

Color	Frequency
black	I
blue	
red	
silver	I
Total	50

- F mean H mode
 G median J range

Spiral Review

Find the GCF of each set of monomials. (Lesson 8-1)

47. $9a, 8ab$

48. $16h, 28hk^2$

49. $3x^2y^2, 9xy, 15x^3y$

Find each product. (Lesson 7-7)

50. $(4s^3 + 3)^2$

51. $(2p + 5q)(2p - 5q)$

52. $(3k + 8)(3k + 8)$

53. **FINANCE** Michael uses at most 60% of his annual Flynn Company stock dividend to purchase more shares of Flynn Company stock. If his dividend last year was \$885 and Flynn Company stock is selling for \$14 per share, what is the greatest number of shares that he can purchase? (Lesson 6-2)

GET READY for the Next Lesson

PREREQUISITE SKILL Find each product. (Lesson 7-6)

54. $(n + 8)(n + 3)$

55. $(x - 4)(x - 5)$

56. $(b - 10)(b + 7)$

57. $(3a + 1)(6a - 4)$

58. $(5p - 2)(9p - 3)$

59. $(2y - 5)(4y + 3)$

Algebra Lab

Factoring Trinomials

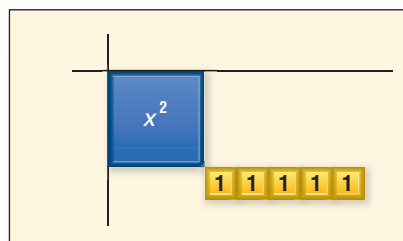
You can use algebra tiles to factor trinomials. If a polynomial represents the area of a rectangle formed by algebra tiles, then the rectangle's length and width are *factors* of the polynomial.

ACTIVITY 1 Use algebra tiles to factor $x^2 + 6x + 5$.

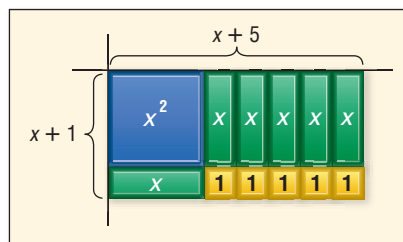
Step 1 Model $x^2 + 6x + 5$.



Step 2 Place the x^2 -tile at the corner of the product mat. Arrange the 1-tiles into a rectangular array. Because 5 is prime, the 5 tiles can be arranged in a rectangle in one way, a 1-by-5 rectangle.



Step 3 Complete the rectangle with the x -tiles. The rectangle has a width of $x + 1$ and a length of $x + 5$. Therefore, $x^2 + 6x + 5 = (x + 1)(x + 5)$.

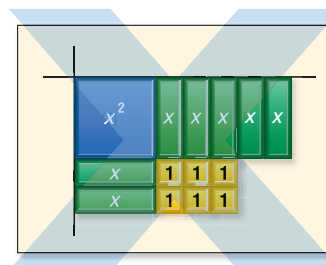
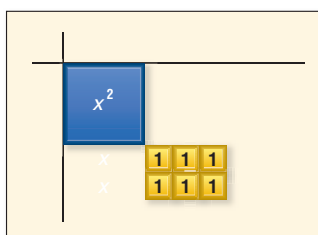


ACTIVITY 2 Use algebra tiles to factor $x^2 + 7x + 6$.

Step 1 Model $x^2 + 7x + 6$.

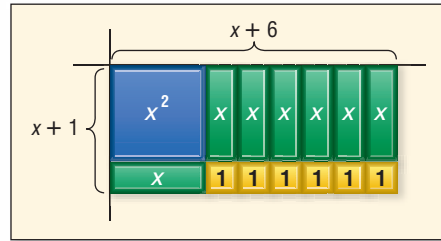


Step 2 Place the x^2 -tile at the corner of the product mat. Arrange the 1-tiles into a rectangular array. Since $6 = 2 \times 3$, try a 2-by-3 rectangle. Try to complete the rectangle. Notice that there are two extra x -tiles.



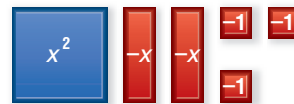
Step 3 Arrange the 1-tiles into a 1-by-6 rectangular array. This time you can complete the rectangle with the x -tiles.

The rectangle has a width of $x + 1$ and a length of $x + 6$. Therefore, $x^2 + 7x + 6 = (x + 1)(x + 6)$.

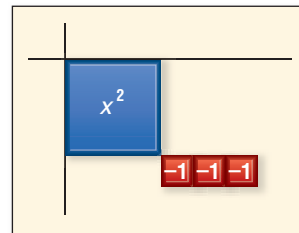


ACTIVITY 3 Use algebra tiles to factor $x^2 - 2x - 3$.

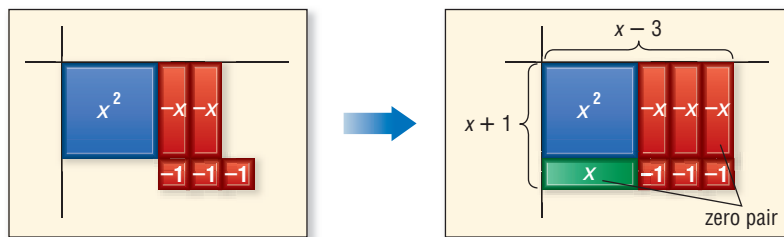
Step 1 Model the polynomial $x^2 - 2x - 3$.



Step 2 Place the x^2 -tile at the corner of the product mat. Arrange the 1-tiles into a 1-by-3 rectangular array as shown.



Step 3 Place the x -tile as shown. Recall that you can add zero-pairs without changing the value of the polynomial. In this case, add a zero pair of x -tiles.



The rectangle has a width of $x + 1$ and a length of $x - 3$. Therefore, $x^2 - 2x - 3 = (x + 1)(x - 3)$.

ANALYZE THE RESULTS

Use algebra tiles to factor each trinomial.

1. $x^2 + 4x + 3$
2. $x^2 + 5x + 4$
3. $x^2 - x - 6$
4. $x^2 - 3x + 2$
5. $x^2 + 7x + 12$
6. $x^2 - 4x + 4$
7. $x^2 - x - 2$
8. $x^2 - 6x + 8$

9. Examine the dimensions of the rectangles in each factored model. How does the sum of the dimensions compare to the coefficient of the x -term? Explain how you could use this observation to factor trinomials.

Factoring Trinomials:

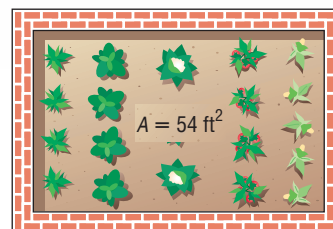
$$x^2 + bx + c$$

Main Ideas

- Factor trinomials of the form $x^2 + bx + c$.
- Solve equations of the form $x^2 + bx + c = 0$.

GET READY for the Lesson

Tamika has enough bricks to make a 30-foot border around the rectangular vegetable garden she is planting. The nursery says that the plants will need a space of 54 square feet to grow. What should the dimensions of her garden be?



To solve this problem, you need to find two numbers with a product of 54 and a sum of 15, half the perimeter of the garden.

Factor $x^2 + bx + c$ When two numbers are multiplied, each number is a factor of the product. Similarly, when two binomials are multiplied, each binomial is a factor of the product. To factor certain types of trinomials, you will use the pattern for multiplying two binomials. Study the following example.

$$\begin{aligned}
 (x + 2)(x + 3) &= (x \cdot x) + (x \cdot 3) + (x \cdot 2) + (2 \cdot 3) && \text{Use the FOIL method.} \\
 &= x^2 + 3x + 2x + 6 && \text{Simplify.} \\
 &= x^2 + (3 + 2)x + 6 && \text{Distributive Property} \\
 &= x^2 + 5x + 6 && \text{Simplify.}
 \end{aligned}$$

Observe the following pattern in this multiplication.

$$\begin{aligned}
 (x + 2)(x + 3) &= x^2 + (3 + 2)x + (2 \cdot 3) \\
 (x + m)(x + n) &= x^2 + (n + m)x + mn && \text{Let } 2 = m \text{ and } 3 = n. \\
 &= x^2 + \underbrace{(m + n)}x + \underbrace{mn} && \text{Commutative (+)} \\
 &= x^2 + bx + c && b = m + n \text{ and } c = mn
 \end{aligned}$$

Notice that the coefficient of the middle term is the sum of m and n and the last term is the product of m and n . This pattern can be used to factor trinomials of the form $x^2 + bx + c$.

KEY CONCEPT

Factoring $x^2 + bx + c$

Words To factor trinomials of the form $x^2 + bx + c$, find two integers, m and n , with a sum equal to b and a product equal to c . Then write $x^2 + bx + c$ as $(x + m)(x + n)$.

Symbols $x^2 + bx + c = (x + m)(x + n)$ when $m + n = b$ and $mn = c$.

Example $x^2 + 5x + 6 = (x + 2)(x + 3)$, since $2 + 3 = 5$ and $2 \cdot 3 = 6$

EXAMPLE b and c are Positive**1** Factor $x^2 + 6x + 8$.

In this trinomial, $b = 6$ and $c = 8$. You need to find two numbers with a sum of 6 and a product of 8. Make an organized list of the factors of 8, and look for the pair of factors with a sum of 6.

Factors of 8	Sum of Factors
1, 8	9
2, 4	6

The correct factors are 2 and 4.

$$\begin{aligned} x^2 + 6x + 8 &= (x + m)(x + n) && \text{Write the pattern.} \\ &= (x + 2)(x + 4) && m = 2 \text{ and } n = 4 \end{aligned}$$

CHECK You can check this result by multiplying the two factors.

$$\begin{aligned} (x + 2)(x + 4) &= \overset{\text{F}}{x^2} + \overset{\text{O}}{4x} + \overset{\text{I}}{2x} + \overset{\text{L}}{8} && \text{FOIL method} \\ &= x^2 + 6x + 8 \checkmark && \text{Simplify.} \end{aligned}$$

CHECK Your Progress Factor each trinomial.

1A. $a^2 + 8a + 15$

1B. $9 + 10t + t^2$

When factoring a trinomial where b is negative and c is positive, use what you know about the product of binomials to narrow the list of possible factors.

EXAMPLE b is Negative and c is Positive**2** Factor $x^2 - 10x + 16$.

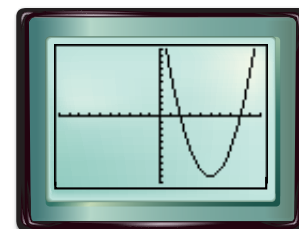
In this trinomial, $b = -10$ and $c = 16$. This means that $m + n$ is negative and mn is positive. So m and n must both be negative. Make a list of the negative factors of 16, and look for the pair with the sum of -10 .

Factors of 16	Sum of Factors
-1, -16	-17
-2, -8	-10
-4, -4	-8

The correct factors are -2 and -8 .

$$\begin{aligned} x^2 - 10x + 16 &= (x + m)(x + n) && \text{Write the pattern.} \\ &= (x - 2)(x - 8) && m = -2 \text{ and } n = -8 \end{aligned}$$

CHECK You can check this result by using a graphing calculator. Graph $y = x^2 - 10x + 16$ and $y = (x - 2)(x - 8)$ on the same screen. Since only one graph appears, the two graphs must coincide. Therefore, the trinomial has been factored correctly. \checkmark



$[-10, 10]$ scl: 1 by $[-10, 10]$ scl: 1

Study Tip**Testing Factors**

Once you find the correct factors, there is no need to test any other factors. Therefore, it is not necessary to test -4 and -4 in Example 2.

CHECK Your Progress Factor each trinomial.

2A. $21 - 22m + m^2$

2B. $s^2 - 11s + 28$

Study Tip

Alternate Method

You can use the opposite of FOIL to factor trinomials. For instance, consider Example 3.

$$x^2 + 2x - 15$$

$$(x + \square)(x + \square)$$

Try factor pairs of -15 until the sum of the products of the Inner and Outer terms is $2x$.

EXAMPLE c is Negative

5 Factor each trinomial.

a. $x^2 + 2x - 15$

Since $b = 2$ and $c = -15$, $m + n$ is positive and mn is negative. So either m or n is negative, but not both. List the factors of -15 , where one factor of each pair is negative. Look for the pair of factors with a sum of 2.

Factors of -15	Sum of Factors
1, -15	-14
-1 , 15	14
3, -5	-2
-3 , 5	2

The correct factors are -3 and 5.

$$x^2 + 2x - 15 = (x + m)(x + n) \quad \text{Write the pattern.}$$

$$= (x - 3)(x + 5) \quad m = -3 \text{ and } n = 5$$

b. $x^2 - 7x - 18$

Since $b = -7$ and $c = -18$, $m + n$ is negative and mn is negative. So either m or n is negative, but not both.

