Quadratic and Exponential Functions

BIG Ideas

- Graph quadratic functions.
- Solve quadratic equations.
- Graph exponential functions.
- Solve problems involving growth and decay.

Key Vocabulary

completing the square (p. 487)

exponential function (p. 502)

parabola (p. 471)

Quadratic Formula (p. 493)

🌐 Real-World Link

Dinosaurs Exponential decay is one type of exponential function. Carbon dating uses exponential decay to determine the age of fossils and dinosaurs.
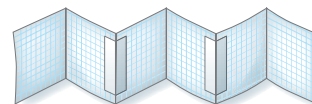
FOLDABLES™
Study Organizer

Quadratic and Exponential Functions Make this Foldable to help you organize your notes. Begin with three sheets of grid paper.
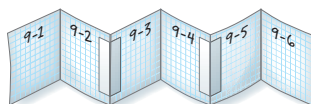
1 **Fold** each sheet in half along the width.
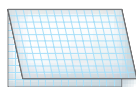
2 **Unfold** each sheet and tape to form one long piece.

3 **Label** each page with the lesson number as shown. Refold to form a booklet.

9-1 9-2 9-3 9-4 9-5 9-6

GET READY for Chapter 9

Diagnose Readiness You have two options for checking Prerequisite Skills.

Option 2

Option 1

Take the Quick Check below. Refer to the Quick Review for help.

QUICK Check

Use a table of values to graph each equation. (Lesson 3-3)

1. $y = x + 5$

2. $y = 2x - 3$

3. $y = 0.5x + 1$

4. $y = -3x - 2$

5. $2x - 3y = 12$

6. $5y = 10 + 2x$

7. SAVINGS Suppose you have already saved $200 toward the cost of a car. You plan to save $35 each month for the next several months. Graph the equation for the total amount T you will have in m months.

Determine whether each trinomial is a perfect square trinomial. If so factor it. (Lesson 8-6)

8. $t^2 + 12t + 36$

9. $a^2 - 14a + 49$

10. $m^2 - 18m + 81$

11. $y^2 + 8y + 12$

12. $9b^2 - 6b + 1$

13. $6x^2 + 4x + 1$

14. $4p^2 + 12p + 9$

15. $16s^2 - 24s + 9$

Find the next three terms of each arithmetic sequence. (Lesson 3-4)

16. $5, 9, 13, 17, \ldots$

17. $12, 5, -2, -9, \ldots$

18. $-4, -1, 2, 5, \ldots$

19. $24, 32, 40, 48, \ldots$

20. GEOMETRY Write a formula that can be used to find the number of sides of a pattern containing n triangles.
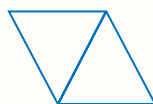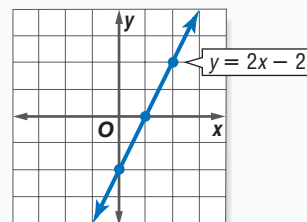

3 sides   5 sides   7 sides

QUICK Review

EXAMPLE 1

Use a table of values to graph $y = 2x - 2$.

x	$y = 2x - 2$	y
-1	$2(-1) - 2$	-4
0	$2(0) - 2$	-2
1	$2(1) - 2$	0
2	$2(2) - 2$	2

EXAMPLE 2

Determine whether $x^2 - 22x + 121$ is a perfect square trinomial. If so, factor it.

1. Is the first term a perfect square?   yes

2. Is the last term a perfect square?   yes

3. Is the middle term equal to $2(1x)(11)$?   yes

$x^2 - 22x + 121$ is a perfect square trinomial.

$x^2 - 22x + 121 = (x - 11)^2$

EXAMPLE 3

Find the next three terms of the arithmetic sequence $-104, -4, 96, 196, \ldots$.

Find the common difference by subtracting successive terms.

$-4 - (-104) = 100$

The common difference is 100.

Add to find the next three terms.

$196 + 100 = 296, 296 + 100 = 396,$
$396 + 100 = 496$

The next three terms are 296, 396, 496.

Graphing Calculator Lab
Exploring Graphs of Quadratic Functions

Not all functions are linear. The graphs of nonlinear functions have different shapes. One type of nonlinear function is a *quadratic function*. The graph of a quadratic function is a *parabola*. You use a data collection device to conduct an experiment and investigate quadratic functions.

▶ **SET UP the Lab**

- Set up the data collection device to collect data every 0.2 second for 4 seconds.
- Connect the motion sensor to your data collection device. Position the motion detector on the floor pointed upward.
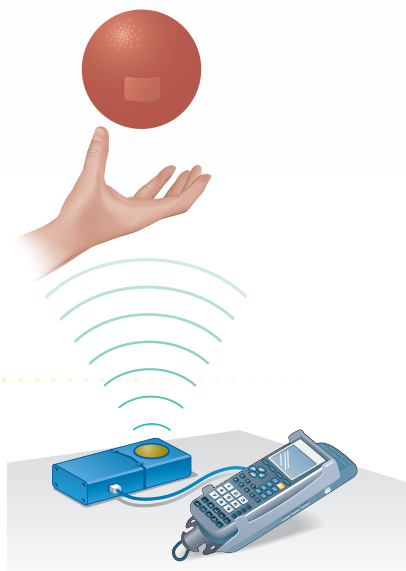
ACTIVITY

Step 1 Have one group member hold a ball about 3 feet above the motion detector. Another group member will operate the data collection device.

Step 2 When the person operating the data collection device says "go," he or she should press the start button to begin data collection. At the same time, the ball should be tossed straight upward.

Step 3 Try to catch the ball at about the same height at which it was tossed. Stop collecting data when the ball is caught.

ANALYZE THE RESULTS

1. The domain contains values represented by the independent variable, time. The range contains values represented by the dependent variable, distance. Use the graphing calculator to graph the data.

2. Write a sentence that describes the shape of the graph. Is the graph linear? Explain.

3. Describe the position of the point on the graph that represents the starting position of the ball.

4. Use the TRACE feature of the calculator to find the maximum height of the ball. At what time was the maximum height achieved?

5. Repeat the experiment and toss the ball higher. Compare and contrast the new graph and the first graph.

6. Conduct an experiment in which the motion detector is held at a height of 4 feet and pointed downward at a dropped ball. How does the graph for this experiment compare to the other graphs?
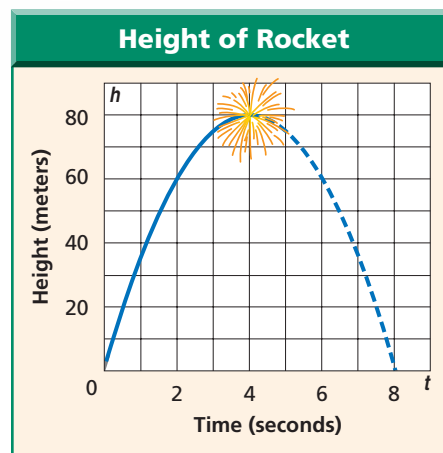
Graphing Quadratic Functions

Main Ideas

- Graph quadratic functions.
- Find the equation of the axis of symmetry and the coordinates of the vertex of a parabola.

New Vocabulary

quadratic function
parabola
minimum
maximum
vertex
symmetry
axis of symmetry

▶ **GET READY for the Lesson**

Boston's Fourth of July celebration includes a fireworks display set to music. If a rocket (firework) is launched with an initial velocity of 39.2 meters per second at a height of 1.6 meters above the ground, the equation $h = -4.9t^2 + 39.2t + 1.6$ represents the rocket's height h in meters after t seconds. The rocket will explode at approximately the highest point.

Height of Rocket

Graph Quadratic Functions The function describing the height of the rocket is an example of a quadratic function. A **quadratic function** can be written in the form $y = ax^2 + bx + c$, where $a \neq 0$. This form of equation is called *standard form*. The graph of a quadratic function is called a **parabola**.

KEY CONCEPT *Quadratic Function*

Words A quadratic function can be described by an equation of the form $y = ax^2 + bx + c$, where $a \neq 0$.

Models
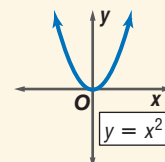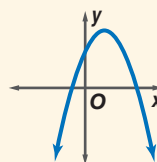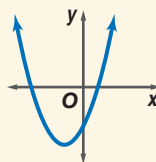
Study Tip

Parent Graph

The parent graph of the family of quadratic functions is $y = x^2$.

EXAMPLE **Graph Opens Upward**

1 Use a table of values to graph $y = 2x^2 - 4x - 5$. What are the domain and range of this function?

Graph these ordered pairs and connect them with a smooth curve. Because the parabola extends infinitely, the domain is all real numbers. The range is all real numbers greater than or equal to −7.

x	y
−2	11
−1	1
0	−5
1	−7
2	−5
3	1
4	11

$y = 2x^2 - 4x - 5$
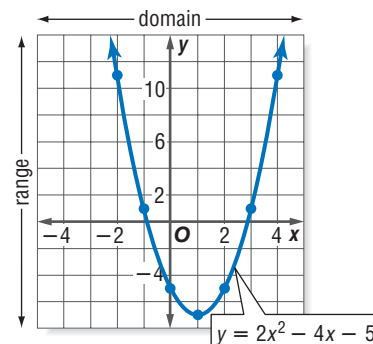
1. Use a table of values to graph $y = x^2 + 3$. What are the domain and range of this function?

Consider the standard form $y = ax^2 + bx + c$. Notice that the value of a in Example 1 is positive and the curve opens upward. The graph of any quadratic function in which a is positive opens upward. The lowest point, or **minimum**, of this graph is located at $(1, -7)$.
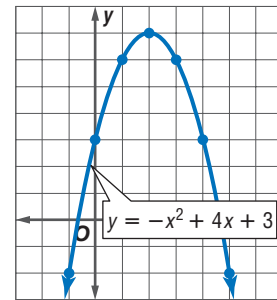
Real-World EXAMPLE   **Graph Opens Downward**

2 **FLYING DISKS** The equation $y = -x^2 + 4x + 3$ represents the height y of a flying disk x seconds after it is tossed.

a. Use a table of values to graph $y = -x^2 + 4x + 3$.

Graph these ordered pairs and connect them with a smooth curve.

x	y
−1	−2
0	3
1	6
2	7
3	6
4	3
5	−2

$y = -x^2 + 4x + 3$

b. What are the domain and range of this function?

D: $\{x | x$ is a real number.$\}$

R: $\{y | y \leq 7\}$

c. Describe reasonable domain and range values for this situation.

The flying disk is in the air for about 4.6 seconds, so a reasonable domain is $\{x | 0 \leq x \leq 4.6\}$. The height of the flying disk ranges from 0 to 7 feet, so a reasonable range is $\{y | 0 \leq y \leq 7\}$.

✓ CHECK Your Progress

2. Use a table of values to graph $y = -2x^2 + x + 1$. What are the domain and range of this function?

Notice that the value of a in Example 2 is negative and the curve opens downward. The graph of any quadratic function in which a is negative opens downward. The highest point, or **maximum**, of the graph is located at $(2, 3)$. The maximum or minimum point of a parabola is called the **vertex**.

Symmetry and Vertices Parabolas possess a geometric property called **symmetry**. Symmetrical figures are those in which each half of the figure matches the other exactly.

The line that divides a parabola into two halves is called the **axis of symmetry**. Each point on the parabola that is on one side of the axis of symmetry has a corresponding point on the parabola on the other side of the axis. The vertex is the only point on the parabola that is on the axis of symmetry. Notice the relationship between the values a and b and the equation of the axis of symmetry.
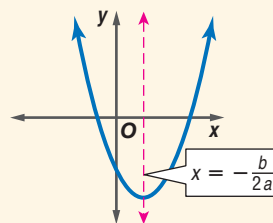

$y = x^2 - x - 6$

axis of symmetry

$x = \frac{1}{2}$
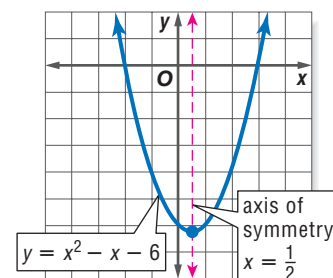
Words — The equation of the axis of symmetry for the graph of $y = ax^2 + bx + c$, where $a \neq 0$, is $x = -\dfrac{b}{2a}$.

Model

$x = -\dfrac{b}{2a}$

EXAMPLE — **Vertex and Axis of Symmetry**

3 Consider the graph of $y = -3x^2 - 6x + 4$.

a. Write the equation of the axis of symmetry.

In $y = -3x^2 - 6x + 4$, $a = -3$ and $b = -6$.

$$x = -\frac{b}{2a} \qquad \text{Equation for the axis of symmetry of a parabola}$$

$$x = -\frac{-6}{2(-3)} \text{ or } -1 \qquad a = -3 \text{ and } b = -6$$

The equation of the axis of symmetry is $x = -1$.

b. Find the coordinates of the vertex.

Since the equation of the axis of symmetry is $x = -1$ and the vertex lies on the axis, the x-coordinate for the vertex is -1.

$$y = -3x^2 - 6x + 4 \qquad \text{Original equation}$$

$$y = -3(-1)^2 - 6(-1) + 4 \qquad x = -1$$

$$y = -3 + 6 + 4 \qquad \text{Simplify.}$$

$$y = 7 \qquad \text{Add.}$$

The vertex is at $(-1, 7)$.

c. Identify the vertex as a maximum or minimum.

Since the coefficient of the x^2 term is negative, the parabola opens downward and the vertex is a maximum point.

d. Graph the function.

You can use the symmetry of the parabola to help you draw its graph. On a coordinate plane, graph the vertex and the axis of symmetry. Choose a value for x other than -1. For example, choose 1 and find the y-coordinate that satisfies the equation.

$$y = -3x^2 - 6x + 4 \qquad \text{Original equation}$$

$$y = -3(1)^2 - 6(1) + 4 \qquad \text{Let } x = 1.$$

$$y = -5 \qquad \text{Simplify.}$$

Graph $(1, -5)$. Since the graph is symmetrical about its axis of symmetry $x = -1$, you can find another point on the other side of the axis of symmetry. The point at $(1, -5)$ is 2 units to the right of the axis. Go 2 units to the left of the axis and plot the point $(-3, -5)$. Repeat this for several other points. Then sketch the parabola.
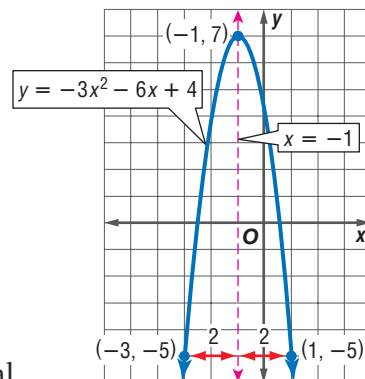
Study Tip

Coordinates of Vertex

Notice that you can find the x-coordinate by knowing the axis of symmetry. However, to find the y-coordinate, you must substitute the value of x into the quadratic equation.

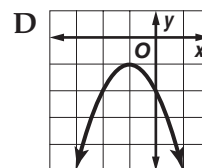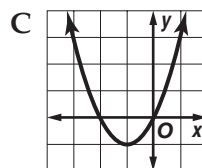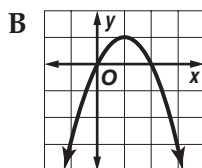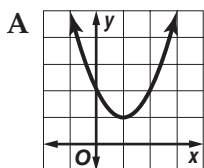**Consider the graph of** $y = x^2 + 2x + 18$.

3A. Write the equation of the axis of symmetry.

3B. Find the coordinates of the vertex.

3C. Identify the vertex as a maximum or minimum.

3D. Graph the function.

STANDARDIZED TEST PRACTICE   **Match Equations and Graphs**

④ Which is the graph of $y + 1 = (x + 1)^2$?

A B C D

Test-Taking Tip

Substituting Values
The ordered pair
$(0, 0)$ satisfies
the equation
$y + 1 = (x + 1)^2$.
Since the point at
$(0, 0)$ is on the
graph, choices A
and D can be
eliminated.

Read the Test Item

You are given a quadratic function, and you are asked to choose its graph.