Factors of -18	Sum of Factors
1, -18	-17
-1 , 18	17
2 , -9	-7

The correct factors are 2 and -9 .

$$x^2 - 7x - 18 = (x + m)(x + n) \quad \text{Write the pattern.}$$

$$= (x + 2)(x - 9) \quad m = 2 \text{ and } n = -9$$

 **CHECK Your Progress** Factor each trinomial.

3A. $h^2 + 3h - 40$

3B. $r^2 - 2r - 24$

Solve Equations by Factoring Some equations of the form $x^2 + bx + c = 0$ can be solved by factoring and then using the Zero Product Property.

EXAMPLE Solve an Equation by Factoring

4 Solve $x^2 + 5x - 6 = 0$. Check the solutions.

$$x^2 + 5x - 6 = 0 \quad \text{Original equation}$$

$$(x - 1)(x + 6) = 0 \quad \text{Factor.}$$

$$x - 1 = 0 \quad \text{or} \quad x + 6 = 0 \quad \text{Zero Product Property}$$

$$x = 1 \quad x = -6 \quad \text{Solve each equation.}$$

The roots are 1 and -6 . Check by substituting 1 and -6 for x in the original equation.

 **CHECK Your Progress**

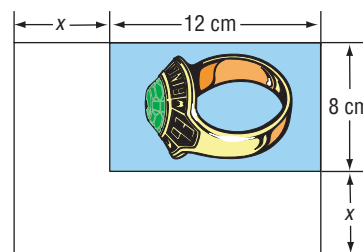
Solve each equation. Check the solutions.

4A. $x^2 + 16x = -28$

4B. $g^2 + 6g = 27$

**Real-World EXAMPLE****Solve a Real-World Problem by Factoring****5**

YEARBOOK DESIGN A sponsor for the school yearbook has asked that the length and width of a photo in their ad be increased by the same amount in order to double the area of the photo. If the original photo is 12 centimeters wide by 8 centimeters long, what should be the new dimensions of the enlarged photo?



Explore Begin by making a diagram like the one shown above, labeling the appropriate dimensions.

Plan Let x = the amount added to each dimension of the photo.

$$\begin{array}{l} \text{The new length} \quad \text{times} \quad \text{the new width} \quad \text{equals} \quad \text{twice the old area.} \\ x + 12 \quad \cdot \quad x + 8 \quad = \quad 2(8)(12) \end{array}$$

Solve $(x + 12)(x + 8) = 2(8)(12)$ Write the equation.

$$x^2 + 20x + 96 = 192 \quad \text{Multiply.}$$

$$x^2 + 20x - 96 = 0 \quad \text{Rewrite the equation so that one side equals 0.}$$

$$(x + 24)(x - 4) = 0 \quad \text{Factor.}$$

$$x + 24 = 0 \quad \text{or} \quad x - 4 = 0 \quad \text{Zero Product Property}$$

$$x = -24 \quad \quad \quad x = 4 \quad \text{Solve each equation.}$$

Check The solution set is $\{-24, 4\}$. In the context of the situation, only 4 is a valid solution because dimensions cannot be negative. Thus, the new dimensions of the photo should be $4 + 12$ or 16 centimeters, and $4 + 8$ or 12 centimeters.

**CHECK Your Progress**

5. GEOMETRY The height of a parallelogram is 18 centimeters less than its base. If the parallelogram has an area of 175 square centimeters, what is its height?



Personal Tutor at algebra1.com

**CHECK Your Understanding**

Examples 1–3 Factor each trinomial.

(pp. 435–436)

1. $x^2 + 11x + 24$

2. $n^2 - 3n + 2$

3. $w^2 + 13w - 48$

4. $p^2 - 2p - 35$

5. $y^2 + y - 20$

6. $72 + 27a + a^2$

Example 4 Solve each equation. Check the solutions.

(p. 436)

7. $n^2 + 7n + 6 = 0$

8. $a^2 + 5a - 36 = 0$

9. $y^2 + 9 = 10y$

10. $d^2 - 3d = 70$

Example 5 **11. NUMBER THEORY** Find two consecutive integers x and $x + 1$ with a product of 156.
(p. 437)

Exercises

HOMEWORK HELP

For Exercises	See Examples
12–23	1–3
24–31	4
32, 33	5

Factor each trinomial.

12. $x^2 + 12x + 27$

13. $c^2 + 12c + 35$

14. $y^2 + 13y + 30$

15. $d^2 - 7d + 10$

16. $p^2 - 17p + 72$

17. $g^2 - 19g + 60$

18. $x^2 + 6x - 7$

19. $n^2 + 3n - 54$

20. $y^2 - y - 42$

21. $z^2 - 18z - 40$

22. $-72 + 6w + w^2$

23. $-30 + 13x + x^2$

Solve each equation. Check the solutions.

24. $b^2 + 20b + 36 = 0$

25. $y^2 + 4y - 12 = 0$

26. $d^2 + 2d - 8 = 0$

27. $m^2 - 19m + 48 = 0$

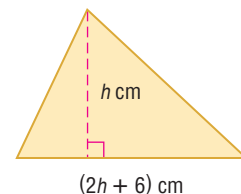
28. $z^2 = 18 - 7z$

29. $h^2 + 15 = -16h$

30. $24 + k^2 = 10k$

31. $c^2 - 50 = -23c$

32. **GEOMETRY** The triangle has an area of 40 square centimeters. Find the height h of the triangle.



33. **SUPREME COURT** When the justices of the Supreme Court assemble each day, each justice shakes hands with each of the other justices. The total number of handshakes h possible for n people is given by $h = \frac{n^2 - n}{2}$. Write and solve an equation to determine the number of justices on the Supreme Court.

RUGBY For Exercises 34 and 35, use the following information.

The length of a Rugby League field is 52 meters longer than its width w .

34. Write an expression for the area of the field.

35. The area of a Rugby League field is 8160 square meters. Find the dimensions of the field.

GEOMETRY Find an expression for the perimeter of a rectangle with the given area.

36. area = $x^2 + 24x - 81$

37. area = $x^2 + 13x - 90$

SWIMMING For Exercises 38–40, use the following information.

The length of a rectangular swimming pool is 20 feet greater than its width. The area of the pool is 525 square feet.

38. Define a variable and write an equation for the area of the pool.

39. Solve the equation.

40. Interpret the solutions. Do they both make sense in the context of the problem? Explain.

41. **REASONING** Explain why, when factoring $x^2 + 6x + 9$, it is not necessary to check the sum of the factor pairs -1 and -9 or -3 and -3 .

42. **OPEN ENDED** Give an example of an equation that can be solved using the factoring techniques presented in this lesson. Then solve your equation.



Real-World Link

The “Conference handshake” has been a tradition since the late 19th century. Each day, there is a total of 36 handshakes by the justices.

Source: supremecourtus.gov

EXTRA PRACTICE

See pages 733, 751.

Math online

Self-Check Quiz at algebra1.com

H.O.T. Problems

- 43. FIND THE ERROR** Peter and Aleta are solving $x^2 + 2x = 15$. Who is correct? Explain your reasoning.

Peter

$$x^2 + 2x = 15$$

$$x(x + 2) = 15$$

$$x = 15 \text{ or } x + 2 = 15$$

$$x = 13$$

Aleta

$$x^2 + 2x = 15$$

$$x^2 + 2x - 15 = 0$$

$$(x - 3)(x + 5) = 0$$

$$x - 3 = 0 \text{ or } x + 5 = 0$$

$$x = 3 \qquad x = -5$$

CHALLENGE Find all values of k so that each trinomial can be factored using integers.

- 44.** $x^2 + kx - 19$ **45.** $x^2 + kx + 14$
46. $x^2 - 8x + k, k > 0$ **47.** $x^2 - 5x + k, k > 0$
- 48. Writing in Math** Use the information about Tamika's garden on page 434 to explain how factoring can be used to find the dimensions of a garden. Explain how your method is related to the process used to factor trinomials of the form $x^2 + bx + c$.



STANDARDIZED TEST PRACTICE

- 49.** Which is a factor of $x^2 + 9x + 18$?
 A $x + 2$
 B $x - 2$
 C $x + 3$
 D $x - 3$
- 50. REVIEW** An 8-foot by 5-foot section of wall is to be covered by square tiles that measure 4 inches on each side. If the tiles are not cut, how many of them will be needed to cover the wall?
 F 30 H 360
 G 240 J 1440

Spiral Review

Solve each equation. Check the solutions. (Lesson 8-2)

- 51.** $(x + 3)(2x - 5) = 0$ **52.** $7b(b - 4) = 0$ **53.** $5y^2 = -9y$

Find the GCF of each set of monomials. (Lesson 8-1)

- 54.** 24, 72 **55.** $9pq^5, 21p^3q^3$ **56.** $30x^2, 75x^3y^4, 20x^4z$

- 57. MUSIC** Albertina practices the guitar 20 minutes each day. She wants to add 5 minutes to her practice time each day until she is practicing at least 45 minutes daily. How many days will it take her to reach her goal? (Lesson 6-3)

GET READY for the Next Lesson

PREREQUISITE SKILL Factor each polynomial. (Lesson 8-2)

- 58.** $3y^2 + 2y + 9y + 6$ **59.** $3a^2 + 2a + 12a + 8$ **60.** $4x^2 + 3x + 8x + 6$
61. $2p^2 - 6p + 7p - 21$ **62.** $3b^2 + 7b - 12b - 28$ **63.** $4g^2 - 2g - 6g + 3$

Factor each monomial completely. (Lesson 8-1)

- $35mn$
- $27r^2$
- $20xy^3$
- $78a^2bc^3$

5. **THEATER** Drama students have 140 chairs to place in front of an outdoor stage. In what ways can they arrange the chairs so that there is the same number in each row, at least 6 rows of chairs, and at least 6 chairs in each row? (Lesson 8-1)

Find the GCF of each set of monomials.

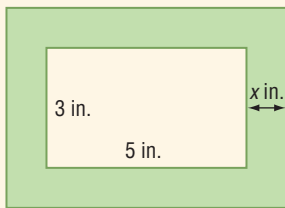
(Lesson 8-1)

- $24ab^2, 21a^3$
- $18n, 25p^2$
- $15q^2r^2, 5r^2s$
- $42x^2y, 30xy^2$

Factor each polynomial. (Lesson 8-2)

- $3m + 18n$
- $4xy^2 - xy$
- $32a^2b + 40b^3 - 8a^2b^2$
- $6pq + 16p - 15q - 40$

14. **PHOTOS** Olinda is placing matting x inches wide around a photo that is 5 inches long and 3 inches wide. Write an expression in factored form for the area of the matting. (Lesson 8-2)

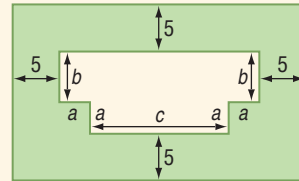


15. **FOOTBALL** In a football game, Darryl punts the ball downfield. The height h of the football above the ground after t seconds can be modeled by $h = 76.8t - 16t^2$. How long was the football in the air? (Lesson 8-2)

16. **MULTIPLE CHOICE** What are the roots of $d^2 - 12d = 0$? (Lesson 8-2)

- A 0 and -12 C -12 and 12
 B 0 and 12 D 12 and 12

17. **GEOMETRY** Write an expression in factored form for the area of the shaded region. (Lesson 8-2)



Solve each equation. Check the solutions. (Lesson 8-2)

- $(8n + 5)(n - 4) = 0$
- $9x^2 - 27x = 0$
- $10x^2 = -3x$

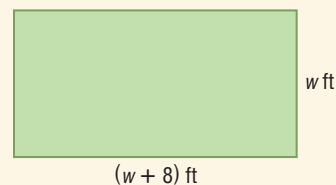
Factor each trinomial. (Lesson 8-3)

- $n^2 - 2n - 48$
- $x^2 - 4xy + 3y^2$
- $a^2 + 5ab + 4b^2$
- $s^2 - 13st + 36t^2$

Solve each equation. Check the solutions. (Lesson 8-3)

- $a^2 + 7a + 10 = 0$
- $n^2 + 4n - 21 = 0$
- $x^2 - 2x - 6 = 74$
- $x^2 - x + 56 = 17x$

29. **GEOMETRY** The rectangle has an area of 180 square feet. Find the width w of the rectangle. (Lesson 8-3)



30. **MULTIPLE CHOICE** Which represents one of the roots of $0 = x^2 + 3x - 18$? (Lesson 8-3)

- F -6 H 6
 G -3 J $\frac{1}{3}$

Factoring Trinomials: $ax^2 + bx + c$

Main Ideas

- Factor trinomials of the form $ax^2 + bx + c$.
- Solve equations of the form $ax^2 + bx + c = 0$.