Solve the Test Item

Step 1 First write the equation in standard form.

$$y + 1 = (x + 1)^2 \qquad \text{Original equation}$$
$$y + 1 = x^2 + 2x + 1 \qquad (x + 1)^2 = x^2 + 2x + 1$$
$$y + 1 - 1 = x^2 + 2x + 1 - 1 \qquad \text{Subtract 1 from each side.}$$
$$y = x^2 + 2x \qquad \text{Simplify.}$$

Step 2 Then find the axis of symmetry of the graph of $y = x^2 + 2x$.

$$x = -\frac{b}{2a} \qquad \text{Equation for the axis of symmetry}$$

$$x = -\frac{2}{2(1)} \text{ or } -1 \qquad a = 1 \text{ and } b = 2$$

The axis of symmetry is $x = -1$. Look at the graphs. Since only choices C and D have $x = -1$ as their axis of symmetry, you can eliminate choices A and B. Since the coefficient of the x^2 term is positive, the graph opens upward. Eliminate choice D. The answer is C.

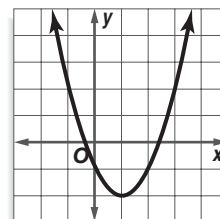**4.** Which is the equation of the graph?

   F $y - 1 = (x + 2)^2$

   G $y - 1 = (x - 2)^2$

   H $y + 2 = (x - 1)^2$

   J $y - 2 = (x + 1)^2$

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Examples 1, 2
(pp. 471–472)

Use a table of values to graph each function.

1. $y = x^2 - 5$ **2.** $y = x^2 + 2$

3. $y = -x^2 + 4x + 5$ **4.** $y = x^2 + x - 1$

Example 3
(pp. 473–474)

Write the equation of the axis of symmetry, and find the coordinates of the vertex of the graph of each function. Identify the vertex as a maximum or minimum. Then graph the function.

5. $y = x^2 + 4x - 9$ **6.** $y = -x^2 + 5x + 6$

7. $y = -(x - 2)^2 + 1$ **8.** $y = (x + 3)^2 - 4$

Example 4
(p. 474)

9. STANDARDIZED TEST PRACTICE Which is the graph of $y = -\frac{1}{2}x^2 + 1$?
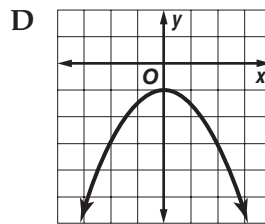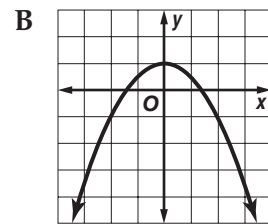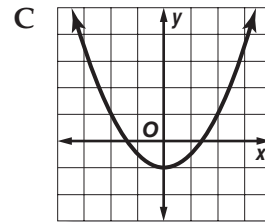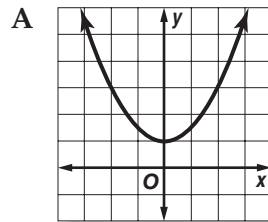
A

C

B

D

Exercises

Use a table of values to graph each function.

10. $y = x^2 - 3$ **11.** $y = -x^2 + 7$

12. $y = x^2 - 2x - 8$ **13.** $y = x^2 - 4x + 3$

14. $y = -3x^2 - 6x + 4$ **15.** $y = -3x^2 + 6x + 1$

Write the equation of the axis of symmetry, and find the coordinates of the vertex of the graph of each function. Identify the vertex as a maximum or minimum. Then graph the function.

16. $y = 4x^2$ **17.** $y = -2x^2$

18. $y = x^2 + 2$ **19.** $y = -x^2 + 5$

20. $y = -x^2 + 2x + 3$ **21.** $y = -x^2 - 6x + 15$

22. $y = 3x^2 - 6x + 4$ **23.** $y = 9 - 8x + 2x^2$

24. What is the equation of the axis of symmetry of the graph of $y = -3x^2 + 2x - 5$?

25. Find the equation of the axis of symmetry of the graph of $y = 4x^2 - 5x + 16$.

ENTERTAINMENT For Exercises 26 and 27, use the following information.
A carnival game involves striking a lever that forces a weight up a tube. If the weight reaches 20 feet to ring the bell, the contestant wins a prize. The equation $h = -16t^2 + 32t + 3$ gives the height of the weight if the initial velocity is 32 feet per second.

26. Find the maximum height of the weight.

27. Will a prize be won? Explain.
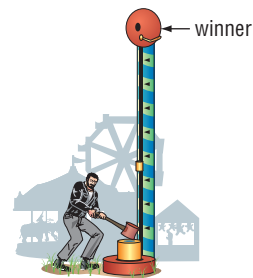
winner

PETS For Exercises 28 and 29, use the following information.
Miriam has 40 meters of fencing to build a pen for her dog.

28. Use the diagram to write an equation for the area A of the pen. Describe a reasonable domain and range for this situation.

29. What value of x will result in the greatest area? What is the greatest possible area of the pen?

$20 - x$

x ⬜ x

$20 - x$

Write the equation of the axis of symmetry, and find the coordinates of the vertex of the graph of each function. Identify the vertex as a maximum or minimum. Then graph the function.
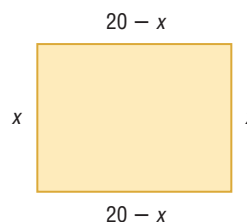
30. $y = -2(x - 4)^2 - 3$

31. $y + 2 = x^2 - 10x + 25$

32. $y - 5 = \frac{1}{3}(x + 2)^2$

33. $y + 1 = \frac{1}{3}(x + 1)^2$

34. The vertex of a parabola is at $(-4, -3)$. If one x-intercept is -11, what is the other x-intercept?

35. What is the equation of the axis of symmetry of a parabola if its x-intercepts are -6 and 4?

ARCHITECTURE For Exercises 36–38, use the following information.
The shape of the Gateway Arch in St. Louis, Missouri, is a *catenary* curve. It resembles a parabola with the equation $h = -0.00635x^2 + 4.0005x - 0.07875$, where h is the height in feet and x is the distance from one base in feet.

36. What is the equation of the axis of symmetry?

37. What is the distance from one end of the arch to the other?

38. What is the maximum height of the arch?

FOOTBALL For Exercises 39–41, use the following information.
A football is kicked from ground level at an initial velocity of 90 feet per second. The equation $h = -16t^2 + 90t$ gives the height h of the football after t seconds.

39. What is the height of the ball after one second?

40. When is the ball 126 feet high?

41. When is the height of the ball zero feet? Describe the events these represent.

H.O.T. Problems

42. **OPEN ENDED** Sketch a parabola that models a real-life situation and describe what the vertex represents. Determine reasonable domain and range values for this type of situation.

43. **REASONING** Sketch the parent graph of the function $y = 3x^2 - 5x - 2$.

REASONING Let $f(x) = x^2 - 9$.

44. What is the domain of $f(x)$?

45. What is the range of $f(x)$?

46. For what values of x is $f(x)$ negative?

47. When x is a real number, what are the domain and range of $f(x) = \sqrt{x^2 - 9}$?

48. REASONING Determine the range of $f(x) = (x - 5)^2 - 6$.

49. CHALLENGE Write and graph a quadratic equation whose graph has the axis of symmetry $x = -\frac{3}{8}$. Summarize the steps that you took to determine the equation.

50. *Writing in Math* Use the information about a rocket's path on page 471 to explain how a fireworks display can be coordinated with recorded music. Include an explanation of how to determine when the rocket will explode and how to find the height of the rocket when it explodes.

STANDARDIZED TEST PRACTICE

51. In the graph of the function $y = x^2 - 3$, which describes the shift in the vertex of the parabola if, in the function, -3 is changed to 1?

A 2 units up

B 4 units up

C 2 units down

D 4 units down

52. REVIEW The costs of two packs of Brand A gum and two packs of Brand B gum are shown in the table. What percent of the cost of Brand B gum does James save by buying two packs of Brand A gum?

Gum	Cost of Two Packs
Brand A	$1.98
Brand B	$2.50

F 11.6% **H** 26.3%

G 20.8% **J** 79.2%

Spiral Review

Factor each polynomial, if possible. (Lessons 8-5 and 8-6)

53. $x^2 + 6x - 9$

54. $a^2 + 22a + 121$

55. $4m^2 - 4m + 1$

56. $4q^2 - 9$

57. $2a^2 - 25$

58. $1 - 16g^2$

Find each sum or difference. (Lesson 7-5)

59. $(13x + 9y) + 11y$

60. $(8 - 2c^2) + (1 + c^2)$

61. $(7p^2 - p - 7) - (p^2 + 11)$

62. RECREATION At a recreation facility, 3 members and 3 nonmembers pay a total of $180 to take an aerobics class. A group of 5 members and 3 nonmembers pay $210 to take the same class. How much does it cost each to take an aerobics class? (Lesson 5-3)

GET READY for the Next Lesson

PREREQUISITE SKILL Find the x-intercept of the graph of each equation. (Lesson 3-3)

63. $3x + 4y = 24$

64. $2x - 5y = 14$

65. $-2x - 4y = 7$

The parent function of the family of quadratic functions is $y = x^2$.

COncepts in MOtion
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ACTIVITY 1

Graph each group of equations on the same screen. Use the standard viewing window. Compare and contrast the graphs.

KEYSTROKES: *Review graphing equations on pages* 162 *and* 163.

a. $y = x^2$, $y = 2x^2$, $y = 4x^2$
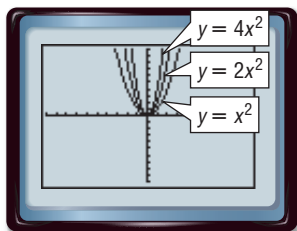
$y = 4x^2$
$y = 2x^2$
$y = x^2$

Each graph opens upward and has its vertex at the origin. The graphs of $y = 2x^2$ and $y = 4x^2$ are narrower than the graph of $y = x^2$.

b. $y = x^2$, $y = 0.5x^2$, $y = 0.2x^2$

$y = x^2$
$y = 0.5x^2$
$y = 0.2x^2$
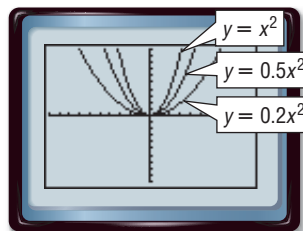
Each graph opens upward and has its vertex at the origin. The graphs of $y = 0.5x^2$ and $y = 0.2x^2$ are wider than the graph of $y = x^2$.

1A. How does the value of a in $y = ax^2$ affect the shape of the graph?

c. $y = x^2$, $y = x^2 + 3$, $y = x^2 - 2$, $y = x^2 - 4$
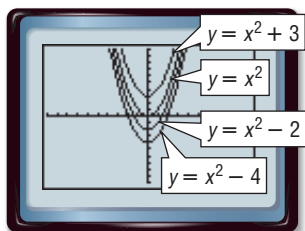
$y = x^2 + 3$
$y = x^2$
$y = x^2 - 2$
$y = x^2 - 4$

Each graph opens upward and has the same shape as $y = x^2$. However, each parabola has a different vertex, located along the *y*-axis.

d. $y = x^2$, $y = (x - 3)^2$, $y = (x + 2)^2$, $y = (x + 4)^2$
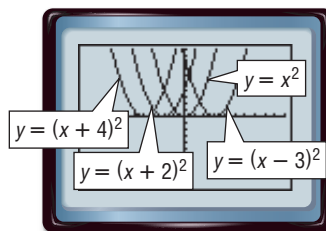
$y = x^2$
$y = (x + 4)^2$
$y = (x + 2)^2$
$y = (x - 3)^2$

Each graph opens upward and has the same shape as $y = x^2$. However, each parabola has a different vertex located along the *x*-axis.

1B. How does the value of the constant affect the position of the graph?

1C. How is the location of the vertex related to the equation of the graph?

Math Online Other Calculator Keystrokes at **algebra1.com**

Suppose you graph the same equation using different windows. How will the appearance of the graph change?

ACTIVITY 2

Graph $y = x^2 - 7$ in each viewing window. What conclusions can you draw about the appearance of a graph in the window used?

a. standard viewing window
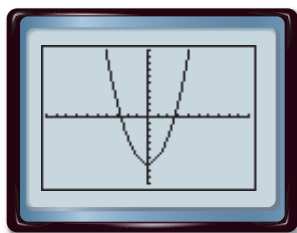
b. [−10, 10] scl: 1 by [−200, 200] scl: 50
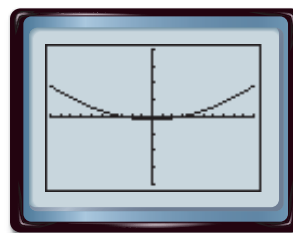
c. [−50, 50] scl: 5 by [−10, 10] scl: 1
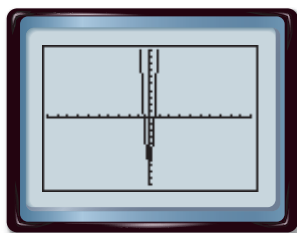
d. [−0.5, 0.5] scl: 0.1 by [−10, 10] scl: 1
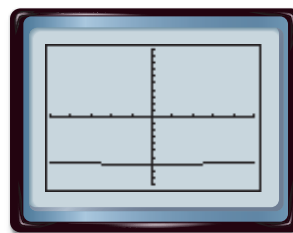
Without knowing the window, graph **b** might be of the family $y = ax^2$, where $0 < a < 1$. Graph **c** looks like a member of $y = ax^2 - 7$, where $a > 1$. Graph **d** looks more like a line. However, all are graphs of the same equation.

EXERCISES

Graph each family of equations on the same screen. Compare and contrast the graphs.

1. $y = -x^2$
$y = -3x^2$
$y = -6x^2$

2. $y = -x^2$
$y = -0.6x^2$
$y = -0.4x^2$

3. $y = -x^2$
$y = -(x + 5)^2$
$y = -(x - 4)^2$

4. $y = -x^2$
$y = -x^2 + 7$
$y = -x^2 - 5$

Use the graphs on page 478 and Exercises 1–4 above to predict the appearance of the graph of each equation. Then draw the graph.

5. $y = -0.1x^2$ **6.** $y = (x + 1)^2$ **7.** $y = 4x^2$ **8.** $y = x^2 - 6$

Describe how each change in $y = x^2$ would affect the graph of $y = x^2$. Be sure to consider all values of a, h, and k.

9. $y = ax^2$ **10.** $y = (x + h)^2$ **11.** $y = x^2 + k$ **12.** $y = (x + h)^2 + k$

Solving Quadratic Equations by Graphing

GET READY for the Lesson

A golf ball follows a path much like a parabola. Because of this property, quadratic functions can be used to simulate parts of a computer golf game. One of the x-intercepts of the quadratic function represents the location where the ball will hit the ground.
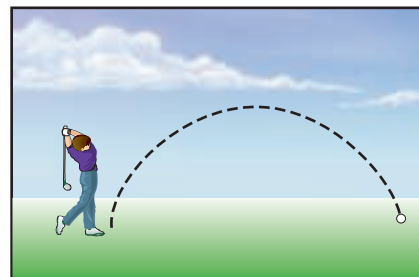
Solve by Graphing A **quadratic equation** is an equation that can be written in the form $ax^2 + bx + c = 0$, where $a \neq 0$. The value of the related quadratic function is 0.

Quadratic Equation	Related Quadratic Function
$x^2 - 2x - 3 = 0$	$f(x) = x^2 - 2x - 3$

The solutions of a quadratic equation are called the **roots** of the equation. The roots of a quadratic equation can be found by finding the x-intercepts or **zeros** of the related quadratic function.

EXAMPLE   Two Roots

① Solve $x^2 + 6x - 7 = 0$ by graphing.

Graph the related function $f(x) = x^2 + 6x - 7$. The equation of the axis of symmetry is $x = -\dfrac{6}{2(1)}$ or $x = -3$. When x equals -3, $f(x)$ equals $(-3)^2 + 6(-3) - 7$ or -16. So, the coordinates of the vertex are $(-3, -16)$. Make a table of values to find other points to sketch the graph.

COncepts in MOtion
Animation **algebra1.com**

x	f(x)
-8	9
-6	-7
-4	-15
-3	-16
-2	-15
0	-7
2	9

Study Tip

x-intercepts
The *x*-intercepts of a graph are also called the *horizontal intercepts*.
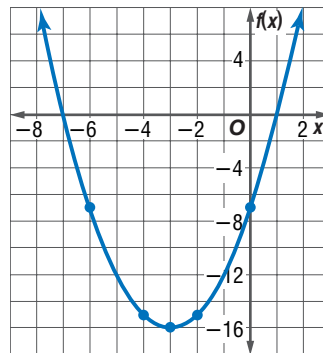
To solve $x^2 + 6x - 7 = 0$, you need to know where the value of $f(x)$ is 0. This occurs at the x-intercepts. The x-intercepts of the parabola appear to be -7 and 1.

Common Misconception

Although solutions found by graphing may appear to be exact, you cannot be sure that they are exact. Solutions need to be verified by substituting into the equation and checking, or by using the algebraic methods that you will learn in this chapter.

CHECK Solve by factoring.

$$x^2 + 6x - 7 = 0 \quad \text{Original equation}$$

$$(x + 7)(x - 1) = 0 \quad \text{Factor.}$$

$$x + 7 = 0 \quad \text{or} \quad x - 1 = 0 \quad \text{Zero Product Property}$$

$$x = -7 ✔ \qquad x = 1 ✔ \quad \text{The solutions are } -7 \text{ and } 1.$$

CHECK Your Progress

1. Solve $-c^2 + 5c - 4 = 0$ by graphing.

Quadratic equations always have two roots. However, these roots are not always two distinct numbers. Sometimes the two roots are the same number, called a **double root**. In other cases the roots are not real numbers.

EXAMPLE **A Double Root**

2 Solve $b^2 + 4b = -4$ by graphing.

First rewrite the equation so one side is equal to zero.

$$b^2 + 4b = -4 \quad \text{Original equation}$$

$$b^2 + 4b + 4 = 0 \quad \text{Add 4 to each side.}$$

Graph the related function $f(b) = b^2 + 4b + 4$.

Notice that the vertex of the parabola is the b-intercept. Thus, one solution is -2. What is the other solution?

Try solving the equation by factoring.

$$b^2 + 4b + 4 = 0 \quad \text{Original equation}$$

$$(b + 2)(b + 2) = 0 \quad \text{Factor.}$$

$$b + 2 = 0 \quad \text{or} \quad b + 2 = 0 \quad \text{Zero Product Property}$$

$$b = -2 \qquad b = -2 \quad \text{The solution is } -2.$$

$$f(b) = b^2 + 4b + 4$$
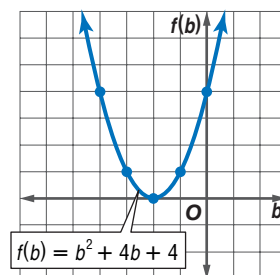
CHECK Your Progress

2. Solve $0 = x^2 - 6x + 9$ by graphing.

EXAMPLE **No Real Roots**

Study Tip

Empty Set
The symbol ∅, indicating an empty set, is often used to represent no real solutions.

3 Solve $x^2 - x + 4 = 0$ by graphing.

Graph the related function $f(x) = x^2 - x + 4$.

The graph has no x-intercept. Thus, there are no real number solutions for this equation.

x	f(x)
−1	6
0	4
1	4
2	6

$$f(x) = x^2 - x + 4$$

CHECK Your Progress

3. Solve $-t^2 - 3t = 5$ by graphing.

Factoring can be used to determine whether the graph of a quadratic function intersects the x-axis in zero, one, or two points.

EXAMPLE Factoring

④ **Use factoring to determine how many times the graph of $f(x) = x^2 + x - 12$ intersects the x-axis. Identify each root.**

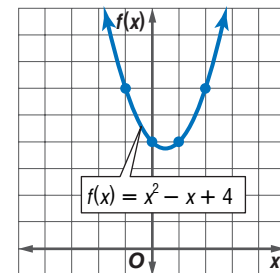The graph intersects the x-axis when $f(x) = 0$.

$$x^2 + x - 12 = 0 \qquad \text{Original equation}$$
$$(x - 3)(x + 4) = 0 \qquad \text{Factor.}$$

Since the trinomial factors into two distinct factors, the graph of the function intersects the x-axis 2 times. The roots are $x = 3$ and $x = -4$.

CHECK Your Progress

4. Use factoring to determine how many times the graph of $f(x) = x^2 - 10x + 25$ intersects the x-axis. Identify each root.

Estimate Solutions In Examples 1 and 2, the roots of the equation were integers. Usually the roots of a quadratic equation are not integers. In these cases, use estimation to approximate the roots of the equation.

EXAMPLE Rational Roots

⑤ **Solve $n^2 + 6n + 7 = 0$ by graphing. If integral roots cannot be found, estimate the roots by stating the consecutive integers between which the roots lie.**

Graph the related function $f(n) = n^2 + 6n + 7$.

n	$f(n)$
-6	7
-5	2
-4	-1
-3	-2
-2	-1
-1	2
0	7

Notice that the value of the function changes from negative to positive between the n values of -5 and -4 and between -2 and -1.

$f(n) = n^2 + 6n + 7$

The n-intercepts are between -5 and -4 and between -2 and -1. So, one root is between -5 and -4, and the other root is between -2 and -1.

CHECK Your Progress
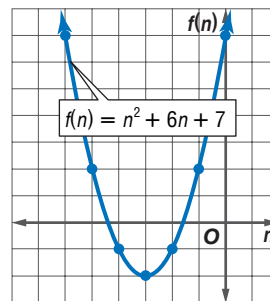
5. Solve $2a^2 + 6a - 3 = 0$ by graphing. If integral roots cannot be found, estimate the roots by stating the consecutive integers between which the roots lie.

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Real-World EXAMPLE

6 **SOCCER** If a goalie kicks a soccer ball with an upward velocity of 65 feet per second and his foot meets the ball 3 feet off the ground, the function $y = -16t^2 + 65t + 3$ represents the height of the ball y in feet after t seconds. Approximately how long is the ball in the air?

You need to find the solution of the equation $0 = -16t^2 + 65t + 3$. Use a graphing calculator to graph the related function $y = -16t^2 + 65t + 3$. The x-intercept is about 4. Therefore, the ball is in the air about 4 seconds.
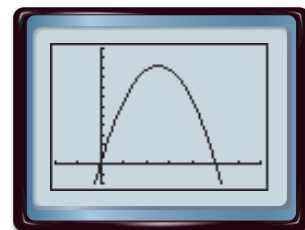
$[-2, 7]$ scl: 1 by $[-20, 80]$ scl: 10

CHECK Your Progress

6. NUMBER THEORY Use a quadratic equation to find two numbers whose sum is 5 and whose product is -24.

CHECK Your Understanding

Examples 1–3
(pp. 480–481)

Solve each equation by graphing.

1. $x^2 - 7x + 6 = 0$ **2.** $-a^2 - 10a = 25$ **3.** $c^2 + 3 = 0$

Example 4
(p. 482)

Use factoring to determine how many times the graph of each function intersects the x-axis. Identify each root.

4. $f(x) = x^2 + 2x - 24$ **5.** $f(x) = x^2 + 14x + 49$

Example 5
(p. 482)

Solve each equation by graphing. If integral roots cannot be found, estimate the roots by stating the consecutive integers between which the roots lie.

6. $-t^2 - 5t + 1 = 0$ **7.** $0 = x^2 - 16$ **8.** $w^2 - 3w = 5$

Example 6
(p. 483)

9. NUMBER THEORY Two numbers have a sum of 4 and a product of -12. Use a quadratic equation to determine the two numbers.

Exercises

HOMEWORK HELP	
For Exercises	**See Examples**
10–17	1–3
18–21	4
22–30	5
31, 32	6

Solve each equation by graphing.

10. $c^2 - 5c - 24 = 0$ **11.** $5n^2 + 2n + 6 = 0$ **12.** $0 = x^2 + 6x + 9$

13. $-b^2 + 4b = 4$ **14.** $x^2 + 2x + 5 = 0$ **15.** $-2r^2 - 6r = 0$

16. The roots of a quadratic equation are -2 and -6. The minimum point of the graph of its related function is at $(-4, -2)$. Sketch the graph of the function and compare the graph to the graph of the parent function $y = x^2$.

17. The roots of a quadratic equation are -6 and 0. The maximum point of the graph of its related function is at $(-3, 4)$. Sketch the graph of the function and compare the graph to the graph of the parent function $y = x^2$.

Use factoring to determine how many times the graph of each function intersects the x-axis. Identify each root.

18. $g(x) = x^2 - 8x + 16$ **19.** $h(x) = x^2 + 12x + 32$

20. $f(x) = x^2 + 3x + 4$ **21.** $g(x) = x^2 + 3x + 4$

Solve each equation by graphing. If integral roots cannot be found, estimate the roots by stating the consecutive integers between which the roots lie.

22. $a^2 - 12 = 0$
23. $-n^2 + 7 = 0$
24. $2c^2 + 20c + 32 = 0$

25. $3s^2 + 9s - 12 = 0$
26. $0 = x^2 + 6x + 6$
27. $0 = -y^2 + 4y - 1$

28. $-a^2 + 8a = -4$
29. $x^2 + 6x = -7$
30. $m^2 - 10m = -21$

31. NUMBER THEORY Use a quadratic equation to find two numbers whose sum is 9 and whose product is 20.

32. COMPUTER GAMES In a computer football game, the function $-0.005d^2 + 0.22d = h$ simulates the path of a football at the kickoff. In this equation, h is the height of the ball and d is the horizontal distance in yards. What is the horizontal distance the ball will travel before it hits the ground?

33. HIKING While hiking in the mountains, Monya and Kishi stop for lunch on a ledge 1000 feet above a valley. Kishi decides to climb to another ledge 20 feet above Monya. Monya throws an apple up to Kishi, but Kishi misses it. The equation $h = -16t^2 + 30t + 1000$ represents the height in feet of the apple t seconds after it was thrown. How long did it take for the apple to reach the ground?

THEATER For Exercises 34–37, use the following information.
The drama club is building a backdrop using arches whose shape can be represented by the function $f(x) = -x^2 + 2x + 8$, where x is the length in feet. The area under each arch is to be covered with fabric.
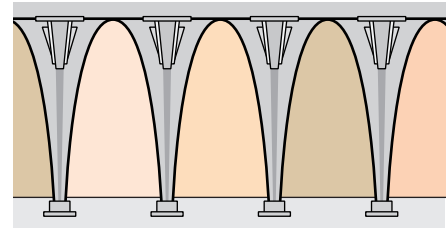
34. Graph the quadratic function and determine its x-intercepts.

35. What is the length of the segment along the floor of each arch?

36. What is the height of the arch?

37. The formula $A = \frac{2}{3}bh$ can be used to estimate the area A under a parabola. In this formula, b represents the length of the base, and h represents the height. If there are five arches, calculate the total amount of fabric that is needed.

WORK For Exercises 38–40, use the following information.
Kirk and Montega mow the soccer playing fields. They must mow an area 500 feet long and 400 feet wide. They agree that each will mow half the area. Kirk will mow around the edge in a path of equal width until half the area is left.
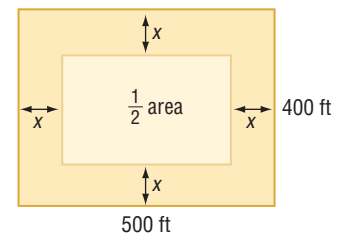
38. What is the area each person will mow?

39. Write a quadratic equation that could be used to find the width x that Kirk should mow. What width should Kirk mow?

40. The mower can mow a path 5 feet wide. To the nearest whole number, how many times should Kirk go around the field?

H.O.T. Problems

41. OPEN ENDED Draw a graph to show a counterexample to the following statement. Explain. *All quadratic equations have two different solutions.*

42. CHALLENGE Describe the zeros of $f(x) = \dfrac{x^3 + 2x^2 - 3x}{x + 5}$. Explain your reasoning.

43. CHALLENGE The graph shown is a *quadratic inequality.* Similar to a linear inequality, the quadratic equation is a boundary between two half-planes. Analyze the graph and determine whether the inequality is *always, sometimes,* or *never* greater than 2. Explain.

44. *Writing in Math* Use the information about computer games on page 480 to explain how quadratic equations can be used in computer simulations. Describe what the roots of a simulation equation for a computer golf game represent.

STANDARDIZED TEST PRACTICE

45. The graph of the equation $y = x^2 + 10x + 21$ is shown. For what value or values of x is $y = 0$?
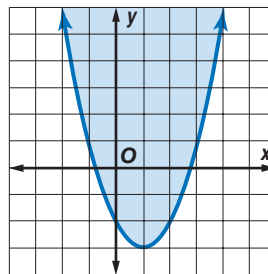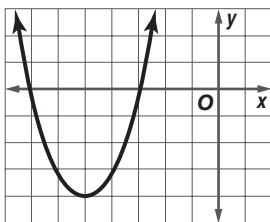

A $x = -4$ **C** $x = 7$ and $x = 3$

B $x = -5$ **D** $x = -7$ and $x = -3$

46. REVIEW Q-Mart has 1200 blue towels in stock. If they sell half of their towels every three months and do not receive any more shipments of towels, how many towels will they have left after a year?

F 60   **H** 150

G 75   **J** 300

Spiral Review

Write the equation of the axis of symmetry, and find the coordinates of the vertex of the graph of each equation. Identify the vertex as a maximum or minimum. Then graph the function. (Lesson 9-1)

47. $y = x^2 + 6x + 9$ **48.** $y = -x^2 + 4x - 3$ **49.** $y = 0.5x^2 - 6x + 5$

Solve each equation. Check your solutions. (Lesson 8-6)

50. $m^2 - 24m = -144$ **51.** $7r^2 = 70r - 175$ **52.** $4d^2 + 9 = -12d$

Simplify. Assume that no denominator is equal to zero. (Lesson 7-2)

53. $\dfrac{10m^4}{30m}$ **54.** $\dfrac{22a^2b^5c^7}{-11abc^2}$ **55.** $\dfrac{-9m^3n^5}{27m^{-2}n^5y^{-4}}$

56. SHIPPING An empty book crate weighs 30 pounds. The weight of a book is 1.5 pounds. For shipping, the crate must weigh at least 55 pounds and no more than 60 pounds. What is the acceptable number of books that can be packed in the crate? (Lesson 6-4)

GET READY for the Next Lesson

PREREQUISITE SKILL Determine whether each trinomial is a perfect square trinomial. If so, factor it. (Lesson 8-6)

57. $a^2 + 14 + 49$ **58.** $m^2 - 10m + 25$ **59.** $t^2 + 16t - 64$ **60.** $4y^2 + 12y + 9$

Solving Quadratic Equations by Completing the Square

Main Ideas

- Solve quadratic equations by finding the square root.
- Solve quadratic equations by completing the square.

New Vocabulary

completing the square

▶ GET READY for the Lesson

Al-Khwarizmi, born in Baghdad in 780, is considered to be one of the foremost mathematicians of all time. He wrote algebra in sentences instead of using equations, and he explained the work with geometric sketches. Al-Khwarizmi would have described $x^2 + 8x = 35$ as "A square and 8 roots are equal to 35 units." He would solve the problem using the following sketch.

The area of the shaded portion is $x^2 + 8x$ or 35. Four squares each with an area of 4 are used to complete the square.
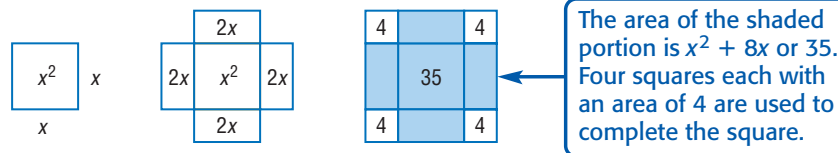
To solve problems this way today, you might use algebra tiles or a method called *completing the square*.

Find the Square Root Some equations can be solved by taking the square root of each side.

EXAMPLE   Irrational Roots

1 Solve $x^2 - 10x + 25 = 7$ by taking the square root of each side. Round to the nearest tenth if necessary.

$$x^2 - 10x + 25 = 7 \qquad \text{Original equation}$$
$$(x - 5)^2 = 7 \qquad x^2 - 10x + 25 \text{ is a perfect square trinomial.}$$
$$\sqrt{(x-5)^2} = \sqrt{7} \qquad \text{Take the square root of each side.}$$
$$|x - 5| = \sqrt{7} \qquad \text{Simplify.}$$
$$x - 5 = \pm\sqrt{7} \qquad \text{Definition of absolute value}$$
$$x - 5 + 5 = \pm\sqrt{7} + 5 \qquad \text{Add 5 to each side.}$$
$$x = 5 \pm \sqrt{7} \qquad \text{Simplify.}$$

Use a calculator to evaluate each value of x.

$$x = 5 + \sqrt{7} \quad \text{or} \quad x = 5 - \sqrt{7} \qquad \text{Write each solution.}$$
$$\approx 7.6 \qquad\qquad\qquad \approx 2.4 \qquad \text{Simplify.}$$

The solution set is $\{2.4, 7.6\}$.