New Vocabulary

prime polynomial

GET READY for the Lesson

The factors of $2x^2 + 7x + 6$ are the dimensions of the rectangle formed by the algebra tiles shown below.



The process you use to form the rectangle is the same mental process you can use to factor this trinomial algebraically.

Factor $ax^2 + bx + c$ For trinomials of the form $x^2 + bx + c$, the coefficient of x^2 is 1. To factor trinomials of this form, you find the factors of c with a sum of b . We can modify this approach to factor trinomials for which the leading coefficient is not 1.

ALGEBRA LAB

1. Complete the following table.

Product of Two Binomials	Use FOIL. $ax^2 + mx + nx + c$	$ax^2 + bx + c$	$m \cdot n$	$a \cdot c$
$(2x + 3)(x + 4)$	$2x^2 + 8x + 3x + 12$	$2x^2 + 11x + 12$	24	24
$(x + 1)(3x + 5)$				
$(2x - 1)(4x + 1)$				
$(3x + 5)(4x - 2)$				

2. How are m and n related to a and c ?

3. How are m and n related to b ?

You can use the pattern in the Algebra Lab and the method of factoring by grouping to factor trinomials. Consider $6x^2 + 17x + 5$. Find two numbers, m and n , with the product of $6 \cdot 5$ or 30 and the sum of 17. The correct factors are 2 and 15.

$$\begin{aligned}
 6x^2 + 17x + 5 &= 6x^2 + mx + nx + 5 && \text{Write the pattern.} \\
 &= 6x^2 + 2x + 15x + 5 && m = 2 \text{ and } n = 15 \\
 &= (6x^2 + 2x) + (15x + 5) && \text{Group terms with common factors.} \\
 &= 2x(3x + 1) + 5(3x + 1) && \text{Factor the GCF from each grouping.} \\
 &= (3x + 1)(2x + 5) && 3x + 1 \text{ is the common factor.}
 \end{aligned}$$

Therefore, $6x^2 + 17x + 5 = (3x + 1)(2x + 5)$.

EXAMPLE Factor $ax^2 + bx + c$ **1** Factor each trinomial.

a. $7x^2 + 29x + 4$

In this trinomial, $a = 7$, $b = 29$, and $c = 4$. You need to find two numbers with a sum of 29 and a product of $7 \cdot 4$ or 28. Make an organized list of the factors of 28 and look for the pair of factors with the sum of 29.

Factors of 28	Sum of Factors
1, 28	29

The correct factors are 1 and 28.

$$7x^2 + 29x + 4 = 7x^2 + mx + nx + 4$$

Write the pattern.

$$= 7x^2 + 1x + 28x + 4$$

 $m = 1$ and $n = 28$

$$= (7x^2 + 1x) + (28x + 4)$$

Group terms with common factors.

$$= x(7x + 1) + 4(7x + 1)$$

Factor the GCF from each grouping.

$$= (7x + 1)(x + 4)$$

Distributive Property

b. $10x^2 - 43x + 28$

In this trinomial, $a = 10$, $b = -43$ and $c = 28$. Since b is negative, $m + n$ is negative. Since c is positive, mn is positive. So, both m and n are negative.

Factors of 280	Sum of Factors
-1, -280	-281
-2, -140	-142
-4, -70	-74
-5, -56	-61
-7, -40	-47
-8, -35	-43

List the negative factors of $10 \cdot 28$ or 280.

Look for the pairs of factors with a sum of -43.

The correct factors are -8 and -35.

$$10x^2 - 43x + 28$$

$$= 10x^2 + mx + nx + 28$$

Write the pattern.

$$= 10x^2 + (-8)x + (-35)x + 28$$

 $m = -8$ and $n = -35$

$$= (10x^2 - 8x) + (-35x + 28)$$

Group terms with common factors.

$$= 2x(5x - 4) + 7(-5x + 4)$$

Factor the GCF from each grouping.

$$= 2x(5x - 4) + 7(-1)(5x - 4)$$

 $-5x + 4 = (-1)(5x - 4)$

$$= 2x(5x - 4) + (-7)(5x - 4)$$

 $7(-1) = -7$

$$= (5x - 4)(2x - 7)$$

Distributive Property

c. $3x^2 + 24x + 45$

The GCF of the terms $3x^2$, $24x$, and 45 is 3. Factor this out first.

$$3x^2 + 24x + 45 = 3(x^2 + 8x + 15)$$

Distributive Property

Now factor $x^2 + 8x + 15$. Since the leading coefficient is 1, find two factors of 15 with a sum of 8. The correct factors are 3 and 5.

So, $x^2 + 8x + 15 = (x + 3)(x + 5)$. Thus, the complete factorization of $3x^2 + 24x + 45$ is $3(x + 3)(x + 5)$.

Study Tip**Finding Factors**

Put pairs in an organized list so you do not miss any possible pairs of factors.

Study Tip**Factoring Completely**

Always check for a GCF first before trying to factor a trinomial.

Check Your Progress

1A. $5x^2 + 13x + 6$

1B. $6x^2 + 22x - 8$

1C. $10y^2 - 35y + 30$

A polynomial that cannot be written as a product of two polynomials with integral coefficients is called a **prime polynomial**.

EXAMPLE Determine Whether a Polynomial Is Prime

2 Factor $2x^2 + 5x - 2$.

In this trinomial, $a = 2$, $b = 5$, and $c = -2$. Since b is positive, $m + n$ is positive. Since c is negative, mn is negative. So either m or n is negative, but not both. Therefore, make a list of the factors of $2(-2)$ or -4 , where one factor in each pair is negative. Look for a pair of factors with a sum of 5.

Factors of -4	Sum of Factors
1, -4	-3
-1 , 4	3
-2 , 2	0

There are no factors with a sum of 5. Therefore, $2x^2 + 5x - 2$ cannot be factored using integers. Thus, $2x^2 + 5x - 2$ is a prime polynomial.

CHECK Your Progress

2A. Is $4r^2 - r + 7$ prime?

2B. Is $2x^2 + 3x - 5$ prime?

Solve Equations by Factoring Some equations of the form $ax^2 + bx + c = 0$ can be solved by factoring and then using the Zero Product Property.

EXAMPLE Solve Equations by Factoring

3 Solve $8a^2 - 9a - 5 = 4 - 3a$. Check the solutions.

$$8a^2 - 9a - 5 = 4 - 3a$$

Write the equation.

$$8a^2 - 6a - 9 = 0$$

Rewrite so that one side equals 0.

$$(4a + 3)(2a - 3) = 0$$

Factor the left side.

$$4a + 3 = 0 \quad \text{or} \quad 2a - 3 = 0$$

Zero Product Property

$$4a = -3$$

$$2a = 3$$

Solve each equation.

$$a = -\frac{3}{4}$$

$$a = \frac{3}{2}$$

The roots are $-\frac{3}{4}$ and $\frac{3}{2}$.

CHECK Check each solution in the original equation.

$$8a^2 - 9a - 5 = 4 - 3a$$

$$8a^2 - 9a - 5 = 4 - 3a$$

$$8\left(-\frac{3}{4}\right)^2 - 9\left(-\frac{3}{4}\right) - 5 \stackrel{?}{=} 4 - 3\left(-\frac{3}{4}\right) \quad 8\left(\frac{3}{2}\right)^2 - 9\left(\frac{3}{2}\right) - 5 \stackrel{?}{=} 4 - 3\left(\frac{3}{2}\right)$$

$$\frac{9}{2} + \frac{27}{4} - 5 \stackrel{?}{=} 4 + \frac{9}{4}$$

$$18 - \frac{27}{2} - 5 \stackrel{?}{=} 4 - \frac{9}{2}$$

$$\frac{25}{4} = \frac{25}{4} \quad \checkmark$$

$$-\frac{1}{2} = -\frac{1}{2} \quad \checkmark$$

CHECK Your Progress

3A. $3x^2 - 5x = 12$

3B. $2x^2 - 30x + 88 = 0$



A model for the vertical motion of a projected object is given by the equation $h = -16t^2 + vt + s$, where h is the height in feet, t is the time in seconds, v is the initial upward velocity in feet per second, and s is the initial height of the object in feet.



Real-World EXAMPLE

4

PEP RALLY At a pep rally, small foam footballs are launched by cheerleaders using a sling-shot. How long is a football in the air if a student catches it on its way down 26 feet above the gym floor?

$$h = -16t^2 + vt + s$$

Vertical motion model

$$26 = -16t^2 + 42t + 6$$

$$h = 26, v = 42, s = 6$$

$$0 = -16t^2 + 42t - 20$$

Subtract 26 from each side.

$$0 = -2(8t^2 - 21t + 10)$$

Factor out -2 .

$$0 = 8t^2 - 21t + 10$$

Divide each side by -2 .

$$0 = (8t - 5)(t - 2)$$

Factor $8t^2 - 21t + 10$.

$$8t - 5 = 0 \quad \text{or} \quad t - 2 = 0$$

Zero Product Property

$$8t = 5$$

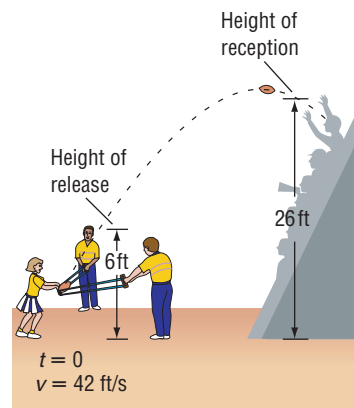
$$t = 2$$

Solve each equation.

$$t = \frac{5}{8}$$

The solutions are $\frac{5}{8}$ second and 2 seconds.

The first time represents how long it takes the football to reach a height of 26 feet on its way up. The later time represents how long it takes the ball to reach a height of 26 feet again on its way down. Thus, the football will be in the air for 2 seconds before the student catches it.



Study Tip

Factoring When a Is Negative

When factoring a trinomial of the form $ax^2 + bx + c$ where a is negative, it is helpful to factor out a negative monomial.



CHECK Your Progress

4. Six times the square of a number plus 11 times the number equals 2. What are possible values of x ?



Personal Tutor at algebra1.com



CHECK Your Understanding

Examples 1–2
(pp. 442–443)

Factor each trinomial, if possible. If the trinomial cannot be factored using integers, write *prime*.

1. $3a^2 + 8a + 4$

2. $2t^2 - 11t + 7$

3. $2p^2 + 14p + 24$

4. $2x^2 + 13x + 20$

5. $6x^2 + 15x - 9$

6. $4n^2 - 4n - 35$

Example 3
(p. 443)

Solve each equation. Check the solutions.

7. $3x^2 + 11x + 6 = 0$

8. $10p^2 - 19p + 7 = 0$

9. $6n^2 + 7n = 20$

Example 4
(p. 444)

10. **CLIFF DIVING** Suppose a diver leaps from the edge of a cliff 80 feet above the ocean with an initial upward velocity of 8 feet per second. How long will it take the diver to enter the water below?

Exercises

HOMEWORK HELP	
For Exercises	See Examples
11–22	1–2
23–30	3
31–32	4

Factor each trinomial, if possible. If the trinomial cannot be factored using integers, write *prime*.

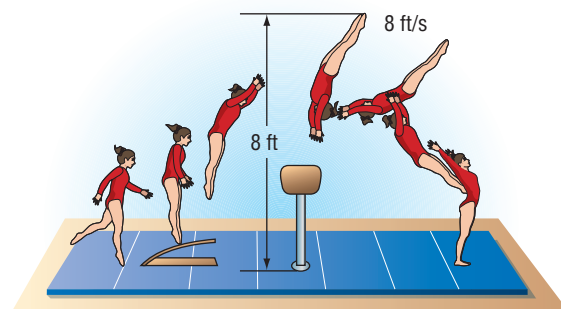
- | | | |
|-----------------------|-----------------------|------------------------|
| 11. $2x^2 + 7x + 5$ | 12. $6p^2 + 5p - 6$ | 13. $5d^2 + 6d - 8$ |
| 14. $8k^2 - 19k + 9$ | 15. $9g^2 - 12g + 4$ | 16. $2a^2 - 9a - 18$ |
| 17. $2x^2 - 3x - 20$ | 18. $5c^2 - 17c + 14$ | 19. $3p^2 - 25p + 16$ |
| 20. $10n^2 - 11n - 6$ | 21. $6r^2 - 14r - 12$ | 22. $30x^2 - 25x - 30$ |

Solve each equation. Check the solutions.