✓ CHECK Your Progress

1. Solve $m^2 + 18m + 81 = 90$ by taking the square root of each side. Round to the nearest tenth if necessary.

Complete the Square In Example 1, the quadratic expression on one side of the equation was a perfect square. However, few quadratic expressions are perfect squares. To make any quadratic expression a perfect square, a method called **completing the square** may be used.

Consider the pattern for squaring a binomial such as $x + 6$.

$$(x + 6)^2 = x^2 + 2(6)(x) + 6^2$$
$$= x^2 + 12x + 36$$

$$\left(\frac{12}{2}\right)^2 \rightarrow 6^2 \quad \text{Notice that one half of 12 is 6 and } 6^2 \text{ is 36.}$$

KEY CONCEPT *Completing the Square*

To complete the square for a quadratic expression of the form $x^2 + bx$, you can follow the steps below.

Step 1 Find $\frac{1}{2}$ of b, the coefficient of x.

Step 2 Square the result of Step 1.

Step 3 Add the result of Step 2 to $x^2 + bx$, the original expression.

EXAMPLE **Complete the Square**

2 Find the value of c that makes $x^2 + 6x + c$ a perfect square.

Method 1 Use algebra tiles.

Arrange the tiles for $x^2 + 6x$ so that the two sides of the figure are congruent.

To make the figure a square, add 9 positive 1-tiles.
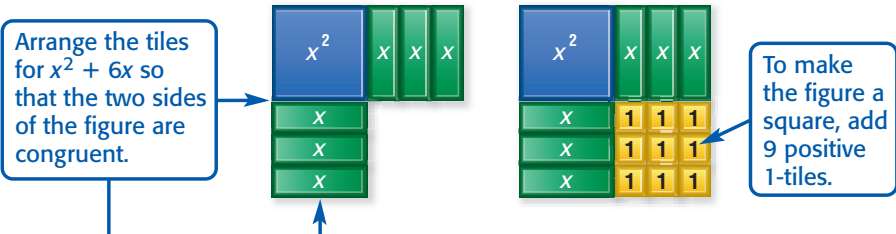
$x^2 + 6x + 9$ is a perfect square.

Method 2 Complete the square.

Step 1 Find $\frac{1}{2}$ of 6. $\quad\quad\quad \frac{6}{2} = 3$

Step 2 Square the result of Step 1. $\quad 3^2 = 9$

Step 3 Add the result of Step 2 to $x^2 + 6x$. $\quad x^2 + 6x + 9$

Thus, $c = 9$. Notice that $x^2 + 6x + 9 = (x + 3)^2$.

CHECK Your Progress

2. Find the value of c that makes $r^2 + 8r + c$ a perfect square.

You can use the technique of completing the square to solve quadratic equations.

Concepts in Motion

Animation
algebra1.com

③ **Solve $a^2 - 14a + 3 = -10$ by completing the square.**

Isolate the a^2 and a terms. Then complete the square and solve.

$a^2 - 14a + 3 = -10$	Original equation
$a^2 - 14a + 3 - 3 = -10 - 3$	Subtract 3 from each side.
$a^2 - 14a = -13$	Simplify.
$a^2 - 14a + 49 = -13 + 49$	Since $\left(\frac{-14}{2}\right)^2 = 49$, add 49 to each side.
$(a - 7)^2 = 36$	Factor $a^2 - 14a + 49$.
$a - 7 = \pm 6$	Take the square root of each side.
$a = 7 \pm 6$	Add 7 to each side.
$a = 7 + 6$ or $a = 7 - 6$	Separate the solutions.
$= 13$ $= 1$	Simplify.

The solution set is $\{1, 13\}$.

✓CHECK Your Progress

3. Solve $x^2 - 8x = 4$ by completing the square. Round to the nearest tenth if necessary.

To solve a quadratic equation in which the leading coefficient is not 1, first divide each term by the coefficient. Then complete the square.

🌐 Real-World EXAMPLE **Solve a Quadratic Equation in Which $a \neq 1$**

④ **ENTERTAINMENT** The path of debris from fireworks when the wind is about 15 miles per hour can be modeled by the quadratic function $h = -0.04x^2 + 2x + 8$, where h is the height and x is the horizontal distance in feet. How far away from the launch site will the debris land?

Explore You know the function that relates the horizontal and vertical distances. You want to know how far away the debris will land.

Plan The debris will hit the ground when $h = 0$. Complete the square to solve $-0.04x^2 + 2x + 8 = 0$.

Solve

$-0.04x^2 + 2x + 8 = 0$	Equation for where debris will land
$\dfrac{-0.04x^2 + 2x + 8}{-0.04} = \dfrac{0}{-0.04}$	Divide each side by –0.04.
$x^2 - 50x - 200 = 0$	Simplify.
$x^2 - 50x - 200 + 200 = 0 + 200$	Add 200 to each side.
$x^2 - 50x = 200$	Simplify.
$x^2 - 50x + 625 = 200 + 625$	Since $\left(\frac{50}{2}\right)^2 = 625$, add 625 to each side.
$x^2 - 50x + 625 = 825$	Simplify.
$(x - 25)^2 = 825$	Factor $x^2 - 50x + 625$.
$x - 25 = \pm\sqrt{825}$	Take the square root of each side.
$x = 25 \pm \sqrt{825}$	Add 25 to each side.

🌐 Real-World Link

One of the exploded fireworks for the Lake Toya Festival in Japan on July 15, 1988, broke a world record. The diameter of the burst was 3937 feet.

Source: *The Guinness Book of Records*

Use a calculator to evaluate each value of x.

$$x = 25 + \sqrt{825} \quad \text{or} \quad x = 25 - \sqrt{825} \qquad \text{Separate the solutions.}$$
$$\approx 53.7 \qquad\qquad\qquad \approx -3.7 \qquad\qquad \text{Evaluate.}$$

Check Since you are looking for a distance, the negative number is not reasonable. The debris will land about 53.7 feet from the launch site.

✓ CHECK Your Progress

4. Solve $3n^2 - 18n = 30$ by completing the square. Round to the nearest tenth if necessary.

🌐 **Personal Tutor at** algebra1.com

✓ CHECK Your Understanding

Example 1
(p. 486)

Solve each equation by taking the square root of each side. Round to the nearest tenth if necessary.

1. $b^2 - 6b + 9 = 25$

2. $m^2 + 14m + 49 = 20$

Example 2
(p. 487)

Find the value of c that makes each trinomial a perfect square.

3. $a^2 - 12a + c$

4. $t^2 + 5t + c$

Example 3
(p. 488)

Solve each equation by completing the square. Round to the nearest tenth if necessary.

5. $c^2 - 6c = 7$

6. $x^2 + 7x = -12$

7. $v^2 + 14v - 9 = 6$

8. $r^2 - 4r = 2$

9. $4a^2 + 9a - 1 = 0$

10. $7 = 2p^2 - 5p + 8$

Example 4
(pp. 488–489)

11. GEOMETRY The area of a square can be doubled by increasing the length by 6 inches and the width by 4 inches. What is the length of the side of the square?

Exercises

HOMEWORK HELP	
For Exercises	**See Examples**
12–15	1
16–19	2
20–27	3
28–33	4

Solve each equation by taking the square root of each side. Round to the nearest tenth if necessary.

12. $b^2 - 4b + 4 = 16$

13. $t^2 + 2t + 1 = 25$

14. $g^2 - 8g + 16 = 2$

15. $w^2 + 16w + 64 = 18$

Find the value of c that makes each trinomial a perfect square.

16. $s^2 - 16s + c$

17. $y^2 - 10y + c$

18. $p^2 - 7p + c$

19. $c + 11k + k^2$

Solve each equation by completing the square. Round to the nearest tenth if necessary.

20. $s^2 - 4s - 12 = 0$

21. $d^2 + 3d - 10 = 0$

22. $y^2 - 19y + 4 = 70$

23. $d^2 + 20d + 11 = 200$

24. $a^2 - 5a = -4$

25. $p^2 - 4p = 21$

26. $x^2 + 4x + 3 = 0$

27. $d^2 - 8d + 7 = 0$

28. $5s^2 - 10s = 23$

29. $9r^2 + 49 = 42r$

30. $4h^2 + 25 = 20h$
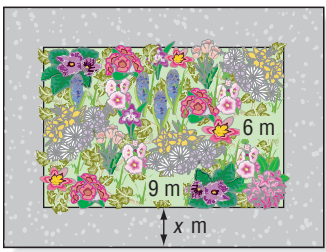
31. $9w^2 - 12w - 1 = 0$

32. PARK PLANNING A rectangular garden of wild flowers is 9 meters long by 6 meters wide. A pathway of constant width goes around the garden. If the area of the path equals the area of the garden, what is the width of the path?

6 m
9 m
x m

33. NUTRITION The consumption of bread and cereal in the United States is increasing and can be modeled by the function $y = 0.059x^2 - 7.423x + 362.1$, where y represents the consumption of bread and cereal in pounds and x represents the number of years since 1900. If this trend continues, in what future year will the average American consume 300 pounds of bread and cereal?

Solve each equation by completing the square. Round to the nearest tenth if necessary.

34. $0.3t^2 + 0.1t = 0.2$

35. $0.4v^2 + 2.5 = 2v$

36. $\frac{1}{2}d^2 - \frac{5}{4}d - 3 = 0$

37. $\frac{1}{3}f^2 - \frac{7}{6}f + \frac{1}{2} = 0$

38. Find all values of c that make $x^2 + cx + 81$ a perfect square.
39. Find all values of c that make $x^2 + cx + 144$ a perfect square.

Solve each equation for x in terms of c by completing the square.

40. $x^2 + 4x + c = 0$

41. $x^2 - 6x + c = 0$

42. PHOTOGRAPHY Emilio is placing a photograph behind a 12-inch-by-12-inch piece of matting. The photograph is to be positioned so that the matting is twice as wide at the top and bottom as it is at the sides. If the area of the photograph is to be 54 square inches, what are the dimensions?

12 in.
2x in.
x in.
x in.
12 in.
2x in.

H.O.T. Problems
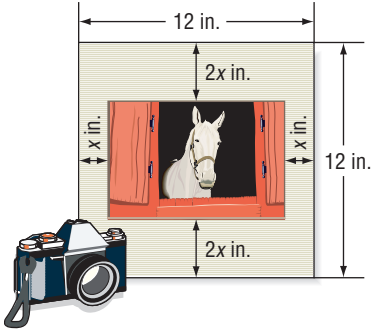
43. OPEN ENDED Make a square using one or more of each of the following types of tiles.

- x^2-tile
- x-tile
- 1-tile

Describe the area of your square using an algebraic expression.

44. REASONING Compare and contrast the following strategies for solving $x^2 - 5x - 7 = 0$: completing the square, graphing the related function, and factoring.

45. CHALLENGE Without graphing, describe the solution of $x^2 + 4x + 12 = 0$. Explain your reasoning. Then describe the graph of the related function.

46. Which One Doesn't Belong? Identify the expression that does not belong with the other three. Explain your reasoning.

| $n^2 - n + \frac{1}{4}$ | $n^2 + n + \frac{1}{4}$ | $n^2 - \frac{2}{3}n + \frac{1}{9}$ | $n^2 + \frac{1}{3}n + \frac{1}{9}$ |

47. *Writing in Math* Use the information about Al-Khwarizmi on page 486 to explain how ancient mathematicians used squares to solve algebraic equations. Include an explanation of Al-Khwarizmi's drawings for $x^2 + 8x = 35$ and a step-by-step algebraic solution with justification for each step of the equation.

48. What are the solutions to the quadratic equation $p^2 - 14p = 32$?

   A 16   **C** $-2, 16$

   B $-3, 14$ **D** $-4, 7$

49. REVIEW If $a = -5$ and $b = 6$, then $3a - 2ab =$

   F -75 **H** 30

   G -55 **J** 45

Spiral Review

Solve each equation by graphing. (Lesson 9-2)

50. $x^2 + 7x + 12 = 0$ **51.** $x^2 - 16 = 0$ **52.** $x^2 - 2x + 6 = 0$

PARKS **For Exercises 53 and 54, use the following information.** (Lesson 9-1)

A city is building a dog park that is rectangular in shape and measures 280 feet around three of the four sides as shown in the diagram.

53. If the width of the park in feet is x, write an equation that models the area A of the park.

54. Analyze the graph of the related function by finding the coordinates of the vertex and describing what this point represents.

Find the GCF for each set of monomials. (Lesson 8-1)

55. $14a^2b^3, 20a^3b^2c, 35ab^3c^2$ **56.** $32m^2n^3, 8m^2n, 56m^3n^2$

Write an inequality for each graph. (Lesson 6-4)

57.

58.
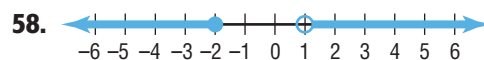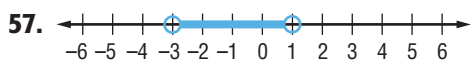
Use substitution to solve each system of equations. If the system does not have exactly one solution, state whether it has no solution or infinitely many solutions. (Lesson 5-2)

59. $y = 2x$
    $x + y = 9$

60. $x = y + 3$
    $2x - 3y = 5$

61. $x - 2y = 3$
    $3x + y = 23$

GET READY for the Next Lesson

PREREQUISITE SKILL Evaluate $\sqrt{b^2 - 4ac}$ for each set of values. Round to the nearest tenth if necessary. (Lesson 1–2)

62. $a = 1, b = -2, c = -15$ **63.** $a = 2, b = 7, c = 3$

64. $a = 1, b = 5, c = -2$ **65.** $a = -2, b = 7, c = 5$

Write the equation of the axis of symmetry, and find the coordinates of the vertex of the graph of each function. Identify the vertex as a maximum or minimum. Then graph the function. (Lesson 9-1)

1. $y = x^2 - x - 6$
2. $y = 2x^2 + 3$
3. $y = -3x^2 - 6x + 5$
4. **MULTIPLE CHOICE** Which graph shows a function $y = x^2 + b$ when $b > 1$? (Lesson 9-1)
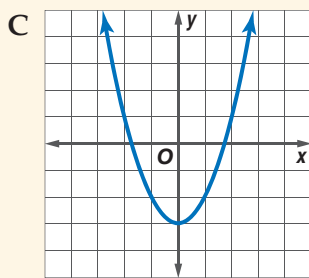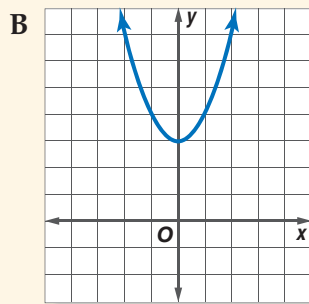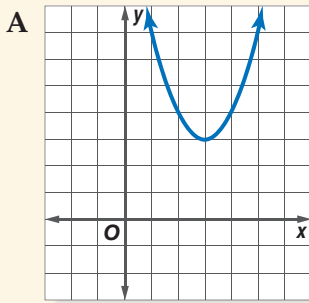
A

B

C

D

Solve each equation by graphing. If integral roots cannot be found, estimate the roots by stating the consecutive integers between which the roots lie. (Lesson 9-2)

5. $x^2 + 6x + 10 = 0$
6. $x^2 - 2x - 1 = 0$
7. $x^2 - 5x - 6 = 0$

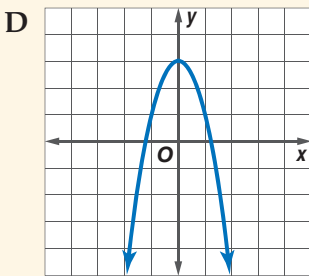8. **SOFTBALL** In a softball game, Lola hit the ball straight up with an initial upward velocity of 47 feet per second. The height h of the softball in feet above ground after t seconds can be modeled by the equation $h = -16t^2 + 47t + 3$. How long was the softball in the air before it hit the ground? (Lesson 9-2)

Solve each equation by completing the square. Round to the nearest tenth if necessary. (Lesson 9-3)

9. $s^2 + 8s = -15$
10. $a^2 - 10a = -24$
11. $y^2 - 14y + 49 = 5$
12. $2b^2 - b - 7 = 14$

13. **ROCKETS** A model rocket is launched from the ground with an initial upward velocity of 475 feet per second. About how many seconds will it take to reach the ground? Use the formula $h = -16t^2 + 175t$, where h is the height of the rocket and t is the time in seconds. Round to the nearest tenth if necessary. (Lesson 9-3)

14. **GEOMETRY** The length and width of the rectangle are increased by the same amount so that the new area is 154 square centimeters. Find the dimensions of the new rectangle. (Lesson 9-3)
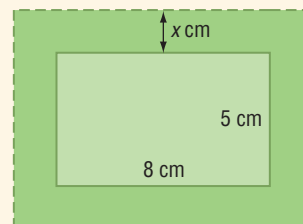
Solving Quadratic Equations by Using the Quadratic Formula

Main Ideas

- Solve quadratic equations by using the Quadratic Formula.
- Use the discriminant to determine the number of solutions for a quadratic equation.

New Vocabulary

Quadratic Formula
discriminant

In the past few decades, there has been a dramatic increase in the percent of people living in the United States who were born in other countries. This trend can be modeled by the quadratic function $P = 0.006t^2 - 0.080t + 5.281$, where P is the percent born outside the United States and t is the number of years since 1960.

Percent Born Outside the U.S.
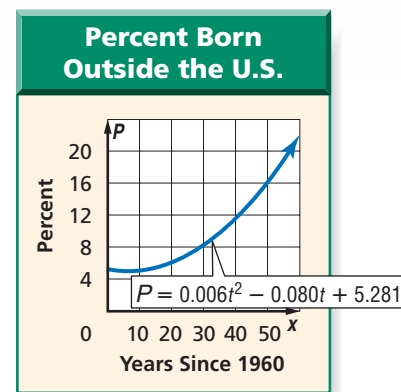
To predict when 15% of the population will be people who were born outside of the U.S., you can solve the equation $15 = 0.006t^2 - 0.080t + 5.281$. This equation would be impossible or difficult to solve using factoring, graphing, or completing the square.

Quadratic Formula You can solve the standard form of the quadratic equation $ax^2 + bx + c = 0$ for x. The result is the **Quadratic Formula.**

KEY CONCEPT   *The Quadratic Formula*

The solutions of a quadratic equation in the form $ax^2 + bx + c = 0$, where $a \neq 0$, are given by the Quadratic Formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

You can solve quadratic equations by factoring, graphing, completing the square, or using the Quadratic Formula.

EXAMPLE   Solve Quadratic Equations

1 **Solve each equation. Round to the nearest tenth if necessary.**

a. $x^2 - 2x - 24 = 0$

Method 1   Factoring

$x^2 - 2x - 24 \ = 0$ Original equation

$(x + 4)(x - 6) = 0$ Factor $x^2 - 2x - 24$.

$x + 4 = 0$ or $x - 6 = 0$ Zero Product Property

$\qquad x = -4 \qquad\qquad x = 6$ Solve for x.

(continued on the next page)

Study Tip

Quadratic Formula

The Quadratic Formula is proved in Lesson 10–1.

Method 2 Quadratic Formula

For this equation, $a = 1$, $b = -2$, and $c = -24$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad \text{Quadratic Formula}$$

$$= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-24)}}{2(1)} \qquad a = 1, b = -2, \text{ and } c = -24$$

$$= \frac{2 \pm \sqrt{4 + 96}}{2} \qquad \text{Multiply.}$$

$$= \frac{2 \pm \sqrt{100}}{2} \quad \text{or} \quad \frac{2 \pm 10}{2} \qquad \text{Add and simplify.}$$

$$x = \frac{2 - 10}{2} \quad \text{or} \quad x = \frac{2 + 10}{2} \qquad \text{Separate the solutions.}$$

$$= -4 \qquad\qquad = 6 \qquad \text{Simplify.}$$

The solution set is $\{-4, 6\}$.

b. $24x^2 - 14x = 6$

Step 1 Rewrite the equation in standard form.

$$24x^2 - 14x = 6 \qquad \text{Original equation}$$

$$24x^2 - 14x - 6 = 0 \qquad \text{Subtract 6 from each side.}$$

Step 2 Apply the Quadratic Formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad \text{Quadratic Formula}$$

$$= \frac{-(-14) \pm \sqrt{(-14)^2 - 4(24)(-6)}}{2(24)} \qquad a = 24, b = -14, \text{ and } c = -6$$

$$= \frac{14 \pm \sqrt{196 + 576}}{48} \qquad \text{Multiply.}$$

$$= \frac{14 \pm \sqrt{772}}{48} \qquad \text{Add.}$$

$$x = \frac{14 - \sqrt{772}}{48} \quad \text{or} \quad x = \frac{14 + \sqrt{772}}{48} \qquad \text{Separate the solutions.}$$

$$\approx -0.3 \qquad\qquad \approx 0.9 \qquad \text{Simplify.}$$

Check the solutions by using the **CALC** menu on a graphing calculator to determine the zeros of the related quadratic function.
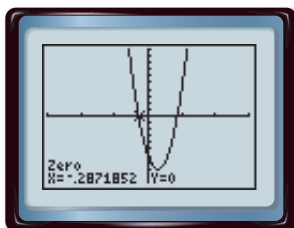
[−3, 3] scl: 1 by [−10. 10] scl: 1
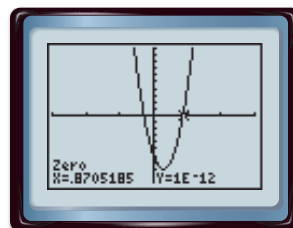
[−3, 3] scl: 1 by [−10. 10] scl: 1

To the nearest tenth, the solution set is $\{-0.3, 0.9\}$.

CHECK Your Progress

1A. $x^2 + 3x - 18 = 0$

1B. $4x^2 + 2x = 17$

The table summarizes the five methods for solving quadratic equations.

CONCEPT SUMMARY — *Solving Quadratic Equations*

Method	Can Be Used	Comments	Lesson(s)
factoring	sometimes	Use if constant term is 0 or factors are easily determined.	8-2 to 8-6
using a table	sometimes	Not always exact; use only when an approximate solution is sufficient.	9-2
graphing	always	Not always exact; use only when an approximate solution is sufficient.	9-2
completing the square	always	Useful for equations of the form $x^2 + bx + c = 0$, where b is an even number.	9-3
Quadratic Formula	always	Other methods may be easier to use in some cases, but this method always gives accurate solutions.	9-4

Real-World EXAMPLE — Use the Quadratic Formula to Solve a Problem

2 **SPACE TRAVEL** The height H of an object t seconds after it is propelled upward with an initial velocity v is represented by $H = -\frac{1}{2}gt^2 + vt + h$, where g is the gravitational pull and h is the initial height. Suppose an astronaut on the Moon throws a baseball upward with an initial velocity of 10 meters per second, letting go of the ball 2 meters above the ground. Use the information at the left to find how much longer the ball will stay in the air than a similarly thrown baseball on Earth.

In order to find when the ball hits the ground, you must find when $H = 0$. Write two equations to represent the situation on the Moon and on Earth.

Baseball Thrown on the Moon

$H = -\frac{1}{2}gt^2 + vt + h$

$0 = -\frac{1}{2}(1.6)t^2 + 10t + 2$

$0 = -0.8t^2 + 10t + 2$

Baseball Thrown on Earth

$H = -\frac{1}{2}gt^2 + vt + h$

$0 = -\frac{1}{2}(9.8)t^2 + 10t + 2$

$0 = -4.9t^2 + 10t + 2$

To find accurate solutions, use the Quadratic Formula.

$t = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{-10 \pm \sqrt{10^2 - 4(-0.8)(2)}}{2(-0.8)}$

$= \dfrac{-10 \pm \sqrt{106.4}}{-1.6}$

$t \approx 12.7 \quad \text{or} \quad t \approx -0.2$

$t = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{-10 \pm \sqrt{10^2 - 4(-4.9)(2)}}{2(-4.9)}$

$= \dfrac{-10 \pm \sqrt{139.2}}{-9.8}$
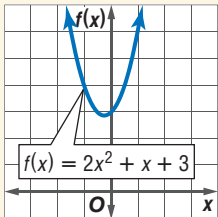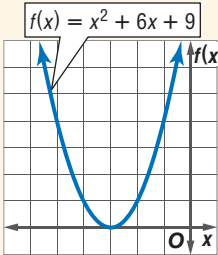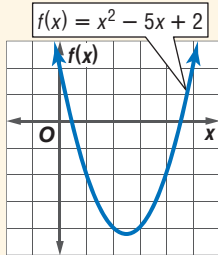
$t \approx 2.2 \quad \text{or} \quad t \approx -0.2$

Since a negative time is not reasonable, use the positive solutions. The ball will stay in the air about $12.7 - 2.2$ or 10.5 seconds longer on the Moon.

CHECK Your Progress

2. GEOMETRY The perimeter of a rectangle is 60 inches. Find the dimensions of the rectangle if its area is 221 square inches.

The Discriminant In the Quadratic Formula, the expression under the radical sign, $b^2 - 4ac$, is called the **discriminant**. The value of the discriminant can be used to determine the number of real roots for a quadratic equation.

KEY CONCEPT			Using the Discriminant
Discriminant	negative	zero	positive
Example	$2x^2 + x + 3 = 0$ $x = \dfrac{-1 \pm \sqrt{1^2 - 4(2)(3)}}{2(2)}$ $= \dfrac{-1 \pm \sqrt{-23}}{4}$ There are no real roots since no real number can be the square root of a negative number.	$x^2 + 6x + 9 = 0$ $x = \dfrac{-6 \pm \sqrt{6^2 - 4(1)(9)}}{2(1)}$ $= \dfrac{-6 \pm \sqrt{0}}{2}$ $= \dfrac{-6}{2}$ or -3 There is a double root, -3.	$x^2 - 5x + 2 = 0$ $x = \dfrac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(2)}}{2(1)}$ $= \dfrac{5 \pm \sqrt{17}}{2}$ There are two roots, $\dfrac{5 + \sqrt{17}}{2}$ and $\dfrac{5 - \sqrt{17}}{2}$.
Graph of Related Function	$f(x) = 2x^2 + x + 3$ The graph does not cross the x-axis.	$f(x) = x^2 + 6x + 9$ The graph touches the x-axis in one place.	$f(x) = x^2 - 5x + 2$ The graph crosses the x-axis twice.
Number of Real Roots	0	1	2

EXAMPLE   Use the Discriminant

3 State the value of the discriminant for each equation. Then determine the number of real roots of the equation.

a. $2x^2 + 10x + 11 = 0$

$b^2 - 4ac = 10^2 - 4(2)(11)$ $a = 2, b = 10$, and $c = 11$

$= 12$ Simplify.

Since the discriminant is positive, the equation has two real roots.

b. $3m^2 + 4m = -2$

Step 1 Rewrite the equation in standard form.

$3m^2 + 4m = -2$ Original equation

$3m^2 + 4m + 2 = -2 + 2$ Add 2 to each side.

$3m^2 + 4m + 2 = 0$ Simplify.

Step 2 Find the discriminant.

$b^2 - 4ac = 4^2 - 4(3)(2)$ $a = 3, b = 4$, and $c = 2$

$= -8$ Simplify.

Since the discriminant is negative, the equation has no real roots.

3A. $4n^2 - 20n + 25 = 0$ **3B.** $5x^2 - 3x + 8 = 0$ **3C.** $2x^2 + 11x + 15 = 0$

CHECK Your Understanding

Example 1
(pp. 493–494)

Solve each equation by using the Quadratic Formula. Round to the nearest tenth if necessary.

1. $x^2 + 7x + 6 = 0$ **2.** $t^2 + 11t = 12$

3. $r^2 + 10r + 12 = 0$ **4.** $3v^2 + 5v + 11 = 0$

Example 2
(p. 495)

5. MANUFACTURING A pan is to be formed by cutting 2-centimeter-by-2-centimeter squares from each corner of a square piece of sheet metal and then folding the sides. If the volume of the pan is to be 441 square centimeters, what should the dimensions of the original piece of sheet metal be?
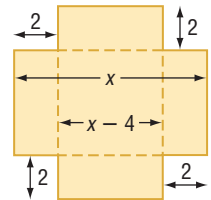
Example 3
(p. 496)

State the value of the discriminant for each equation. Then determine the number of real roots of the equation.

6. $m^2 + 5m - 6 = 0$ **7.** $s^2 + 8s + 16 = 0$ **8.** $2z^2 + z = -50$

Exercises

HOMEWORK HELP	
For Exercises	See Examples
9–20	1, 2
21, 22	3
23–28	4

Solve each equation by using the Quadratic Formula. Round to the nearest tenth if necessary.

9. $v^2 + 12v + 20 = 0$ **10.** $3t^2 - 7t - 20 = 0$ **11.** $5y^2 - y - 4 = 0$

12. $x^2 - 25 = 0$ **13.** $r^2 + 25 = 0$ **14.** $2x^2 + 98 = 28x$

15. $4s^2 + 100 = 40s$ **16.** $2r^2 + r - 14 = 0$ **17.** $2n^2 - 7n - 3 = 0$

18. $5v^2 - 7v = 1$ **19.** $11z^2 - z = 3$ **20.** $2w^2 = -(7w + 3)$

21. GEOMETRY What are the dimensions of rectangle *ABCD*?

Rectangle *ABCD*	
perimeter	42 cm
area	80 cm²

22. PHYSICAL SCIENCE A projectile is shot vertically up in the air from ground level. Its distance *s*, in feet, after *t* seconds is given by $s = 96t - 16t^2$. Find the values of *t* when *s* is 96 feet.

State the value of the discriminant for each equation. Then determine the number of real roots of the equation.

23. $x^2 + 3x - 4 = 0$ **24.** $y^2 + 3y + 1 = 0$ **25.** $4p^2 + 10p = -6.25$

26. $1.5m^2 + m = -3.5$ **27.** $2r^2 = \frac{1}{2}r - \frac{2}{3}$ **28.** $\frac{4}{3}n^2 + 4n = -3$

Solve each equation by using the Quadratic Formula. Round to the nearest tenth if necessary.

29. $1.34d^2 - 1.1d = -1.02$ **30.** $-2x^2 + 0.7x = -0.3$

31. $2y^2 - \frac{5}{4}y = \frac{1}{2}$ **32.** $w^2 + \frac{2}{25} = \frac{3}{5}w$

Without graphing, determine the x-intercepts of the graph of each function.

33. $f(x) = 4x^2 - 9x + 4$

34. $f(x) = 13x^2 - 16x - 4$

Without graphing, determine the number of x-intercepts of the graph of each function.

35. $f(x) = 7x^2 - 3x - 1$

36. $f(x) = x^2 + 4x + 7$

RECREATION For Exercises 37 and 38, use the following information.
As Darius is skiing down a ski slope, Jorge is on the chairlift on the same slope. The chair lift has stopped. Darius stops directly below Jorge and attempts to toss a disposable camera up to him. If the camera is thrown with an initial velocity of 35 feet per second, the equation for the height of the camera is $h = -16t^2 + 35t + 5$, where h represents the height in feet and t represents the time in seconds.
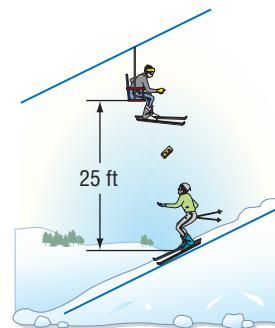
37. If the chairlift is 25 feet above the ground, will Jorge have 0, 1, or 2 chances to catch the camera?

38. If Jorge is unable to catch the camera, when will it hit the ground?

39. AMUSEMENT PARKS The Demon Drop ride at Cedar Point takes riders to the top of a tower and drops them 60 feet at speeds reaching 80 feet per second. A function that models this ride is $h = -16t^2 + 64t - 60$, where h is the height in feet and t is the time in seconds. About how many seconds does it take for riders to drop from 60 feet to 0 feet?

40. REASONING Use the Quadratic Formula to show that $f(x) = 3x^2 - 2x - 4$ has two real roots.

41. FIND THE ERROR Lakeisha and Juanita are determining the number of solutions of $5y^2 - 3y = 2$. Who is correct? Explain your reasoning.

Lakeisha
$5y^2 - 3y = 2$
$b^2 - 4ac = (-3)^2 - 4(5)(2)$
$= -31$
Since the discriminant is negative, there are no real solutions.