- | | |
|------------------------------|-----------------------------|
| 23. $5x^2 + 27x + 10 = 0$ | 24. $24x^2 - 14x - 3 = 0$ |
| 25. $12a^2 - 13a = 35$ | 26. $6x^2 - 14x = 12$ |
| 27. $21x^2 - 6 = 15x$ | 28. $24x^2 - 46x = 18$ |
| 29. $17x^2 - 11x + 2 = 2x^2$ | 30. $24x^2 - 30x + 8 = -2x$ |

31. ROCK CLIMBING Damaris is rock climbing at Joshua Tree National Park in the Mojave Desert. She launches a grappling hook from a height of 6 feet with an initial upward velocity of 56 feet per second. The hook just misses the stone ledge that she wants to scale. As it falls, the hook anchors on a ledge 30 feet above the ground. How long was the hook in the air?

32. GYMNASTICS The feet of a gymnast making a vault leave the horse at a height of 8 feet with an initial upward velocity of 8 feet per second. Use the model for vertical motion to find the time t in seconds it takes for the gymnast's feet to reach the mat. (*Hint: Let $h = 0$, the height of the mat.*)



33. GEOMETRY A square has an area of $9x^2 + 30xy + 25y^2$ square inches. What is the perimeter of the square? Explain.

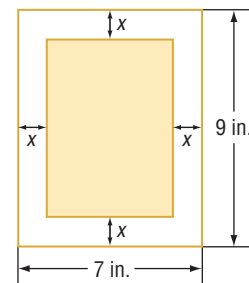
Solve each equation. Check the solutions.

- | | |
|---|--|
| 34. $\frac{x^2}{12} - \frac{2x}{3} - 4 = 0$ | 35. $t^2 - \frac{t}{6} = \frac{35}{6}$ |
| 36. $(3y + 2)(y + 3) = y + 14$ | 37. $(4a - 1)(a - 2) = 7a - 5$ |

GEOMETRY For Exercises 38 and 39, use the following information.

A rectangle 35 square inches in area is formed by cutting off strips of equal width from a rectangular piece of paper.

38. Find the width of each strip.
39. Find the dimensions of the new rectangle.



40. **OPEN ENDED** Create a trinomial that can be factored using a pair of numbers with a sum of 9 and a product of 14.
41. **CHALLENGE** Find all values of k so that $2x^2 + kx + 12$ can be factored as two binomials using integers.

EXTRA PRACTICE
See pages 734, 751.
Math online
Self-Check Quiz at algebra1.com

H.O.T. Problems

42. **FIND THE ERROR** Dasan and Luther are factoring $2x^2 + 11x + 18$. Who is correct? Explain your reasoning.

Dasan

$$2x^2 + 11x + 18 = 2(x^2 + 11x + 18)$$

$$= 2(x + 9)(x + 2)$$

Luther
 $2x^2 + 11x + 18$ is prime.

43. **Writing in Math** Explain how to determine which values should be chosen for m and n when factoring a polynomial of the form $ax^2 + bx + c$.

STANDARDIZED TEST PRACTICE

44. Which of the following shows $6x^2 + 24x + 18$ factored completely?
- A $(3x + 6)^2$
 B $(3x + 3)(2x + 6)$
 C $(3x + 2)(2x + 9)$
 D $(6x + 3)(x + 6)$
45. **REVIEW** An oak tree grew 18 inches per year from 1985 to 2006. If the tree was 25 feet tall in 1985, what was the height of the tree in 2001?
- F 31.0 ft
 G 49.0 ft
 H 56.5 ft
 J 80.5 ft

Spiral Review

Factor each trinomial, if possible. If the trinomial cannot be factored using integers, write *prime*. (Lesson 8-3)

46. $a^2 - 4a - 21$

47. $t^2 + 2t + 2$

48. $d^2 + 15d + 44$

Solve each equation. Check the solutions. (Lesson 8-2)

49. $(y - 4)(5y + 7) = 0$

50. $(2k + 9)(3k + 2) = 0$

51. $12u = u^2$

CAMERAS For Exercises 52 and 53, use the graph at the right. (Lessons 2-7 and 7-3)

52. Find the percent of increase in the number of digital cameras sold from 1999 to 2003.
53. Use the answer from Exercise 52 to verify the statement that digital camera sales increased more than 9 times from 1999 to 2003 is correct.

GET READY for the Next Lesson

PREREQUISITE SKILL Find the principal square root of each number. (Lesson 1-8)

54. 49

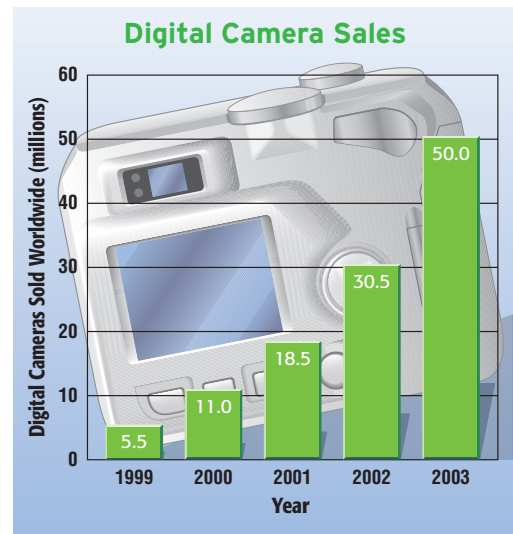
55. 36

56. 100

57. 121

58. 169

59. 225



Source: Digital Photography Review

Factoring Differences of Squares

Main Ideas

- Factor binomials that are the differences of squares.
- Solve equations involving the differences of squares.

GET READY for the Lesson

A basketball player's *hang time* is the length of time he or she is in the air after jumping. Given the maximum height h a player can jump, you can determine his or her hang time t in seconds by solving $4t^2 - h = 0$. If h is a perfect square, this equation can be solved by factoring, using the pattern for the difference of squares.

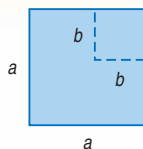


Factor $a^2 - b^2$ A geometric model can be used to factor the difference of squares.

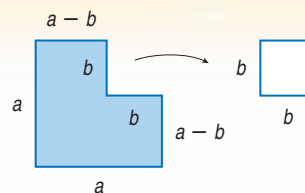
ALGEBRA LAB

Difference of Squares

Step 1 Use a straightedge to draw two squares similar to those shown below. Choose any measures for a and b .



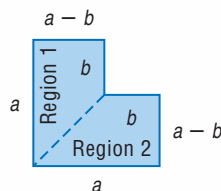
Step 2 Cut the small square from the large square.



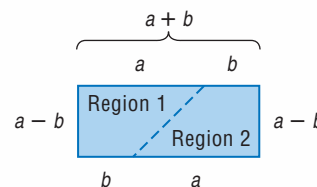
Notice that the area of the large square is a^2 , and the area of the small square is b^2 .

The area of the remaining irregular region is $a^2 - b^2$.

Step 3 Cut the irregular region into two congruent pieces as shown below.



Step 4 Rearrange the two pieces to form a rectangle with length $a + b$ and width $a - b$.



ANALYZE THE RESULTS

- Write an expression representing the area of the rectangle.
- Explain why $a^2 - b^2 = (a + b)(a - b)$.

Study Tip

Look Back

To review the product of a sum and a difference, see Lesson 7-7.

The Algebra Lab leads to the following rule for finding the difference of two squares.

KEY CONCEPT

Difference of Squares

Symbols $a^2 - b^2 = (a + b)(a - b)$ or $(a - b)(a + b)$

Examples $x^2 - 9 = (x + 3)(x - 3)$ or $(x - 3)(x + 3)$

EXAMPLE Factor the Difference of Squares

1 Factor each binomial.

a. $n^2 - 25$

$$\begin{aligned} n^2 - 25 &= n^2 - 5^2 && \text{Write in the form } a^2 - b^2. \\ &= (n + 5)(n - 5) && \text{Factor the difference of squares.} \end{aligned}$$

b. $36x^2 - 49y^2$

$$\begin{aligned} 36x^2 - 49y^2 &= (6x)^2 - (7y)^2 && 36x^2 = 6x \cdot 6x \text{ and } 49y^2 = 7y \cdot 7y \\ &= (6x + 7y)(6x - 7y) && \text{Factor the difference of squares.} \end{aligned}$$

c. $48a^3 - 12a$

If the terms of a binomial have a common factor, the GCF should be factored out first before trying to apply any other factoring technique.

$$\begin{aligned} 48a^3 - 12a &= 12a(4a^2 - 1) && \text{The GCF of } 48a^3 \text{ and } -12a \text{ is } 12a. \\ &= 12a[(2a)^2 - 1^2] && 4a^2 = 2a \cdot 2a \text{ and } 1 = 1 \cdot 1 \\ &= 12a(2a + 1)(2a - 1) && \text{Factor the difference of squares.} \end{aligned}$$

CHECK Your Progress

1A. $81 - t^2$

1B. $64g^2 - h^2$

1C. $9x^3 - 4x$

1D. $-4y^3 + 9y$

Occasionally, the difference of squares pattern needs to be applied more than once to factor a polynomial completely.

EXAMPLE Apply a Factoring Technique More Than Once

2 Factor $x^4 - 81$.

$$\begin{aligned} x^4 - 81 &= [(x^2)^2 - 9^2] && x^4 = x^2 \cdot x^2 \text{ and } 81 = 9 \cdot 9 \\ &= (x^2 + 9)(x^2 - 9) && \text{Factor the difference of squares.} \\ &= (x^2 + 9)(x^2 - 3^2) && x^2 = x \cdot x \text{ and } 9 = 3 \cdot 3 \\ &= (x^2 + 9)(x + 3)(x - 3) && \text{Factor the difference of squares.} \end{aligned}$$

CHECK Your Progress

Factor each binomial.

2A. $y^4 - 1$

2B. $4a^4 - 4b^4$

Study Tip

Common Misconception

Remember that the sum of two squares, $x^2 + a^2$, is not factorable. $x^2 + a^2$ is a prime polynomial.

EXAMPLE Apply Several Different Factoring Techniques**3** Factor $5x^3 + 15x^2 - 5x - 15$.

$$\begin{aligned}
 5x^3 + 15x^2 - 5x - 15 & \quad \text{Original polynomial} \\
 = 5(x^3 + 3x^2 - x - 3) & \quad \text{Factor out the GCF.} \\
 = 5[(x^3 - x) + (3x^2 - 3)] & \quad \text{Group terms with common factors.} \\
 = 5[x(x^2 - 1) + 3(x^2 - 1)] & \quad \text{Factor each grouping.} \\
 = 5(x^2 - 1)(x + 3) & \quad x^2 - 1 \text{ is the common factor.} \\
 = 5(x + 1)(x - 1)(x + 3) & \quad \text{Factor the difference of squares, } x^2 - 1, \text{ into } (x + 1)(x - 1).
 \end{aligned}$$

CHECK Your Progress Factor each polynomial.

3A. $2x^3 + x^2 - 50x - 25$

3B. $r^3 + 6r^2 + 11r + 6$

Solve Equations by Factoring You can apply the Zero Product Property to an equation that is written as the product of factors set equal to 0.**STANDARDIZED TEST EXAMPLE** Solve Equations by Factoring**4** In the equation $y = x^2 - \frac{9}{16}$, which is a value of x when $y = 0$?

A $-\frac{9}{4}$

B 0

C $\frac{3}{4}$

D $\frac{9}{4}$

Read the Test ItemFactor $x^2 - \frac{9}{16}$ as the difference of squares. Then find the values of x .**Solve the Test Item**

$y = x^2 - \frac{9}{16}$ Original equation

$0 = x^2 - \frac{9}{16}$ Replace y with 0.

$0 = x^2 - \left(\frac{3}{4}\right)^2$ $x^2 = x \cdot x$ and $\frac{9}{16} = \frac{3}{4} \cdot \frac{3}{4}$

$0 = \left(x + \frac{3}{4}\right)\left(x - \frac{3}{4}\right)$ Factor the difference of squares.

$0 = x + \frac{3}{4}$ or $0 = x - \frac{3}{4}$ Zero Product Property

$-\frac{3}{4} = x$ $\frac{3}{4} = x$ Solve each equation.

The solutions are $-\frac{3}{4}$ and $\frac{3}{4}$. The correct answer is C.**CHECK Your Progress****4.** Which are the solutions of $18x^3 = 50x$?

F $0, \frac{5}{3}$

G $-\frac{5}{3}, \frac{5}{3}$

H $-\frac{5}{3}, 0, \frac{5}{3}$

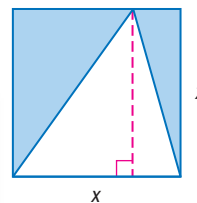
J $-\frac{5}{3}, 1, \frac{5}{3}$

Test-Taking Tip

When working with a difference of two squares, the solutions will be a number and its opposite. Therefore, choices A and D can be eliminated because if one of them is a solution then the other is also a solution.

EXAMPLE Use Differences of Two Squares

- 5 GEOMETRY** The area of the shaded part of the square is 72 square inches. Find the dimensions of the square.



Words The area of the square less the area of the triangle equals the area of the shaded region.

Variable Let x = the side length of the square.

Equation $x^2 - \frac{1}{2}x^2 = 72$

$$x^2 - \frac{1}{2}x^2 = 72 \quad \text{Original equation}$$

$$\frac{1}{2}x^2 = 72 \quad \text{Combine like terms.}$$

$$\frac{1}{2}x^2 - 72 = 0 \quad \text{Subtract 72 from each side.}$$

$$x^2 - 144 = 0 \quad \text{Multiply each side by 2 to remove the fraction.}$$

$$(x - 12)(x + 12) = 0 \quad \text{Factor the difference of squares.}$$

$$x - 12 = 0 \quad \text{or} \quad x + 12 = 0 \quad \text{Zero Product Property}$$

$$x = 12 \quad \quad \quad x = -12 \quad \text{Solve each equation.}$$

Since length cannot be negative, the only reasonable solution is 12. The dimensions of the square are 12 inches by 12 inches. *Is this solution reasonable in the context of the original problem?*

CHECK Your Progress

- 5. DRIVING** The formula $\frac{1}{24}s^2 = d$ approximates a vehicle's speed s in miles per hour given the length d in feet of skid marks on dry concrete. If skid marks on dry concrete are 54 feet long, how fast was the car traveling when the brakes were applied?

CHECK Your Understanding

Examples 1–3
(pp. 448–449)

Factor each polynomial, if possible. If the polynomial cannot be factored, write *prime*.

1. $n^2 - 81$

2. $4 - 9a^2$

3. $2x^5 - 98x^3$

4. $32x^4 - 2y^4$

5. $4t^2 - 27$

6. $x^3 - 3x^2 - 9x + 27$

Example 4
(p. 449)

Solve each equation by factoring. Check the solutions.

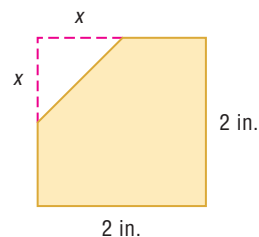
7. $4y^2 = 25$

8. $x^2 - \frac{1}{36} = 0$

9. $121a = 49a^3$

Example 5
(p. 450)

- 10. GEOMETRY** A corner is cut off a 2-inch by 2-inch square piece of paper as shown. What value of x will result in an area that is $\frac{7}{9}$ the area of the original square?



Exercises

HOMEWORK HELP	
For Exercises	See Examples
11–22	1–3
23–30	4
31, 32	5

Factor each polynomial, if possible. If the polynomial cannot be factored, write *prime*.

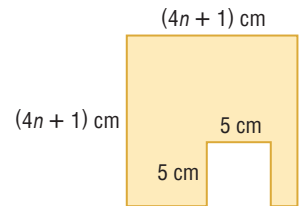
- | | | |
|-----------------------------|--------------------|----------------------------|
| 11. $x^2 - 49$ | 12. $n^2 - 36$ | 13. $81 + 16k^2$ |
| 14. $-16 + 49h^2$ | 15. $75 - 12p^2$ | 16. $-18r^3 + 242r$ |
| 17. $144a^2 - 49b^2$ | 18. $9x^2 - 10y^2$ | 19. $n^3 + 5n^2 - 4n - 20$ |
| 20. $3x^3 + x^2 - 75x - 25$ | 21. $z^4 - 16$ | 22. $256g^4 - 1$ |

Solve each equation by factoring. Check the solutions.

- | | | | |
|------------------------------|--------------------------------|-------------------------------|-------------------------------|
| 23. $25x^2 = 36$ | 24. $9y^2 = 64$ | 25. $12 - 27n^2 = 0$ | 26. $50 - 8a^2 = 0$ |
| 27. $w^2 - \frac{4}{49} = 0$ | 28. $\frac{81}{100} - p^2 = 0$ | 29. $36 - \frac{1}{9}r^2 = 0$ | 30. $\frac{1}{4}x^2 - 25 = 0$ |

31. BOATING The basic breaking strength b in pounds for a natural fiber line is determined by the formula $900c^2 = b$, where c is the circumference of the line in inches. What circumference of natural line would have 3600 pounds of breaking strength?

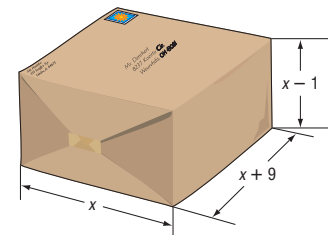
32. GEOMETRY Find the dimensions of a rectangle with the same area as the shaded region in the drawing. Assume that the dimensions of the rectangle must be represented by binomials with integral coefficients.



33. AERODYNAMICS The pressure difference P above and below a wing is described by the formula $P = \frac{1}{2}dv_1^2 - \frac{1}{2}dv_2^2$, where d is the density of the air, v_1 is the velocity of the air passing above, and v_2 is the velocity of the air passing below. Write this formula in factored form.

EXTRA PRACTICE
See pages 734, 751.
Math online
Self-Check Quiz at algebra1.com

34. PACKAGING The width of a box is 9 inches more than its length. The height of the box is 1 inch less than its length. If the box has a volume of 72 cubic inches, what are the dimensions of the box?



H.O.T. Problems

- 35. OPEN ENDED** Create a binomial that is the difference of two squares. Then factor your binomial.
- 36. CHALLENGE** Show that $a^2 - b^2 = (a + b)(a - b)$ algebraically. (*Hint:* Rewrite $a^2 - b^2$ as $a^2 - ab + ab - b^2$.)
- 37. FIND THE ERROR** Manuel and Jessica are factoring $64x^2 + 16y^2$. Who is correct? Explain your reasoning.

Manuel

$$64x^2 + 16y^2$$

$$= 16(4x^2 + y^2)$$

Jessica

$$64x^2 + 16y^2$$

$$= 16(4x^2 + y^2)$$

$$= 16(2x + y)(2x - y)$$

- 38. REASONING** The following statements appear to prove that 2 is equal to 1. Find the flaw in this “proof.”

Suppose a and b are real numbers such that $a = b$, $a \neq 0$, $b \neq 0$.

- | | | |
|-----|-----------------------------|--------------------------------|
| (1) | $a = b$ | Given. |
| (2) | $a^2 = ab$ | Multiply each side by a . |
| (3) | $a^2 - b^2 = ab - b^2$ | Subtract b^2 from each side. |
| (4) | $(a - b)(a + b) = b(a - b)$ | Factor. |
| (5) | $a + b = b$ | Divide each side by $a - b$. |
| (6) | $a + a = a$ | Substitution Property; $a = b$ |
| (7) | $2a = a$ | Combine like terms. |
| (8) | $2 = 1$ | Divide each side by a . |

- 39. Writing in Math** Use the information about basketball on page 447 to explain how to determine a basketball player’s hang time. Include a maximum height that is a perfect square and that would be considered a reasonable distance for a student athlete to jump. Describe how to find the hang time for this height.

STANDARDIZED TEST PRACTICE

- 40.** What are the solutions to the quadratic equation $25b^2 - 1 = 0$?

- A $0, \frac{1}{5}$
 B $-\frac{1}{5}, 0$
 C $\frac{1}{5}, 1$
 D $-\frac{1}{5}, \frac{1}{5}$

- 41. REVIEW** Carla’s Candle Shop sells 3 small candles for a total of \$5.94. Which expression can be used to find the total cost c of x candles?

- F $\frac{5.94}{x}$
 G $5.94x$
 H $\frac{x}{1.98}$
 J $1.98x$

Spiral Review

Factor each trinomial, if possible. If the trinomial cannot be factored using integers, write *prime*. (Lesson 8-4)

42. $2n^2 + 5n + 7$

43. $6x^2 - 11x + 4$

44. $21p^2 + 29p - 10$

Solve each equation. Check the solutions. (Lesson 8-3)

45. $y^2 + 18y + 32 = 0$

46. $k^2 - 8k = -15$

47. $b^2 - 8 = 2b$

- 48. STATISTICS** Amy’s scores on the first three of four 100-point biology tests were 88, 90, and 91. To get a B+ in the class, her average must be between 88 and 92, inclusive, on all tests. What score must she receive on the fourth test to get a B+ in biology? (Lesson 5-4)

GET READY for the Next Lesson

PREREQUISITE SKILL Find each product. (Lesson 7-7)

49. $(x + 1)(x + 1)$

50. $(x + 8)^2$

51. $(3x - 4)(3x - 4)$

52. $(5x - 2)^2$

READING MATH

Proofs

When you solve an equation by factoring, you are using a deductive argument. Each step can be justified by an algebraic property.

Solve $4x^2 - 324 = 0$.

$4x^2 - 324 = 0$	Original equation
$(2x)^2 - 18^2 = 0$	$4x^2 = (2x)^2$ and $324 = 18^2$
$(2x + 18)(2x - 18) = 0$	Factor the difference of squares.
$2x + 18 = 0$ or $2x - 18 = 0$	Zero Product Property
$x = -9$ $x = 9$	Solve each equation.

Notice that the column on the left is a step-by-step process that leads to a solution. The column on the right contains the reasons for each statement. A *two-column proof* is a deductive argument that contains statements and reasons.

Two-Column Proof

Given: a , x , and y are real numbers such that $a \neq 0$, $x \neq 0$, and $y \neq 0$.

Prove: $ax^4 - ay^4 = a(x^2 + y^2)(x + y)(x - y)$

There is a reason for each statement.

	Statements	Reasons
The first statement contains the given information.	1. a , x , and y are real numbers such that $a \neq 0$, $x \neq 0$, and $y \neq 0$.	1. Given
	2. $ax^4 - ay^4 = a(x^4 - y^4)$	2. The GCF of ax^4 and ay^4 is a .
	3. $ax^4 - ay^4 = a[(x^2)^2 - (y^2)^2]$	3. $x^4 = (x^2)^2$ and $y^4 = (y^2)^2$
	4. $ax^4 - ay^4 = a(x^2 + y^2)(x^2 - y^2)$	4. Factor the difference of squares.
The last statement is what you want to prove.	5. $ax^4 - ay^4 = a(x^2 + y^2)(x + y)(x - y)$	5. Factor the difference of squares.

Reading to Learn

- Solve $\frac{1}{16}t^2 - 100 = 0$ by using a two-column proof.
- Write a two-column proof using the following information. (*Hint:* Group terms with common factors.)

Given: c and d are real numbers such that $c \neq 0$ and $d \neq 0$.

Prove: $c^3 - cd^2 - c^2d + d^3 = (c + d)(c - d)(c - d)$

- Explain how the process used to write two-column proofs can be useful in solving Find the Error exercises, such as Exercise 37 on page 451.

Perfect Squares and Factoring

Main Ideas

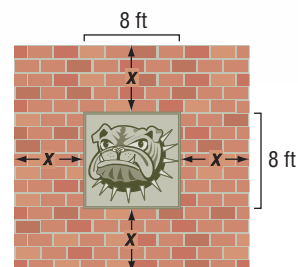
- Factor perfect square trinomials.
- Solve equations involving perfect squares.

New Vocabulary

perfect square trinomials

GET READY for the Lesson

The senior class has decided to build an outdoor pavilion. It will have an 8-foot by 8-foot portrayal of the school's mascot in the center. The class is selling bricks with students' names on them to finance the project. If they sell enough bricks to cover 80 square feet and want to arrange the bricks around the art, how wide should the border of bricks be?



To solve this problem, you need to solve the equation $(8 + 2x)^2 = 144$.

Factor Perfect Square Trinomials Numbers like 16, 49, and 144 are perfect squares, since each can be expressed as the square of an integer.

$$16 = 4 \cdot 4 \text{ or } 4^2 \quad 49 = 7 \cdot 7 \text{ or } 7^2 \quad 144 = 12 \cdot 12 \text{ or } 12^2$$

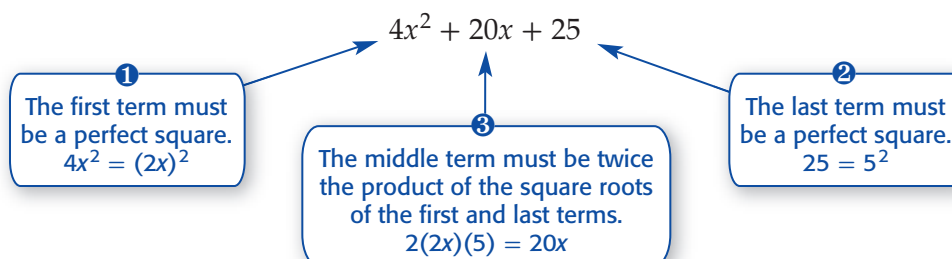
Products of the form $(a + b)^2$ and $(a - b)^2$, such as $(8 + 2x)^2$, are also perfect squares. Recall that these are special products that follow specific patterns.

$$\begin{aligned} (a + b)^2 &= (a + b)(a + b) & (a - b)^2 &= (a - b)(a - b) \\ &= a^2 + ab + ab + b^2 & &= a^2 - ab - ab + b^2 \\ &= a^2 + 2ab + b^2 & &= a^2 - 2ab + b^2 \end{aligned}$$

These patterns can help you factor **perfect square trinomials**, which are trinomials that are the squares of binomials.