Juanita
$5y^2 - 3y = 2$
$5y^2 - 3y - 2 = 0$
$b^2 - 4ac = (-3)^2 - 4(5)(-2)$
$= 49$
Since the discriminant is positive, there are two real roots.

42. OPEN ENDED Write a quadratic equation with no real solutions. Explain how you know there are no solutions.

43. REASONING Use factoring techniques to determine the number of real roots of the function $f(x) = x^2 - 8x + 16$. Compare this method to using the discriminant.

44. *Writing in Math* Describe three different ways to solve $x^2 - 2x - 15 = 0$. Which method do you prefer and why?

25 ft

45. Which statement *best* describes why there is no real solution to the quadratic equation $y = x^2 - 6x + 13$?

 A The value of $(-6)^2 - 4 \cdot 1 \cdot 13$ is a perfect square.

 B The value of $(-6)^2 - 4 \cdot 1 \cdot 13$ is equal to zero.

 C The value of $(-6)^2 - 4 \cdot 1 \cdot 13$ is negative.

 D The value of $(-6)^2 - 4 \cdot 1 \cdot 13$ is positive.

46. REVIEW In the system of equations $6x - 3y = 12$ and $2x + 5y = 9$, which expression can be correctly substituted for y in the equation $2x + 5y = 9$?

 F $12 + 2x$

 G $12 - 2x$

 H $-4 + 2x$

 J $4 - 2x$

Spiral Review

Solve each equation by completing the square. Round to the nearest tenth if necessary. (Lesson 9-3)

47. $x^2 - 8x = -7$

48. $a^2 + 2a + 5 = 20$

49. $n^2 - 12n = 5$

Solve each equation by graphing. If integral roots cannot be found, estimate the roots by stating the consecutive integers between which the roots lie. (Lesson 9-2)

50. $x^2 - x = 6$

51. $2x^2 + x = 2$

52. $-x^2 + 3x + 6 = 0$

53. GEOMETRY The triangle has an area of 96 square centimeters. Find the base b of the triangle. (Lesson 8-3)

$(2b - 8)$ cm
b cm
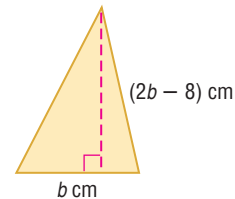
Factor each polynomial. (Lesson 8-2)

54. $24r + 6s$

55. $15xy^3 + y^4$

56. $2ax + 6xc + ba + 3bc$

Solve each inequality. Then check your solution. (Lesson 6-3)

57. $2m + 7 > 17$

58. $-2 - 3x \geq 2$

59. $-20 \geq 8 + 7k$

Write an equation of the line that passes through each point with the given slope. (Lesson 4-4)

60. $(2, 13)$, $m = 4$

61. $(-2, -7)$, $m = 0$

62. $(-4, 6)$, $m = \dfrac{3}{2}$

GET READY for the Next Lesson

PREREQUISITE SKILL Evaluate $c(a^x)$ for each of the given values. (Lesson 1-1)

63. $a = 2, c = 1, x = 4$

64. $a = 7, c = 3, x = 2$

65. $a = 5, c = 2, x = 3$

**Algebra Lab
Applying Quadratic Equations**

Many of the real-world problems you solved in Chapters 8 and 9 were physical problems involving the path of an object that is influenced by gravity. These paths, called **trajectories,** can be modeled by a quadratic function. The formula relating the height of the object $H(t)$ and time t is shown below.

$$H(t) = -\tfrac{1}{2}gt^2 + vt + h$$

Initial height of the object

Height of the object

Acceleration due to gravity

Initial velocity of the object
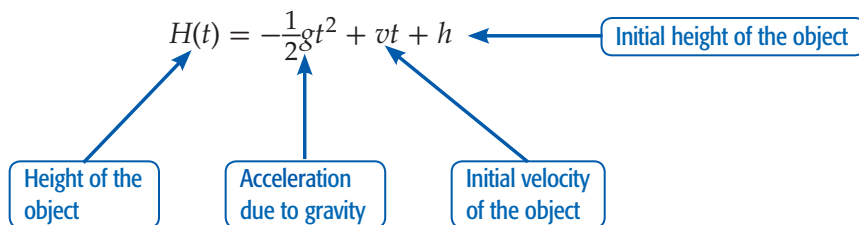
The acceleration due to gravity is 9.8 meters per second squared or 32 feet per second squared.

EXAMPLE 1

Juan kicks a football at a velocity of 25 meters per second. If the ball makes contact with his foot 0.5 meter off the ground, how long will the ball stay in the air?

We want to find the time t when $H(t)$ is 0. First substitute the known values into the motion formula. Since the known measures are written in terms of meters and meters per second, use 9.8 meters per second squared for the acceleration due to gravity.

$H(t) = -\tfrac{1}{2}gt^2 + vt + h$ Motion Formula

$0 = -\tfrac{1}{2}(9.8)t^2 + 25t + 0.5$ $H(t) = 0, g = 9.8, v = 25, h = 0.5$

$0 = -4.9t^2 + 25t + 0.5$ Simplify.

Use the Quadratic Formula to solve for t.

$t = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ Quadratic Formula

$= \dfrac{-25 \pm \sqrt{25^2 - 4(-4.9)(0.5)}}{2(-4.9)}$ $a = -4.9, b = 25, c = 0.5$

$= \dfrac{-25 \pm \sqrt{634.8}}{-9.8}$ Simplify.

$t \approx -0.02$ or $t \approx 5.12$ Use a calculator.

Since time cannot be a negative value, discard the negative solution. The football will be in the air about 5 seconds.

If an object were projected downward, the initial velocity of the object is negative.

EXAMPLE 2

Katharine is on a bridge 12 feet above a pond. She throws a handful of fish food straight down with a velocity of 8 feet per second. In how many seconds will it reach the surface of the water?

Since the units given are in feet, use $g = 32$ ft/s^2. Katharine throws the food down, so the initial velocity is negative. When the food hits the water, $H(t)$ will be 0 feet.

$H(t) = -\frac{1}{2}gt^2 + vt + h$ **Motion Formula**

$0 = -\frac{1}{2}(32)t^2 - 8t + 12$ **$H(t) = 0$, $g = 32$, $v = -8$, $h = 12$**

$0 = -16t^2 - 8t + 12$ **Simplify.**

$0 = -4t^2 - 2t + 3$ **Divide each side by 4.**

Use the Quadratic Formula to solve for t.

$t = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ **Quadratic Formula**

$t = \dfrac{2 \pm \sqrt{(-2)^2 - 4(-4)(3)}}{2(-4)}$ **$a = -4$, $b = -2$, $c = 3$**

$t = \dfrac{2 \pm \sqrt{52}}{-8}$ **Simplify.**

$t \approx -1.15$ or $t \approx 0.65$ **Use a calculator.**

Discard the negative solution. The fish food will hit the water in 0.65 second.

EXERCISES

1. Darren swings at a golf ball on the ground with a velocity of 10 feet per second. How long was the ball in the air?

2. Amalia hits a volleyball at a velocity of 15 meters per second. If the ball was hit from a height of 1.8 meters, determine the time it takes for the ball to land on the floor. Assume that the ball is not hit by another player.

3. Michael is repairing the roof on a shed. He accidentally dropped a box of nails from a height of 14 feet. How long did it take for the box to land on the ground? Since the box was dropped and not thrown, $v = 0$.

4. Carmen threw a penny into a fountain. She threw it from a height of 1.2 meters and at a velocity of 6 meters per second. How long did it take for the penny to hit the surface of the water?

Exponential Functions

Main Ideas

- Graph exponential functions.
- Identify data that displays exponential behavior.

New Vocabulary

exponential function

GET READY for the Lesson

Earnest "Mooney" Warther was a whittler and a carver. For one of his most unusual carvings, Mooney carved a large pair of pliers in a tree.
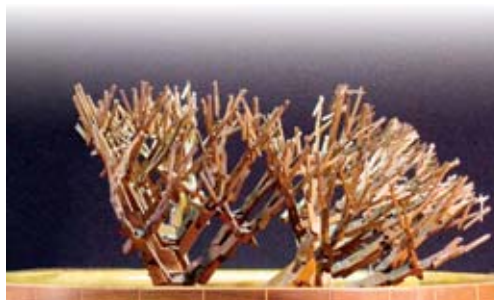
From this original carving, he carved another pair of pliers in each handle of the original. Then he carved another pair of pliers in each of those handles. He continued this pattern to create the original pliers and 8 more layers of pliers. Even more amazing is the fact that all of the pliers work.

Graph Exponential Functions The number of pliers on each level is given in the table below.

Level	Number of Pliers	Power of 2
Original	1	2^0
First	1(2) = 2	2^1
Second	2(2) = 4	2^2
Third	2(2)(2) = 8	2^3
Fourth	2(2)(2)(2) = 16	2^4
Fifth	2(2)(2)(2)(2) = 32	2^5
Sixth	2(2)(2)(2)(2)(2) = 64	2^6
Seventh	2(2)(2)(2)(2)(2)(2) = 128	2^7
Eighth	2(2)(2)(2)(2)(2)(2)(2) = 256	2^8

Study the last column above. Notice that the exponent matches the level. So we can write an equation to describe y, the number of pliers for any given level x as $y = 2^x$. This function is neither linear nor quadratic. It is in the class of functions called **exponential functions** in which the variable is the exponent.

KEY CONCEPT
Exponential Function

An exponential function is a function that can be described by an equation of the form $y = a^x$, where $a > 0$ and $a \neq 1$.

As with other functions, you can use ordered pairs to graph an exponential function.

1 **a. Graph $y = 4^x$. State the y-intercept.**

x	4^x	y
-2	4^{-2}	$\frac{1}{16}$
-1	4^{-1}	$\frac{1}{4}$
0	4^0	1
1	4^1	4
2	4^2	16
3	4^3	64

Study Tip

Exponential Graphs

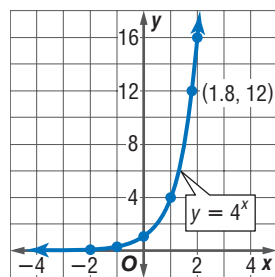Notice that the y-values change little for small values of x, but they increase quickly as the values of x become greater.

Graph the ordered pairs and connect the points with a smooth curve. The y-intercept is 1.

b. Use the graph to determine the approximate value of $4^{1.8}$.

The graph represents all real values of x and their corresponding values of y for $y = 4^x$. So, the value of y is about 12 when $x = 1.8$. Use a calculator to confirm this value. $4^{1.8} \approx 12.12573253$

CHECK Your Progress

1A. Graph $y = 7^x$. State the y-intercept.

1B. Use the graph to determine the approximate value of $y = 7^{0.1}$ to the nearest tenth. Use a calculator to confirm the value.

The graphs of functions of the form $y = a^x$, where $a > 1$, all have the same shape as the graph in Example 1, rising faster and faster as you move from left to right.

2 **a. Graph $y = \left(\frac{1}{2}\right)^x$. State the y-intercept.**

x	$\left(\frac{1}{2}\right)^x$	y
-3	$\left(\frac{1}{2}\right)^{-3}$	8
-2	$\left(\frac{1}{2}\right)^{-2}$	4
-1	$\left(\frac{1}{2}\right)^{-1}$	2
0	$\left(\frac{1}{2}\right)^{0}$	1
1	$\left(\frac{1}{2}\right)^{1}$	$\frac{1}{2}$
2	$\left(\frac{1}{2}\right)^{2}$	$\frac{1}{4}$
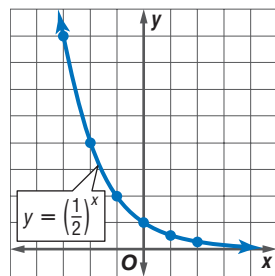
The y-intercept is 1. *Notice that the y-values decrease less rapidly as x increases.*

b. Use the graph to determine the approximate value of $\left(\frac{1}{2}\right)^{-2.5}$.

The value of y is about $5\frac{1}{2}$ when $x = -2.5$. Use a calculator to confirm this value. $\left(\frac{1}{2}\right)^{-2.5} \approx 5.656854249$

**2A.** Graph $y = \left(\frac{1}{3}\right)^x + 2$. State the y-intercept.

2B. Use the graph to determine the approximate value of $y = \left(\frac{1}{3}\right)^{-1.5} + 2$ to the nearest tenth. Use a calculator to confirm the value.

GRAPHING CALCULATOR LAB

Transformations of Exponential Functions

The graphs of $y = 2^x$, $y = 3 \cdot 2^x$, and $y = 0.5 \cdot 2^x$ are shown at the right. Notice that the y-intercept of $y = 2^x$ is 1, the y-intercept of $y = 3 \cdot 2^x$ is 3, and the y-intercept of $y = 0.5 \cdot 2^x$ is 0.5. The graph of $y = 3 \cdot 2^x$ is steeper than the graph of $y = 2^x$. The graph of $y = 0.5 \cdot 2^x$ is not as steep as the graph of $y = 2^x$.
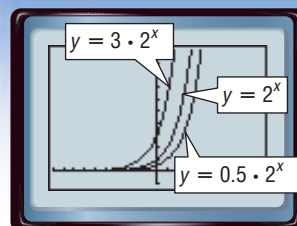
$y = 3 \cdot 2^x$
$y = 2^x$
$y = 0.5 \cdot 2^x$

[−10, 10] scl: 1 by [−1, 10] scl: 1

THINK AND DISCUSS

Graph each set of equations on the same screen. Compare and contrast the graphs.

1. $y = 2^x$
$y = 2^x + 3$
$y = 2^x - 4$

2. $y = 2^x$
$y = 2^{x + 5}$
$y = 2^{x - 4}$

3. $y = 2^x$
$y = 3^x$
$y = 5^x$

4. $y = 3 \cdot 2^x$
$y = 3(2^x - 1)$
$y = 3(2^x + 1)$

🌐 **Real-World EXAMPLE** **Use Exponential Functions to Solve Problems**

3 MOTION PICTURES Movie ticket sales decrease each weekend after an opening. The function $E = 49.9 \cdot 0.692^w$ models the earnings of a popular movie. In this equation, E represents earnings in millions of dollars and w represents the weekend number.

a. Graph the function. What values of E and w are meaningful in the context of the problem?

Use a graphing calculator to graph the function. Only values where $E \leq 49.9$ and $w > 0$ are meaningful in the context of the problem.
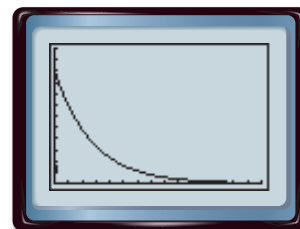
[0, 15] scl: 1 by [0, 60] scl: 5

b. How much did the movie make on the first weekend?

$E = 49.9 \cdot 0.692^w$ Original equation

$= 49.9 \cdot 0.692^1$ $w = 1$

$= 34.5308$ Use a calculator.

On the first weekend, the movie grossed about $34.53 million.

Real-World Link

The first successful photographs of motion were made in 1877. Today, the motion picture industry is big business, with the highest-grossing movie making $1,835,300,000.

Source: imdb.com

c. How much did it make on the fifth weekend?

$$E = 49.9 \cdot 0.692^w \qquad \text{Original equation}$$
$$= 49.9 \cdot 0.692^5 \qquad w = 5$$
$$\approx 7.918282973 \qquad \text{Use a calculator.}$$

On the fifth weekend, the movie grossed about $7.92 million.

✓ **CHECK Your Progress**

3. BIOLOGY A certain bacteria doubles every 20 minutes. How many will there be after 2 hours?

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Identify Exponential Behavior How do you know if a set of data is exponential? One method is to observe the shape of the graph. Another way is to use the problem-solving strategy *look for a pattern*.

EXAMPLE **Identify Exponential Behavior**

④ **Determine whether the set of data at the right displays exponential behavior. Explain why or why not.**

x	0	10	20	30	40	50
y	80	40	20	10	5	2.5

Method 1 Look for a Pattern

The domain values are at regular intervals of 10. Look for a common factor among the range values.

$$80 \quad 40 \quad 20 \quad 10 \quad 5 \quad 2.5$$
$$\times\tfrac{1}{2} \ \times\tfrac{1}{2} \ \times\tfrac{1}{2} \ \times\tfrac{1}{2} \ \times\tfrac{1}{2}$$

Since the domain values are at regular intervals and the range values differ by a common factor, the data are probably exponential. Its equation may involve $\left(\tfrac{1}{2}\right)^x$.