Squaring a Binomial	Factoring a Perfect Square
$(x + 7)^2 = x^2 + 2(x)(7) + 7^2$ $= x^2 + 14x + 49$	$x^2 + 14x + 49 = x^2 + 2(x)(7) + 7^2$ $= (x + 7)^2$
$(3x - 4)^2 = (3x)^2 - 2(3x)(4) + 4^2$ $= 9x^2 - 24x + 16$	$9x^2 - 24x + 16 = (3x)^2 - 2(3x)(4) + 4^2$ $= (3x - 4)^2$

For a trinomial to be factorable as a perfect square, three conditions must be satisfied as illustrated in the example below.



KEY CONCEPT

Factoring Perfect Square Trinomials

Words If a trinomial can be written in the form $a^2 + 2ab + b^2$ or $a^2 - 2ab + b^2$, then it can be factored as $(a + b)^2$ or as $(a - b)^2$, respectively.

Symbols $a^2 + 2ab + b^2 = (a + b)^2$ and $a^2 - 2ab + b^2 = (a - b)^2$

EXAMPLE Factor Perfect Square Trinomials

1 Determine whether each trinomial is a perfect square trinomial. If so, factor it.

a. $16x^2 + 32x + 64$

- 1 Is the first term a perfect square? Yes, $16x^2 = (4x)^2$.
 - 2 Is the last term a perfect square? Yes, $64 = 8^2$.
 - 3 Is the middle term equal to $2(4x)(8)$? No, $32x \neq 2(4x)(8)$.
- $16x^2 + 32x + 64$ is not a perfect square trinomial.

b. $9y^2 - 12y + 4$

- 1 Is the first term a perfect square? Yes, $9y^2 = (3y)^2$.
- 2 Is the last term a perfect square? Yes, $4 = 2^2$.
- 3 Is the middle term equal to $2(3y)(2)$? Yes, $12y = 2(3y)(2)$.

$9y^2 - 12y + 4$ is a perfect square trinomial.

$$\begin{aligned} 9y^2 - 12y + 4 &= (3y)^2 - 2(3y)(2) + 2^2 && \text{Write as } a^2 - 2ab + b^2. \\ &= (3y - 2)^2 && \text{Factor using the pattern.} \end{aligned}$$

CHECK Your Progress

1A. $n^2 - 24n + 144$

1B. $x^2 + 9x + 81$

You have learned various techniques for factoring polynomials. The Concept Summary can help you decide when to use a specific technique.

CONCEPT SUMMARY		Factoring Polynomials	
Number of Terms	Factoring Technique	Example	
2 or more	greatest common factor	$3x^2 + 6x^2 - 15x = 3x(x^2 + 2x - 5)$	
2	difference of squares $a^2 - b^2 = (a + b)(a - b)$	$4x^2 - 25 = (2x + 5)(2x - 5)$	
3	perfect square trinomial $a^2 + 2ab + b^2 = (a + b)^2$ $a^2 - 2ab + b^2 = (a - b)^2$	$x^2 + 6x + 9 = (x + 3)^2$ $4x^2 - 4x + 1 = (2x - 1)^2$	
	$x^2 + bx + c$ $x^2 + bx + c = (x + m)(x + n)$ when $m + n = b$ and $mn = c$.	$x^2 - 9x + 20 = (x - 5)(x - 4)$	
	$ax^2 + bx + c$ $ax^2 + bx + c = ax^2 + mx + nx + c$ when $m + n = b$ and $mn = ac$. Then use factoring by grouping.	$6x^2 - x - 2 = 6x^2 + 3x - 4x - 2$ $= 3x(2x + 1) - 2(2x + 1)$ $= (2x + 1)(3x - 2)$	
4 or more	factoring by grouping $ax + bx + ay + by$ $= x(a + b) + y(a + b)$ $= (a + b)(x + y)$	$3xy - 6y + 5x - 10$ $= (3xy - 6y) + (5x - 10)$ $= 3y(x - 2) + 5(x - 2)$ $= (x - 2)(3y + 5)$	

Study Tip

Factoring Methods

When there is a GCF other than 1, it is usually easier to factor it out first. Then, check the appropriate factoring methods in the order shown in the table.

EXAMPLE Factor Completely

2 Factor each polynomial.

a. $4x^2 - 36$

First check for a GCF. Then, since the polynomial has two terms, check for the difference of squares.

$$\begin{aligned}4x^2 - 36 &= 4(x^2 - 9) && \text{4 is the GCF.} \\ &= 4(x^2 - 3^2) && x^2 = x \cdot x \text{ and } 9 = 3 \cdot 3 \\ &= 4(x + 3)(x - 3) && \text{Factor the difference of squares.}\end{aligned}$$

b. $25x^2 + 5x - 6$

This is not a perfect square trinomial. It is of the form $ax^2 + bx + c$. Are there two numbers m and n with a product of $25(-6)$ or -150 and a sum of 5? Yes, the product of 15 and -10 is -150 and the sum is 5.

$$\begin{aligned}25x^2 + 5x - 6 &= 25x^2 + mx + nx - 6 && \text{Write the pattern.} \\ &= 25x^2 + 15x - 10x - 6 && m = 15 \text{ and } n = -10 \\ &= (25x^2 + 15x) + (-10x - 6) && \text{Group terms with common factors.} \\ &= 5x(5x + 3) - 2(5x + 3) && \text{Factor out the GCF from each grouping.} \\ &= (5x + 3)(5x - 2) && 5x + 3 \text{ is the common factor.}\end{aligned}$$

CHECK Your Progress

2A. $2x^2 - 32$

2B. $9t^2 - 3t - 20$

Online Personal Tutor at algebra1.com

Solve Equations with Perfect Squares When solving equations involving repeated factors, it is only necessary to set one of the repeated factors equal to zero.

EXAMPLE Solve Equations with Repeated Factors

3 Solve $x^2 - x + \frac{1}{4} = 0$.

$$x^2 - x + \frac{1}{4} = 0 \quad \text{Original equation}$$

$$x^2 - 2(x)\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 = 0 \quad \text{Recognize } x^2 - x + \frac{1}{4} \text{ as a perfect square trinomial.}$$

$$\left(x - \frac{1}{2}\right)^2 = 0 \quad \text{Factor the perfect square trinomial.}$$

$$x - \frac{1}{2} = 0 \quad \text{Set repeated factor equal to zero.}$$

$$x = \frac{1}{2} \quad \text{Solve for } x.$$

CHECK Your Progress

Solve each equation. Check the solutions.

3A. $a^2 + 12a + 36 = 0$

3B. $y^2 - \frac{4}{3}y + \frac{4}{9} = 0$

Reading Math

Square Root

Solutions $\pm\sqrt{36}$ is read as *plus or minus the square root of 36*.

You have solved equations like $x^2 - 36 = 0$ by factoring. You can also use the definition of a square root to solve this equation.

$$\begin{aligned}x^2 - 36 &= 0 && \text{Original equation} \\x^2 &= 36 && \text{Add 36 to each side.} \\x &= \pm\sqrt{36} && \text{Take the square root of each side.}\end{aligned}$$

Remember that there are two square roots of 36, namely 6 and -6 . Therefore, the solution set is $\{-6, 6\}$. You can express this as $\{\pm 6\}$.

KEY CONCEPT

Square Root Property

Symbols For any number $n > 0$, if $x^2 = n$, then $x = \pm\sqrt{n}$.

Example $x^2 = 9$

$$x = \pm\sqrt{9} \text{ or } \pm 3$$

EXAMPLE

- 4 PHYSICAL SCIENCE** During an experiment, a ball is dropped from a height of 205 feet. The formula $h = -16t^2 + h_0$ can be used to approximate the number of seconds t it takes for the ball to reach height h from an initial height h_0 in feet. Find the time it takes the ball to reach the ground.

$$\begin{aligned}h &= -16t^2 + h_0 && \text{Original formula} \\0 &= -16t^2 + 205 && \text{Replace } h \text{ with 0 and } h_0 \text{ with 205.} \\-205 &= -16t^2 && \text{Subtract 205 from each side.} \\12.8125 &= t^2 && \text{Divide each side by } -16. \\ \pm 3.6 &\approx t && \text{Take the square root of each side.}\end{aligned}$$

Since a negative number does not make sense in this situation, the solution is 3.6. This means that it takes about 3.6 seconds for the ball to reach the ground.

CHECK Your Progress

4. Find the time it takes a ball to reach the ground if it is dropped from a bridge that is half as high as the one described above.

EXAMPLE

Use the Square Root Property to Solve Equations

- 5** Solve each equation. Check the solutions.

a. $(a + 4)^2 = 49$

$$\begin{aligned}(a + 4)^2 &= 49 && \text{Original equation} \\a + 4 &= \pm\sqrt{49} && \text{Square Root Property} \\a + 4 &= \pm 7 && 49 = 7 \cdot 7 \\a &= -4 \pm 7 && \text{Subtract 4 from each side.} \\a &= -4 + 7 \quad \text{or} \quad a = -4 - 7 && \text{Separate into two equations.} \\&= 3 && = -11 && \text{Simplify.}\end{aligned}$$

The roots are -11 and 3 . Check in the original equation.

(continued on the next page)



b. $(x - 3)^2 = 5$

$(x - 3)^2 = 5$ Original equation

$x - 3 = \pm\sqrt{5}$ Square Root Property


$x = 3 \pm \sqrt{5}$ Add 3 to each side.

Since 5 is not a perfect square, the roots are $3 \pm \sqrt{5}$. Using a calculator, the roots are $3 + \sqrt{5}$ or about 5.24 and $3 - \sqrt{5}$ or about 0.76.

 CHECK Your Progress

5A. $z^2 + 2z + 1 = 16$

5B. $(y - 8)^2 = 7$

 CHECK Your Understanding

Example 1
(p. 455)

Determine whether each trinomial is a perfect square trinomial. If so, factor it.

1. $y^2 + 8y + 16$

2. $9x^2 - 30x + 10$

Example 2
(p. 456)

Factor each polynomial, if possible. If the polynomial cannot be factored, write *prime*.

3. $2x^2 + 18$

4. $c^2 - 5c + 6$

5. $8x^2 - 18x - 35$

6. $9g^2 + 12g - 4$

Examples 3, 5
(pp. 456–458)

Solve each equation. Check the solutions.

7. $4y^2 + 24y + 36 = 0$

8. $3n^2 = 48$

9. $a^2 - 6a + 9 = 16$

10. $(m - 5)^2 = 13$

Example 4
(p. 457)

11. **HISTORY** Galileo showed that objects of different weights fall at the same velocity by dropping two objects of different weights from the top of the Leaning Tower of Pisa. A model for the height h in feet of an object dropped from an initial height h_0 feet is $h = -16t^2 + h_0$, where t is the time in seconds after the object is dropped. Use this model to determine approximately how long it took for objects to hit the ground if Galileo dropped them from a height of 180 feet.

Exercises

HOMEWORK HELP	
For Exercises	See Examples
12–15	1
16–23	2
24–33	3, 5
34–37	4

Determine whether each trinomial is a perfect square trinomial. If so, factor it.

12. $4y^2 - 44y + 121$

13. $2c^2 + 10c + 25$

14. $9n^2 + 49 + 42n$

15. $25a^2 - 120ab + 144b^2$

Factor each polynomial, if possible. If the polynomial cannot be factored, write *prime*.

16. $4k^2 - 100$

17. $4a^2 - 36b^2$

18. $x^2 + 6x - 9$

19. $50g^2 + 40g + 8$

20. $9t^3 + 66t^2 - 48t$

21. $20n^2 + 34n + 6$

22. $5y^2 - 90$

23. $18y^2 - 48y + 32$



Real-World Link

Some amusement park free-fall rides can seat 4 passengers across per coach and reach speeds of up to 62 miles per hour.

Source: pgathrills.com

Solve each equation. Check the solutions.

24. $3x^2 + 24x + 48 = 0$

25. $7r^2 = 70r - 175$

26. $49a^2 + 16 = 56a$

27. $18y^2 + 24y + 8 = 0$

28. $y^2 - \frac{2}{3}y + \frac{1}{9} = 0$

29. $a^2 + \frac{4}{5}a + \frac{4}{25} = 0$

30. $x^2 + 10x + 25 = 81$

31. $(w + 3)^2 = 2$

32. $p^2 + 2p + 1 = 6$

33. $x^2 - 12x + 36 = 11$

34. **FORESTRY** The number of board feet B that a log will yield can be estimated by using the formula $B = \frac{L}{16}(D^2 - 8D + 16)$, where D is the diameter in inches and L is the log length in feet. For logs that are 16 feet long, what diameter will yield approximately 256 board feet?

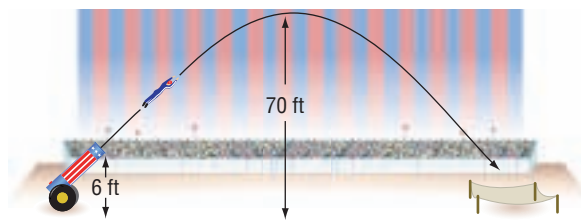
FREE-FALL RIDE For Exercises 35 and 36, use the following information.

The height h in feet of a car above the exit ramp of an amusement park's free-fall ride can be modeled by $h = -16t^2 + s$, where t is the time in seconds after the car drops and s is the starting height of the car in feet.

35. How high above the car's exit ramp should the ride's designer start the drop in order for riders to experience free fall for at least 3 seconds?

36. Approximately how long will riders be in free fall if their starting height is 160 feet above the exit ramp?

37. **HUMAN CANNONBALL** A circus acrobat is shot out of a cannon with an initial upward velocity of 64 feet per second. If the acrobat leaves the cannon 6 feet above the ground, will he reach a height of 70 feet? If so, how long will it take him to reach that height? Use the model for vertical motion.



Factor each polynomial, if possible. If the polynomial cannot be factored, write *prime*.

38. $4a^3 + 3a^2b^2 + 8a + 6b^2$

39. $5a^2 + 7a + 6b^2 - 4b$

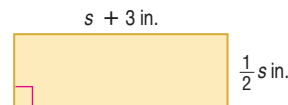
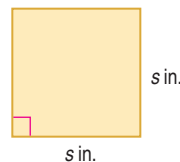
40. $x^2y^2 - y^2 - z^2 + x^2z^2$

41. $4m^4n + 6m^3n - 16m^2n^2 - 24mn^2$

42. **GEOMETRY** The volume of a rectangular prism is $x^3y - 63y^2 + 7x^2 - 9xy^3$ cubic meters. Find the dimensions of the prism if they can be represented by binomials with integral coefficients.

EXTRA PRACTICE
See pages 734, 751.
Math online
Self-Check Quiz at algebra1.com

43. **GEOMETRY** If the area of the square shown is $16x^2 - 56x + 49$ square inches, what is the area of the rectangle in terms of x ?



H.O.T. Problems

44. **REASONING** Determine whether the following statement is *sometimes*, *always*, or *never* true. Explain your reasoning.

$$a^2 - 2ab - b^2 = (a - b)^2, b \neq 0$$

45. **OPEN ENDED** Create a polynomial that requires at least two different factoring techniques to factor it completely. Then factor the polynomial completely, describing the techniques that were used.
46. **Which One Doesn't Belong?** Identify the trinomial that does not belong with the other three. Explain your reasoning.

$$4x^2 - 36x + 81$$

$$25x^2 + 10x + 1$$

$$4x^2 + 10x + 4$$

$$9x^2 - 24x + 16$$

CHALLENGE Determine all values of k that make each of the following a perfect square trinomial.

47. $4x^2 + kx + 1$

48. $x^2 - 18x + k$

49. $x^2 + 20x + k$

50. *Writing in Math* Use the information about the project on page 454 to explain how factoring can be used to design a pavilion. Explain how the equation $(8 + 2x)^2 = 144$ models the given situation, solve this equation, and interpret its solutions.

STANDARDIZED TEST PRACTICE

51. What are the solutions for the equation $3(5x - 1)^2 = 27$?

A $-\frac{9}{5}$ and 2

B -2 and $\frac{9}{5}$

C $-\frac{2}{5}$ and $\frac{4}{5}$

D $-\frac{1}{5}$ and $\frac{3}{5}$

52. **REVIEW** Marta has a bag of 8 marbles. There are 3 red marbles, 2 blue marbles, 2 white marbles, and 1 black marble. If she picks one marble without looking, what is the probability that it is either black or white?

F $\frac{1}{8}$

H $\frac{3}{8}$

G $\frac{1}{4}$

J $\frac{5}{8}$

Spiral Review

Solve each equation. Check the solutions. (Lessons 8-4 and 8-5)

53. $9x^2 - 16 = 0$

54. $49m^2 = 81$

55. $8k^2 + 22k - 6 = 0$

56. $12w^2 + 23x = -5$

Solve each inequality. Check the solution. (Lesson 6-2)

57. $\frac{r}{5} > -11$

58. $8 > \frac{2}{3}n$

59. $76 < 4t$

60. $-14c \leq 84$

61. **BUSINESS** Jake's Garage charges \$180 for a two-hour repair job and \$375 for a five-hour repair job. Write a linear equation that Jake can use to bill customers for repair jobs of any length of time. (Lesson 4-3)

62. **MODEL TRAINS** One of the most popular sizes of model trains is called the HO. Every dimension of the HO model measures $\frac{1}{87}$ times that of a real engine. The HO model of a modern diesel locomotive is about 8 inches long. About how many feet long is the real locomotive? (Lesson 3-6)



FOLDABLES™

Study Organizer

GET READY to Study

Be sure the following
Key Concepts are noted
in your Foldable.

8-1	F a c t o r i n g
8-2	
8-3	
8-4	
8-5	
8-6	

Key Concepts

Monomials and Factoring (Lesson 8-1)

- The greatest common factor (GCF) of two or more monomials is the product of their common prime factors.

Factoring Using the Distributive Property

(Lesson 8-2)

- Using the Distributive Property to factor polynomials with four or more terms is called factoring by grouping.

$$ax + bx + ay + by = x(a + b) + y(a + b) \\ = (a + b)(x + y)$$

- Factoring can be used to solve some equations. According to the Zero Product Property, for any real numbers a and b , if $ab = 0$, then either $a = 0$, $b = 0$, or both a and b equal zero.

Factoring Trinomials and Differences of Squares (Lessons 8-3, 8-4, and 8-5)

- To factor $x^2 + bx + c$, find m and n with a sum of b and a product of c . Then write $x^2 + bx + c$ as $(x + m)(x + n)$.
- To factor $ax^2 + bx + c$, find m and n with a product of ac and a sum of b . Then write as $ax^2 + mx + nx + c$ and factor by grouping.
 $a^2 - b^2 = (a + b)(a - b)$ or $(a - b)(a + b)$

Perfect Squares and Factoring (Lesson 8-6)

- $a^2 + 2ab + b^2 = (a + b)^2$ and $a^2 - 2ab + b^2 = (a - b)^2$
- For a trinomial to be a perfect square, the first and last terms must be perfect squares, and the middle term must be twice the product of the square roots of the first and last terms.
- For any number $n > 0$, if $x^2 = n$, then $x = \pm\sqrt{n}$.

Key Vocabulary

composite number (p. 420)	perfect square trinomials (p. 454)
factored form (p. 421)	prime factorization (p. 421)
factoring (p. 426)	prime number (p. 420)
factoring by grouping (p. 427)	prime polynomial (p. 443)
greatest common factor (p. 422)	roots (p. 428)

Vocabulary Check

State whether each sentence is *true* or *false*. If *false*, replace the underlined word, phrase, expression, or number to make a true sentence.

- The number 27 is an example of a prime number.
- $2x$ is the greatest common factor of $12x^2$ and $14xy$.
- 66 is an example of a perfect square.
- 61 is a factor of 183.
- The prime factorization of 48 is $3 \cdot 4^2$.
- $x^2 - 25$ is an example of a perfect square trinomial.
- The number 35 is an example of a composite number.
- $x^2 - 3x - 70$ is an example of a prime polynomial.
- Expressions with four or more unlike terms can sometimes be factored by grouping.
- $(b - 7)(b + 7)$ is the factorization of a difference of squares.



Lesson-by-Lesson Review

8-1

Monomials and Factoring (pp. 420–424)

Factor each monomial completely.

11. $28n^3$

12. $-33a^2b$

13. $150st$

14. $-378pq^2r^2$

Find the GCF of each set of monomials.

15. 35, 30

16. 12, 18, 40

17. $12ab$, $4a^2b^2$

18. $16mrt$, $30m^2r$

19. $20n^2$, $25np^5$

20. $60x^2y^2$, $15xyz$, $35xz^3$

21. **HOME IMPROVEMENT** A landscape architect is designing a stone path to cover an area 36 inches by 120 inches. What is the maximum size square stone that can be used so that none of the stones have to be cut?

Example 1 Factor $68cd^2$ completely.

$$\begin{aligned} 68cd^2 &= 4 \cdot 17 \cdot c \cdot d \cdot d & 68 &= 4 \cdot 17, d^2 = d \cdot d \\ &= 2 \cdot 2 \cdot 17 \cdot c \cdot d \cdot d & 4 &= 2 \cdot 2 \end{aligned}$$

Thus, $68cd^2$ in factored form is $2 \cdot 2 \cdot 17 \cdot c \cdot d \cdot d$.**Example 2** Find the GCF of $15x^2y$ and $45xy^2$.

$$\begin{array}{l} 15x^2y = \textcircled{3} \cdot \textcircled{5} \cdot x \cdot x \cdot y \\ 45xy^2 = \textcircled{3} \cdot 3 \cdot \textcircled{5} \cdot x \cdot y \cdot y \end{array} \quad \begin{array}{l} \text{Factor each number.} \\ \text{Circle the common} \\ \text{prime factors.} \end{array}$$

The GCF is $3 \cdot 5 \cdot x \cdot y$ or $15xy$.

8-2

Factoring Using the Distributive Property (pp. 426–431)

Factor each polynomial.

22. $13x + 26y$

23. $a^2 - 4ac + ab - 4bc$

24. $24a^2b^2 - 18ab$

25. $26ab + 18ac + 32a^2$

26. $4rs + 12ps + 2mr + 6mp$

27. $24am - 9an + 40bm - 15bn$

Solve each equation. Check the solutions.

28. $x(2x - 5) = 0$

29. $4x^2 = -7x$

30. $(3n + 8)(2n - 6) = 0$

31. **EXERCISE** A gymnast jumps on a trampoline traveling at 12 feet per second. Her height h in feet above the trampoline after t seconds is given by the formula $h = 12t - 16t^2$. How long is the gymnast in the air before returning to the trampoline?

Example 3 Factor $2x^2 - 3xz - 2xy + 3yz$.

$$\begin{aligned} 2x^2 - 3xz - 2xy + 3yz & \\ &= (2x^2 - 3xz) + (-2xy + 3yz) \\ &= x(2x - 3z) - y(2x - 3z) \\ &= (x - y)(2x - 3z) \end{aligned}$$

Example 4 Solve $x^2 = 5x$. Check the solutions.Write the equation so that it is of the form $ab = 0$.

$$\begin{array}{ll} x^2 = 5x & \text{Original equation} \\ x^2 - 5x = 0 & \text{Subtract } 5x \text{ from each side.} \\ x(x - 5) = 0 & \text{Factor using the GCF, } x. \\ x = 0 \text{ or } x - 5 = 0 & \text{Zero Product Property} \\ x = 5 & \text{Solve the equation.} \end{array}$$

The roots are 0 and 5. Check by substituting 0 and 5 for x in the original equation.

8-3 Factoring Trinomials: $x^2 + bx + c$ (pp. 434-439)

Factor each trinomial.

32. $y^2 + 7y + 12$ 33. $x^2 - 9x - 36$
34. $b^2 + 5b - 6$ 35. $18 - 9r + r^2$

Solve each equation. Check the solutions.

36. $y^2 + 13y + 40 = 0$
37. $x^2 - 5x - 66 = 0$

38. **SOCCER** In order for a town to host an international soccer game, its field's length must be 110-120 yards, and its width must be 70-80 yards. Green Meadows soccer field is 30 yards longer than it is wide. Write an expression for the area of the rectangular field. If the area of the field is 8800 square yards, will Green Meadows be able to host an international game? Explain.

Example 5 Factor $x^2 - 9x + 20$.

$b = -9$ and $c = 20$, so $m + n$ is negative and mn is positive. Therefore, m and n must both be negative. List the negative factors of 20, and look for the pair of factors with a sum of -9 .

Factors of 20	Sum of Factors
-1, -20	-21
-2, -10	-12
-4, -5	-9

The correct factors are -4 and -5 .

$$\begin{aligned} x^2 - 9x + 20 &= (x + m)(x + n) \quad \text{Write the pattern.} \\ &= (x - 4)(x - 5) \quad m = -4 \text{ and } n = -5 \end{aligned}$$

8-4 Factoring Trinomials: $ax^2 + bx + c$ (pp. 441-446)

Factor each trinomial, if possible. If the trinomial cannot be factored using integers, write *prime*.