Method 2 Graph the Data
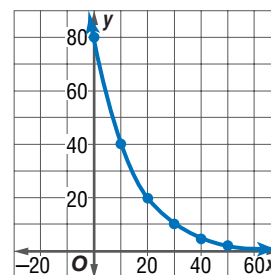
The graph shows a rapidly decreasing value of y as x increases. This is a characteristic of exponential behavior.

✓ **CHECK Your Progress**

4. Determine whether the set of data displays exponential behavior. Explain why or why not.

x	0	10	20	30	40	50
y	15	21	27	33	39	45

✓ **CHECK Your Understanding**

Examples 1, 2
(pp. 503–504)

Graph each function. State the y-intercept. Then use the graph to determine the approximate value of the given expression. Use a calculator to confirm the value.

1. $y = 3^x; 3^{1.2}$

2. $y = \left(\tfrac{1}{4}\right)^x; \left(\tfrac{1}{4}\right)^{1.7}$

3. $y = 9^x; 9^{0.8}$

Graph each function. State the y-intercept.

4. $y = 2 \cdot 3^x$

5. $y = 4(5^x - 10)$

Lesson 9-5 Exponential Functions **505**

6. BIOLOGY The function $f(t) = 100 \cdot 1.05^t$ models the growth of a fruit fly population, where $f(t)$ is the number of flies and t is time in days.

  a. What values for the domain and range are reasonable in the context of this situation? Explain.

  b. After two weeks, approximately how many flies are in this population?

Determine whether the data in each table display exponential behavior. Explain why or why not.

7.

x	0	1	2	3	4	5
y	1	6	36	216	1296	7776

8.

x	4	6	8	10	12	14
y	5	9	13	17	21	25

Exercises

Graph each function. State the y-intercept. Then use the graph to determine the approximate value of the given expression. Use a calculator to confirm the value.

9. $y = 5^x; 5^{1.1}$

10. $y = 10^x; 10^{0.3}$

11. $y = \left(\frac{1}{10}\right)^x; \left(\frac{1}{10}\right)^{-1.3}$

12. $y = \left(\frac{1}{5}\right)^x; \left(\frac{1}{5}\right)^{0.5}$

13. $y = 6^x; 6^{0.3}$

14. $y = 8^x; 8^{0.8}$

Graph each function. State the y-intercept.

15. $y = 5(2^x)$

16. $y = 3(5^x)$

17. $y = 3^x - 7$

18. $y = 2^x + 4$

BIOLOGY For Exercises 19 and 20, use the following information.

A population of bacteria in a culture increases according to the model $p = 300 \cdot 2.7^{0.02t}$, where t is the number of hours and $t = 0$ corresponds to 9:00 A.M.

19. Use this model to estimate the number of bacteria at 11 A.M.

20. Graph the function and name the y-intercept. Describe what the y-intercept represents and describe a reasonable domain and range for this situation.

BUSINESS For Exercises 21 and 22, use the following information.

The amount of money spent at West Outlet Mall in Midtown continues to increase. The total $T(x)$ in millions of dollars can be estimated by the function $T(x) = 12(1.12)^x$, where x is the number of years after it opened in 1995.

21. According to the function, find the amount of sales for the mall in the years 2005, 2006, and 2007.

22. Graph the function and name the y-intercept. What does the y-intercept represent in this problem?

Determine whether the data in each table display exponential behavior. Explain why or why not.

23.

x	−2	−1	0	1
y	−5	−2	1	4

24.

x	0	1	2	3
y	1	0.5	0.25	0.125

25.

x	10	20	30	40
y	16	12	9	6.75

26.

x	−1	0	1	2
y	−0.5	1.0	−2.0	4.0

Graph each function. State the y-intercept.

27. $y = 2(3^x) - 1$

28. $y = 2(3^x + 1)$

29. $y = 3(2^x - 5)$

Real-World Link

Identify each function as *linear, quadratic,* or *exponential.*

30. $y = 4^x + 3$

31. $y = 2x(x - 1)$

32. $5x + y = 8$

33.

34.

35.
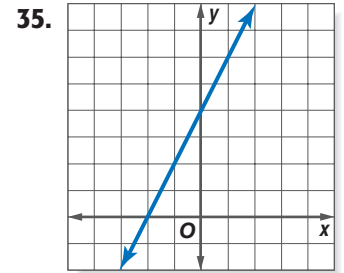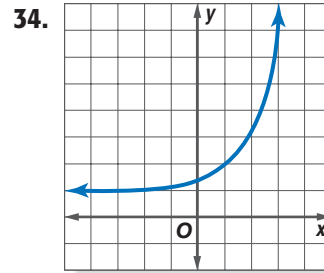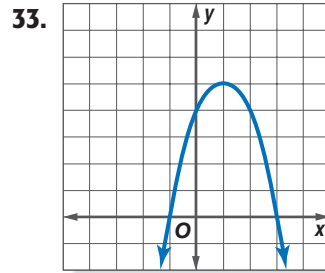
TOURNAMENTS For Exercises 36–38, use the following information.
In a quiz bowl competition, three schools compete, and the winner advances to the next round. Therefore, after each round, only $\frac{1}{3}$ of the schools remain in the competition for the next round. Suppose 729 schools start the competition.

36. Write an exponential function to describe the number of schools remaining after x rounds.

37. How many schools are left after 3 rounds?

38. How many rounds will it take to declare a champion?

ANALYZE TABLES For Exercises 39 and 40, use the following information.
A runner is training for a marathon, running a total of 20 miles per week on a regular basis. She plans to increase the distance $D(x)$ in miles according to the function $D(x) = 20(1.1)^x$, where x represents the number of weeks of training.

39. Copy and complete the table showing the number of miles she plans to run.

40. The runner's goal is to work up to 50 miles per week. What is the first week that the total will be 50 miles or more?

Week	Distance (miles)
1	
2	
3	
4	

H.O.T. Problems

41. REASONING Determine whether the graph of $y = a^x$, where $a > 0$ and $a \neq 1$, *sometimes, always,* or *never* has an x-intercept. Explain your reasoning.

42. OPEN ENDED Choose an exponential function that represents a real-world situation and graph the function. Analyze the graph.

43. FIND THE ERROR Amalia and Hannah are graphing $y = \left(\frac{1}{3}\right)^x$. Who is correct? Explain your reasoning.

Amalia

Hannah
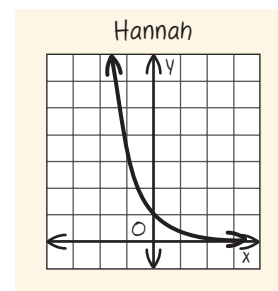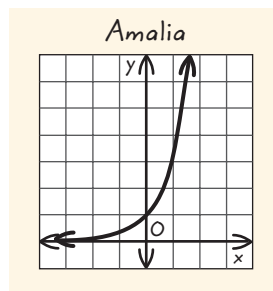
CHALLENGE Describe the graph of each equation as a transformation of the graph of $y = 5^x$.

44. $y = \left(\frac{1}{5}\right)^x$

45. $y = 5^x + 2$

46. $y = 5^x - 4$

47. *Writing in Math* Use the information about the carving on page 502 to explain how exponential functions can be used in art. Include the exponential function representing the pliers, an explanation of which x and y values are meaningful, and the graph of this function.

48. Compare the graphs of $y = 2^x$ and $y = 6^x$.

  A The graph of $y = 6^x$ increases at a faster rate than the graph of $y = 2^x$.

  B The graph of $y = 2^x$ increases at a faster rate than the graph of $y = 6^x$.

  C The graph of $y = 6^x$ is the graph of $y = 2^x$ translated 4 units up.

  D The graph of $y = 6^x$ is the graph of $y = 2^x$ translated 3 units up.

49. REVIEW $\triangle KLM$ is similar to $\triangle HIJ$. Which scale factor is used to transform $\triangle KLM$ to $\triangle HIJ$?
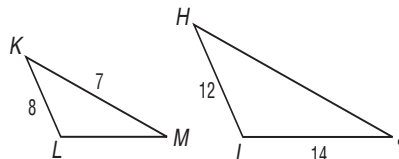

  F $\frac{1}{2}$ **H** $1\frac{1}{2}$

  G 1 **J** 2

Spiral Review

Solve each equation by using the Quadratic Formula. Round to the nearest tenth if necessary. (Lesson 9-4)

50. $x^2 - 9x - 36 = 0$

51. $2t^2 + 3t - 1 = 0$

52. $5y^2 + 3 = y$

Solve each equation by completing the square. Round to the nearest tenth if necessary. (Lesson 9-3)

53. $x^2 - 7x = -10$

54. $a^2 - 12a = 3$

55. $t^2 + 6t + 3 = 0$

Factor each trinomial, if possible. If the trinomial cannot be factored using integers, write *prime*. (Lesson 8-3)

56. $m^2 - 14m + 40$

57. $t^2 - 2t + 35$

58. $z^2 - 5z - 24$

Solve each inequality. (Lesson 6-1)

59. $x + 7 > 2$

60. $10 \geq x + 8$

61. $y - 7 < -12$

62. NUMBER THEORY Three times one number equals twice a second number. Twice the first number is 3 more than the second number. Find the numbers. (Lesson 5-4)

GET READY for the Next Lesson

PREREQUISITE SKILL Evaluate $p(1 + r)^t$ for each of the given values. (Lesson 1-1)

63. $p = 5, r = \frac{1}{2}, t = 2$

64. $p = 300, r = \frac{1}{4}, t = 3$

65. $p = 100, r = 0.2, t = 2$

66. $p = 6, r = 0.5, t = 3$

Algebra Lab

Investigating Exponential Functions

ACTIVITY

Step 1 Cut a sheet of notebook paper in half.

Step 2 Stack the two halves, one on top of the other.

Step 3 Make a table like the one at the right and record the number of sheets of paper you have in the stack after one cut.

Number of Cuts	Number of Sheets
0	1
1	
2	

Step 4 Cut the two stacked sheets in half, placing the resulting pieces in a single stack. Record the number of sheets of paper in the new stack after 2 cuts.

Step 5 Continue cutting the stack in half, each time putting the resulting piles in a single stack and recording the number of sheets in the stack. Stop when the resulting stack is too thick to cut.

ANALYZE THE RESULTS

1. Write a list of ordered pairs (x, y), where x is the number of cuts and y is the number of sheets in the stack. Notice that the list starts with the ordered pair $(0, 1)$, which represents the single sheet of paper before any cuts were made.

2. Continue the list beyond the point where you stopped cutting, until you reach the ordered pair for 7 cuts. Explain how you calculated the last y values for your list after you had stopped cutting.

3. Plot the ordered pairs in your list on a coordinate grid. Be sure to choose a scale for the y-axis so that you can plot all of the points.

4. Write a function that expresses y as a function of x.

5. Evaluate the function you wrote in Exercise 4 for $x = 8$ and $x = 9$. Does it give the correct number of sheets in the stack after 8 and 9 cuts?

6. Notebook paper usually stacks about 500 sheets to the inch. How thick would your stack of paper be if you had been able to make 9 cuts?

7. Suppose each cut takes about 5 seconds. If you had been able to keep cutting, you would have made 36 cuts in three minutes. At 500 sheets to the inch, make a conjecture as to how thick you think the stack would be after 36 cuts.

8. Calculate the thickness of your stack after 36 cuts. Write your answer in miles.

9. Use the results of the Activity to complete a table like the one at the right for 0 to 7 cuts. Then write a function to describe the area y after x cuts.

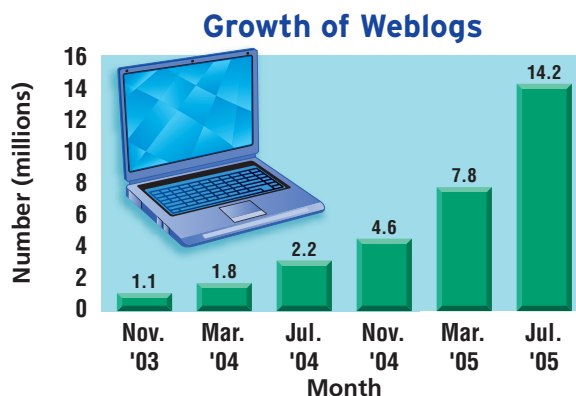| Number of Cuts	Area of Sheet
0	1
1	0.5
2	
...	

Growth and Decay

New Vocabulary

exponential growth
compound interest
exponential decay

GET READY for the Lesson

The number of Weblogs or "blogs" increased at a monthly rate of about 13.7% between November 2003 and July 2005. Let y represent the total number of blogs in millions, and let t represent the number of months since November 2003. Then the average number per month can be modeled by $y = 1.1(1 + 0.137)^t$ or $y = 1.1(1.137)^t$.

Growth of Weblogs

Number (millions)

| Nov. '03 | Mar. '04 | Jul. '04 | Nov. '04 | Mar. '05 | Jul. '05 |
| 1.1 | 1.8 | 2.2 | 4.6 | 7.8 | 14.2 |

Month

Source: Technoration

Exponential Growth The equation for the number of blogs is in the form $y = C(1 + r)^t$. This is the general equation for **exponential growth** in which the initial amount C increases by the same percent over a given period of time.

KEY CONCEPT   *General Equation for Exponential Growth*

The general equation for exponential growth is $y = C(1 + r)^t$ where y represents the final amount, C represents the initial amount, r represents the rate of change expressed as a decimal, and t represents time.

Real-World EXAMPLE   **Exponential Growth**

1 **SPORTS** In 1971, there were 294,105 females in high school sports. Since then, the number has increased an average of 8.5% per year.

a. **Write an equation to represent the number of females participating in high school sports since 1971.**

$$y = C(1 + r)^t \qquad \text{General equation for exponential growth}$$
$$= 294{,}105(1 + 0.085)^t \qquad C = 294{,}105 \text{ and } r = 8.5\% \text{ or } 0.085$$
$$= 294{,}105(1.085)^t \qquad \text{Simplify.}$$

An equation to represent the number of females participating in high school sports is $y = 294{,}105(1.085)^t$, where y is the number of female athletes and t is the number of years since 1971.

b. **According to the equation, how many females participated in high school sports in 2005?**

$$y = 294{,}105(1.085)^t \quad \text{Equation for females participating in sports}$$
$$= 294{,}105(1.085)^{34} \quad t = 2005 - 1971 \text{ or } 34$$
$$\approx 4{,}711{,}004 \quad \text{Use a calculator.}$$

In 2005, about 4,711,004 females participated.

CHECK Your Progress

TECHNOLOGY Computer use has risen 19% annually since 1980.

1A. If 18.9 million computers were in use in 1980, write an equation for the number of computers in use for t years after 1980.

1B. Predict the number of computers in 2015.

Online Personal Tutor at algebra1.com

One special application of exponential growth is **compound interest.** The equation for compound interest is $A = P\left(1 + \frac{r}{n}\right)^{nt}$, where A is the current amount of the investment, P is the principal (initial amount of the investment), r represents the annual rate of interest expressed as a decimal, n represents the number of times the interest is compounded each year, and t represents the number of years that the money is invested.

Real-World EXAMPLE **Compound Interest**

2 **COLLEGE** Maria's parents invested $14,000 at 6% per year compounded monthly. How much money will there be in 10 years?

$$A = P\left(1 + \frac{r}{n}\right)^{nt} \quad \text{Compound interest equation}$$
$$= 14{,}000\left(1 + \frac{0.06}{12}\right)^{12(10)} \quad P = 14{,}000, r = 6\% \text{ or } 0.06, n = 12, \text{ and } t = 10$$
$$= 14{,}000(1.005)^{120} \quad \text{Simplify.}$$
$$\approx 25{,}471.55 \quad \text{Use a calculator.}$$

There will be about $25,471.55.

CHECK Your Progress

2. **MONEY** Determine the amount of an investment if $300 is invested at an interest rate of 3.5% compounded monthly for 22 years.

Exponential Decay A variation of the growth equation can be used as the general equation for exponential decay. In **exponential decay,** the original amount decreases by the same percent over a period of time.

KEY CONCEPT *General Equation for Exponential Decay*

The general equation for exponential decay is $y = C(1 - r)^t$, where y represents the final amount, C represents the initial amount, r represents the rate of decay expressed as a decimal, and t represents time.