39. $2a^2 - 9a + 3$ 40. $2m^2 + 13m - 24$
41. $12b^2 + 17b + 6$ 42. $3n^2 - 6n - 45$

Solve each equation. Check the solutions.

43. $2r^2 - 3r - 20 = 0$ 44. $40x^2 + 2x = 24$

45. **BASEBALL** Victor hit a baseball into the air that modeled the equation $h = -16t^2 + 36t + 1$, where h is the height in feet and t is the time in seconds. How long was the ball in the air if Casey caught the ball 9 feet above the ground on its way down?

Example 6 Factor $12x^2 + 22x - 14$.

$$12x^2 + 22x - 14 = 2(6x^2 + 11x - 7) \quad \text{Factor.}$$

So, $a = 6$, $b = 11$, and $c = -7$. Since b is positive, $m + n$ is positive. Since c is negative, mn is negative. So either m or n is negative. List the factors of $6(-7)$ or -42 , where one factor in each pair is negative. The correct factors are -3 and 14 .

$$\begin{aligned} 6x^2 + 11x - 7 &= 6x^2 + mx + nx - 7 \\ &= 6x^2 - 3x + 14x - 7 \\ &= (6x^2 - 3x) + (14x - 7) \\ &= 3x(2x - 1) + 7(2x - 1) \\ &= (2x - 1)(3x + 7) \end{aligned}$$

Thus, the complete factorization of $12x^2 + 22x - 14$ is $2(2x - 1)(3x + 7)$.

8-5

Factoring Differences of Squares (pp. 447-452)

Factor each polynomial, if possible. If the polynomial cannot be factored, write *prime*.

46. $64 - 4s^2$

47. $2y^3 - 128y$

48. $9b^2 - 20$

49. $\frac{1}{4}n^2 - \frac{9}{16}r^2$

Solve each equation by factoring. Check the solutions.

50. $b^2 - 16 = 0$

51. $25 - 9y^2 = 0$

52. $16a^2 = 81$

53. $\frac{25}{49} - r^2 = 0$

54. **EROSION** A boulder breaks loose from the face of a mountain and falls toward the water 576 feet below. The distance d that the boulder falls in t seconds is given by the equation $d = 16t^2$. How long does it take the boulder to hit the water?

Example 7 Solve $y^2 + 9 = 90$ by factoring.

$$y^2 + 9 = 90 \quad \text{Original equation}$$

$$y^2 - 81 = 0 \quad \text{Subtract 90 from each side.}$$

$$y^2 - (9)^2 = 0 \quad y^2 = y \cdot y \text{ and } 81 = 9 \cdot 9$$

$$(y + 9)(y - 9) = 0 \quad \text{Factor the difference of squares.}$$

$$y + 9 = 0 \text{ or } y - 9 = 0 \quad \text{Zero Product Property}$$

$$y = 9 \quad y = -9 \quad \text{Solve each equation.}$$

The roots are -9 and 9 .

8-6

Perfect Squares and Factoring (pp. 454-460)

Factor each polynomial, if possible. If the polynomial cannot be factored, write *prime*.

55. $a^2 + 18a + 81$

56. $9k^2 - 12k + 4$

57. $4 - 28r + 49r^2$

58. $32n^2 - 80n + 50$

Solve each equation. Check the solutions.

59. $6b^3 - 24b^2 + 24b = 0$

60. $144b^2 = 36$

61. $49m^2 - 126m + 81 = 0$

62. $(c - 9)^2 = 144$

63. **PICTURE FRAMING** A picture that measures 7 inches by 7 inches is being framed. The area of the frame is 32 square inches. What is the width of the frame?

Example 8 Solve $(x - 4)^2 = 121$.

$$(x - 4)^2 = 121 \quad \text{Original equation}$$

$$x - 4 = \pm\sqrt{121} \quad \text{Square Root Property}$$

$$x - 4 = \pm 11 \quad 121 = 11 \cdot 11$$

$$x = 4 \pm 11 \quad \text{Add 4 to each side.}$$

$$x = 4 + 11 \text{ or } x = 4 - 11 \quad \text{Separate into two equations.}$$

$$= 15 \quad = -7$$

The roots are -7 and 15 .

Factor each monomial completely.

1. $9g^2h$ 2. $-40ab^3c$

Find the GCF of each set of monomials.

3. $16c^2, 4cd^2$ 4. $12r, 35st$
 5. $10xyz, 15x^2y$ 6. $18a^2b^2, 28a^3b^2$

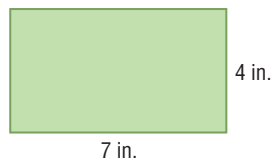
Factor each polynomial, if possible. If the polynomial cannot be factored using integers, write *prime*.

7. $x^2 + 14x + 24$ 8. $28m^2 + 18m$
 9. $a^2 - 11ab + 18b^2$ 10. $2h^2 - 3h - 18$
 11. $6x^3 + 15x^2 - 9x$ 12. $15a^2b + 5a^2 - 10a$

13. **MULTIPLE CHOICE** What are the roots of $x^2 - 3x - 4 = 0$?

- A -4 and -1
 B -4 and 1
 C 4 and -1
 D 4 and 1

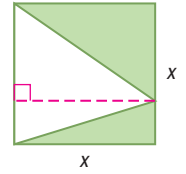
14. **GEOMETRY** When the length and width of the rectangle are increased by the same amount, the area is increased by 26 square inches. What are the dimensions of the new rectangle?



Factor each polynomial, if possible. If the polynomial cannot be factored using integers, write *prime*.

15. $a^2 - 4$
 16. $t^2 - 16t + 64$
 17. $64p^2 - 63p + 16$
 18. $36m^2 + 60mn + 25n^2$
 19. $x^3 - 4x^2 - 9x + 36$
 20. $4my - 20m + 3py - 15p$

21. **ART** An artist is designing square tiles like the one shown at the right. The area of the shaded part of each tile is 98 square centimeters. Find the dimensions of the tile.



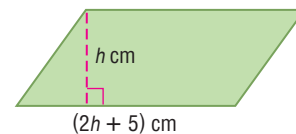
22. **CONSTRUCTION** A sidewalk will be built along the inside edges of all four sides of the rectangular lawn described in the table. The remaining lawn will have an area of 425 square feet. How wide will the walk be?

Dimensions of Lawn	
length	32 ft
width	24 ft

Solve each equation. Check the solutions.

23. $(4x - 3)(3x + 2) = 0$
 24. $4x^2 = 36$
 25. $18s^2 + 72s = 0$
 26. $t^2 + 25 = 10t$
 27. $a^2 - 9a - 52 = 0$
 28. $x^3 - 5x^2 - 66x = 0$
 29. $2x^2 = 9x + 5$
 30. $3b^2 + 6 = 11b$

31. **GEOMETRY** The parallelogram has an area of 52 square centimeters. Find the height h of the parallelogram.



32. **MULTIPLE CHOICE** Which represents one of the roots of $0 = 2x^2 + 9x - 5$?

- F -5 H $\frac{5}{2}$
 G $-\frac{1}{2}$ J 5

Standardized Test Practice

Cumulative, Chapters 1–8

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. Marlo bought 6 notebooks, 12 pencils, 8 pens, 1 backpack, 2 binders and 1 calendar. According to the chart below, which equation best represents the total amount she spent?

Item	Cost
Notebooks	2 for \$4.50
Pencils	4 for \$1.25
Pens	2 for \$1.00
Backpacks	2 for \$35.00
Binders	1 for \$2.50
Calendars	3 for \$21.00

- A Cost = $6(4.50) + 12(1.25) + 8(1.00) + 1(35.00) + 2(2.50) + 1(21.00)$
- B Cost = $2(4.50) + 4(1.25) + 2(1.00) + 2(35.00) + 1(2.50) + 3(21.00)$
- C Cost = $3(4.50) + 3(1.25) + 4(1.00) + \frac{1}{2}(35.00) + 2(2.50) + \frac{1}{3}(21.00)$
- D Cost = $\frac{1}{3}(4.50) + \frac{1}{3}(1.25) + \frac{1}{4}(1.00) + 2(35.00) + \frac{1}{2}(2.50) + 3(21.00)$
2. The area of a rectangle is $24a^6b^{13}$ square units. If the width of the rectangle is $8a^5b^7$ units, how many units long is the rectangle? ($a \neq 0$ and $b \neq 0$)
- F $3a^{11}b^{20}$ H $3ab^6$
- G $16a^{11}b^{20}$ J $32ab^6$

TEST-TAKING TIP

Question 2 When answering a multiple-choice question, first find an answer on your own. Then, compare your answer to the choices given in the item. If your answer does not match any of the answer choices, check your calculations.

3. **GRIDDABLE** Kayla is making a 120-inch by 144-inch quilt with quilt squares that measure 6 inches on a side. If the squares are not cut, how many of them will be needed to make the quilt?
4. The area of a rectangle is $2x^2 - 5x - 3$, and the width is $2x + 1$. Which expression best describes the rectangle's length?
A $x + 3$ B $2x - 3$ C $x - 3$ D $2x - 1$
5. Aliya used algebra tiles to model the trinomial $x^2 - 3x - 4$ as shown below.

x^2				
	$-x$	$-x$	$-x$	$-x$
x	-1	-1	-1	-1

What are the factors of this trinomial?

- F $(x - 4)(x + 1)$ H $(x - 2)(x + 2)$
- G $(x + 4)(x - 1)$ J $(x - 2)(x - 2)$
6. A music store surveyed 100 of its customers about their preferred styles of music. The results of the survey are shown in the table.

Favorite Style of Music	
Style	Frequency
Country	25
Rock	38
Jazz	18
Classical	12
Other	7

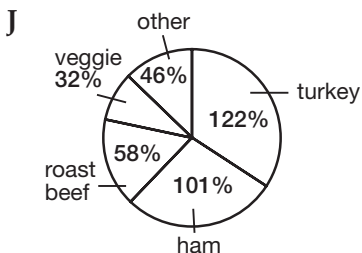
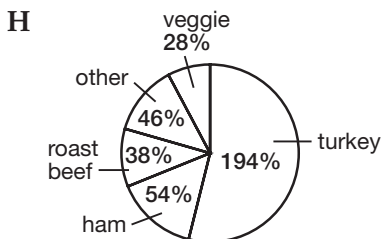
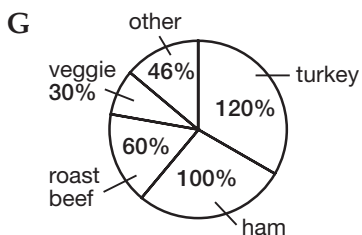
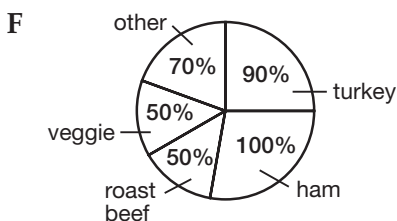
What conclusion can be drawn if the store only uses this data to order new CDs?

- A More than half of each order should be country and rock CDs.
- B More than half of each order should be rock CDs.
- C Only country, rock, and jazz CDs should be ordered.
- D About a fourth of each order should be classical music CDs.

7. At Haulalani's Sandwich Shop, Haulalani made a chart of the percentage of each type of sandwich sold. Below is her chart.

Sandwich Type	Percent of Sales
Turkey	34
Ham	28
Roast beef	16
Veggie	9
Other	13

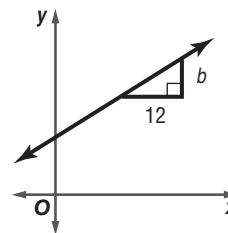
Which circle graph represents this situation?



8. Which of the following shows $16x^2 + 24x + 9$ factored completely?

- A $(4x + 3)^2$
B $(4x + 9)(4x + 1)$
C $(16x + 9)(x + 1)$
D $16x^2 + 24x + 9$

9. The slope of the line below is $\frac{3}{4}$.



What is the value of b ?

- F 3 G 4 H 9 J 16

10. **GRIDDABLE** A cylindrical grain silo has a radius of 5.5 feet and a height of 20 feet. If grain is poured in at 5 cubic feet per minute, about how long, in minutes, will it take to fill the empty silo? Round to the nearest tenth.

Pre-AP

Record your answers on a sheet of paper.
Show your work.

11. Madison is building a fenced, rectangular dog pen. The width of the pen will be 3 yards less than the length. The total area enclosed is 28 square yards.
- Using L to represent the length of the pen, write an equation showing the area of the pen in terms of its length.
 - What is the length of the pen?
 - How many yards of fencing will Madison need to enclose the pen completely?

NEED EXTRA HELP?											
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