3 **SWIMMING** A fully inflated raft loses 6.6% of its air every day. The raft originally contains 4500 cubic inches of air.

a. **Write an equation to represent the loss of air.**

$y = C(1 - r)^t$ General equation for exponential decay

$\quad = 4500(1 - 0.066)^t$ $C = 4500$ and $r = 6.6\%$ or 0.066

$\quad = 4500(0.934)^t$ Simplify.

An equation to represent the loss of air is $y = 4500(0.934)^t$, where y represents the amount of air in the raft in cubic inches and t represents the number of days.

b. **Estimate the amount of air that will be lost after 7 days.**

$y = 4500(0.934)^t$ Equation for air loss

$\quad = 4500(0.934)^7$ $t = 7$

$\quad \approx 2790$ Use a calculator.

The amount of air lost after 7 days will be about 2790 cubic inches.

CHECK Your Progress

POPULATION During the past several years, the population of Campbell County, Kentucky, has been decreasing at an average rate of about 0.3% per year. In 2000, its population was 88,647.

3A. Write an equation to represent the population since 2000.

3B. If the trend continues, predict the population in 2010.

CHECK Your Understanding

Example 1
(pp. 510–511)

ANALYZE GRAPHS For Exercises 1 and 2, use the graph at the right and the following information.
The median house price in the United States increased an average of 8.9% each year between 2002 and 2004. Assume this pattern continues.

1. Write an equation for the median house price for t years after 2004.

2. Predict the median house price in 2009.

Median House Price

2002	$187,600
2003	$195,000
2004	$221,000

Source: RealEstateJournal.com

Example 2
(p. 511)

3. **INVESTMENTS** Determine the amount of an investment if $400 is invested at an interest rate of 7.25% compounded quarterly for 7 years.

Example 3
(p. 512)

4. **POPULATION** In 1995, the population of West Virginia reached 1,821,000, its highest in the 20th century. During the rest of the 20th century, its population decreased 0.2% each year. If this trend continues, predict the population of West Virginia in 2010.

WEIGHT TRAINING For Exercises 5 and 6, use the following information.
In 1997, there were 43.2 million people who used free weights.

5. Assuming the use of free weights increases 6% annually, write an equation for the number of people using free weights t years from 1997.

6. Predict the number of people using free weights in 2007.

7. **POPULATION** The population of Mexico has been increasing 1.7% annually. If the population was 100,350,000 in 2000, predict the population in 2012.

8. **ANALYZE GRAPHS** The increase in the number of visitors to the Grand Canyon National Park is similar to an exponential function. If the average visitation has increased 5.63% annually since 1920, predict the number of visitors to the park in 2020.

9. **INVESTMENTS** Determine the amount of an investment if $500 is invested at an interest rate of 5.75% compounded monthly for 25 years.

10. **INVESTMENTS** Determine the amount of an investment if $250 is invested at an interest rate of 7.3% compounded quarterly for 40 years.

Grand Canyon National Park Annual Visitors

4.8 million

71,601
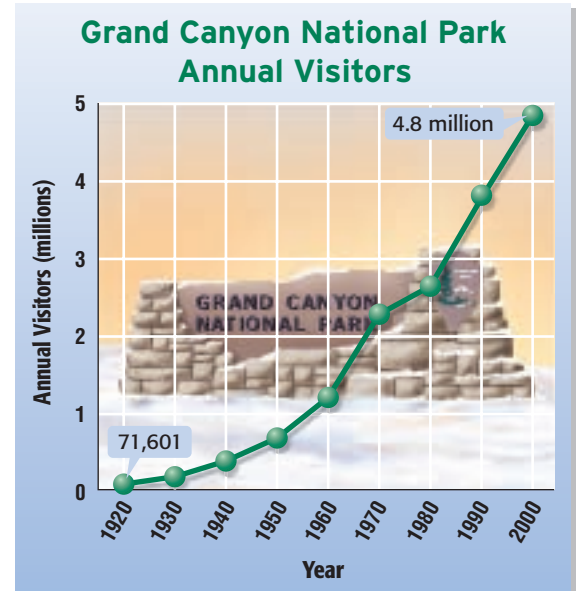
Source: Grand Canyon National Park

11. **POPULATION** The country of Latvia has been experiencing a 1.1% annual decrease in population. In 2005, its population was 2,290,237. If the trend continues, predict Latvia's population in 2015.

12. **MUSIC** In 1994, the sales of music cassettes reached its peak at $2,976,400,000. Since then, cassette sales have been declining. If the annual percent of decrease in sales is 18.6%, predict the sales of cassettes in the year 2009.

ARCHAEOLOGY For Exercises 13–15, use the following information.
The *half-life* of a radioactive element is defined as the time that it takes for one-half a quantity of the element to decay. Radioactive Carbon-14 is found in all living organisms and has a half-life of 5730 years. Archaeologists use this information to estimate the age of fossils. Consider a living organism with an original Carbon-14 content of 256 grams. The number of grams remaining in the fossil of the organism after t years would be $256(0.5)^{\frac{t}{5730}}$.

13. If the organism died 5730 years ago, what is the amount of Carbon-14 today?

14. If an organism died 10,000 years ago, what is the amount of Carbon-14 today?

15. If the fossil has 32 grams of Carbon-14 remaining, how long ago did it live? (*Hint:* Make a table.)

16. RESEARCH Find the enrollment of your school district each year for the last decade. Find the rate of change from one year to the next. Then, determine the average annual rate of change for those years. Use this information to estimate the enrollment for your school district in ten years.

17. OPEN ENDED Create a compound interest problem that could be solved by the equation $A = 500\left(1 + \frac{0.07}{4}\right)^{4(6)}$.

18. *Writing in Math* Use the information about Weblogs on page 510 to explain how exponential growth can be used to predict future blogs. Include an explanation of the equation $y = 1.1(1 + 0.137)^t$ and an estimate of the number of blogs in January 2010.

STANDARDIZED TEST PRACTICE

19. Lorena is investing a $5000 inheritance from her aunt in a certificate of deposit that matures in 4 years. The interest rate is 6.25% compounded quarterly. What is the balance of the account after 4 years?

A $5078.13

B $5319.90

C $5321.82

D $6407.73

20. REVIEW Diego is building a 10-foot ramp for loading heavy equipment into the back of a semi-truck. If the floor of the truck is 3.5 feet off the ground, about how far from the truck should the ramp be?

F 9 ft

G 10 ft

H 10.6 ft

J 11 ft

Spiral Review

Graph each function. State the y-intercept. (Lesson 9-5)

21. $y = \left(\frac{1}{8}\right)^x$

22. $y = 2^x - 5$

23. $y = 4(3^x - 6)$

Solve each equation by using the Quadratic Formula. Round to the nearest tenth if necessary. (Lesson 9-4)

24. $m^2 - 9m - 10 = 0$

25. $2t^2 - 4t = 3$

26. $7x^2 + 3x + 1 = 0$

27. SKIING A course for cross-country skiing is regulated so that the slope of any hill cannot be greater than 0.33. A hill rises 60 meters over a horizontal distance of 250 meters. Does the hill meet the requirements? Explain. (Lesson 4-1)

Cross-Curricular Project

Algebra and Science

Out of This World It is time to complete your project. Use the infomation and data you have gathered about the solar system to prepare a brochure, poster, or Web page. Be sure to include the three graphs, tables, diagrams, or calculations in the presentation.
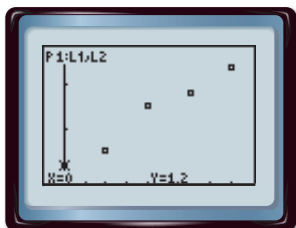
Math Online **Cross-Curricular Project at** algebra1.com
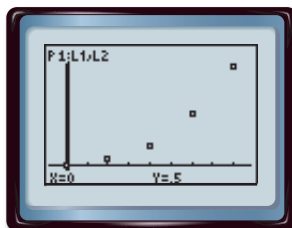
Graphing Calculator Lab
Curve Fitting

If there is a constant increase or decrease in data values, there is a linear trend.
If the values are increasing or decreasing more and more rapidly, there may be
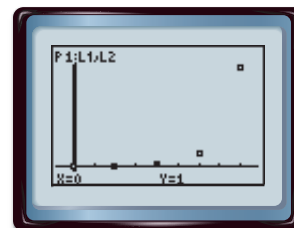a quadratic or exponential trend.

Linear Trend	Quadratic Trend	Exponential Trend

With a TI-83/84 Plus, you can find the appropriate regression equation.

ACTIVITY 1

FARMING A study is conducted in which groups of 25 corn plants are given
a different amount of fertilizer and the gain in height after a certain time
is recorded. The table below shows the results.

Fertilizer (mg)	0	20	40	60	80
Gain in Height (in.)	6.48	7.35	8.73	9.00	8.13

Step 1 Make a scatter plot.

• Enter the fertilizer in L1 and the height
in L2.

KEYSTROKES: *Review entering a list on
page 234.*

• Use **STAT PLOT** to graph the scatter plot.

KEYSTROKES: *Review statistical plots on page 234.
Use* ZOOM *9 to graph.*
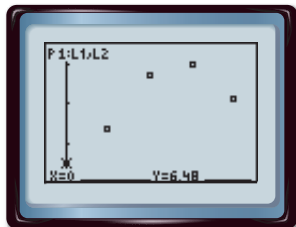
[−8, 88] scl: 5 by [6.0516, 9.4284] scl: 1

The graph appears to be a quadratic
regression.

Step 2 Find the regression equation.

• Select DiagnosticOn from the CATALOG.

• Select QuadReg on the STAT CALC
menu.

KEYSTROKES: STAT ▶ 5 ENTER

The equation is
in the form
$y = ax^2 + bx + c$.

```
QuadReg
 y=ax²+bx+c
 a=-8.196429ᴇ-4
 b=.0903214286
 c=6.292285714
 R²=.9276727663
```
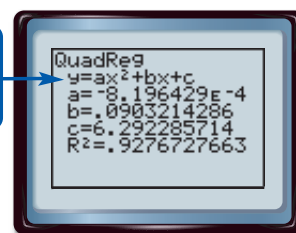
The equation is about
$y = -0.0008x^2 + 0.1x + 6.3$.

R^2 is the **coefficient of determination**. The
closer R^2 is to 1, the better the model. To
choose a quadratic or exponential model,
fit both and use the one with the R^2 value
closer to 1.

Step 3 Graph the regression equation.

• Copy the equation to the Y= list and graph.

KEYSTROKES: [Y=] [VARS] 5 [▶] [▶] 1 [ZOOM] 9

[−8, 88] scl: 5 by [6.0516, 9.4284] scl: 1

Step 4 Predict using the equation.

• Find the amount of fertilizer that produces the maximum gain in height.

KEYSTROKES: [2nd] [CALC] 4

[−8, 88] scl: 5 by [6.0516, 9.4284] scl: 1

According to the graph, on average about 55 milligrams of the fertilizer produces the maximum gain.
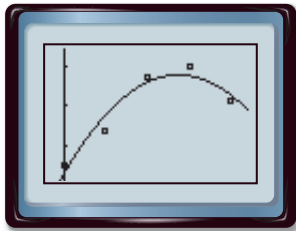
EXERCISES

Plot each set of data points. Determine whether to use a *linear*, *quadratic*, or *exponential* regression equation. State the coefficient of determination.

1.

x	y
0.0	2.98
0.2	1.46
0.4	0.90
0.6	0.51
0.8	0.25
1.0	0.13

2.

x	y
1	25.9
2	22.2
3	20.0
4	19.3
5	18.2
6	15.9

3.

x	y
10	35
20	50
30	70
40	88
50	101
60	120

4.

x	y
1	3.67
3	5.33
5	6.33
7	5.67
9	4.33
11	2.67

TECHNOLOGY **DVD players were introduced in 1997. For Exercises 5–8, use the table at the right.**

5. Make a scatter plot of the data.

6. Find an appropriate regression equation, and state the coefficient of determination.

7. Use the regression equation to predict the number of DVD players that will sell in 2008.

8. Do you believe your equation would be accurate for a year beyond the range of the data, such as 2020? Explain.

Year	DVD Players Sold (millions)
1997	0.32
1998	1.09
1999	4.02
2000	8.50
2001	12.71
2002	17.09
2003	21.99
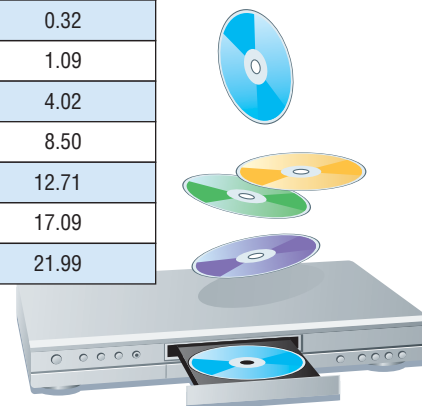
Source: Consumer Electronics Association

CHAPTER 9 Study Guide and Review
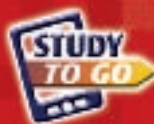
FOLDABLES™ Study Organizer

GET READY to Study

9-1 | 9-2 | 9-3 | 9-4 | 9-5 | 9-6

Be sure the following Key Concepts are noted in your Foldable.
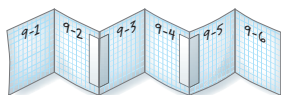
Key Concepts

Graphing Quadratic Functions (Lesson 9-1)

- A quadratic function can be described by an equation of the form $y = ax^2 + bx + c$, where $a \neq 0$.

- The axis of symmetry for the graph of $y = ax^2 + bx + c$, where $a \neq 0$, is $x = -\frac{b}{2a}$.

Solving Quadratic Equations (Lessons 9-2, 9-3, and 9-4)

- The solutions of a quadratic equation are called the roots of the equation. They are the x-intercepts or zeros of the related quadratic function.

- Quadratic equations can be solved by completing the square. To complete the square for $x^2 + bx$, find $\frac{1}{2}$ of b, square this result, and then add the final result to $x^2 + bx$.

- The solutions of a quadratic equation can be found by using the Quadratic Formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Exponential Functions (Lessons 9-5 and 9-6)

- An exponential function can be described by an equation of the form $y = a^x$, where $a > 0$ and $a \neq 1$.

- The general equation for exponential growth is $y = C(1 + r)^t$ and the general equation for exponential decay is $y = C(1 - r)^t$, where $y =$ the final amount, $C =$ the initial amount, $r =$ the rate of change, and $t =$ the time.

Key Vocabulary

axis of symmetry (p. 472)

completing the square (p. 487)

compound interest (p. 511)

discriminant (p. 496)

exponential function (p. 502)

general equation for exponential decay (p. 511)

general equation for exponential growth (p. 510)

maximum (p. 472)

minimum (p. 472)

parabola (p. 471)

quadratic equation (p. 480)

Quadratic Formula (p. 493)

quadratic function (p. 471)

roots (p. 480)

symmetry (p. 472)

vertex (p. 472)

zeros (p. 480)

Vocabulary Check

State whether each sentence is *true* or *false*. If *false*, replace the underlined word or phrase to make a true sentence.

1. The graph of a quadratic function is a <u>parabola</u>.

2. The solutions of a quadratic equation are called <u>roots</u>.

3. The <u>zeros</u> of a quadratic function can be found by using the equation $x = -\frac{b}{2a}$.

4. The <u>vertex</u> is the maximum or minimum point of a parabola.

5. The <u>exponential decay</u> equation is $y = C(1 + r)^t$.

6. An example of a <u>quadratic function</u> is $y = 8^x$.

7. <u>Symmetry</u> is a geometric property possessed by parabolas.

8. The graph of a quadratic function has a <u>minimum</u> if the coefficient of the x^2 term is negative.

9. The expression $b^2 - 4ac$ is called the <u>discriminant</u>.

10. A quadratic equation whose graph has two x-intercepts has <u>no</u> real roots.

Lesson-by-Lesson Review

9-1 **Graphing Quadratic Functions** (pp. 471–477)

Write the equation of the axis of symmetry, and find the coordinates of the vertex of the graph of each function. Identify the vertex as a maximum or minimum. Then graph the function.

11. $y = x^2 + 2x$ **12.** $y = -3x^2 + 4$

13. $y = x^2 - 3x - 4$ **14.** $y = 3x^2 + 6x - 17$

15. $y = -2x^2 + 1$ **16.** $y = -x^2 - 3x$

PHYSICAL SCIENCE For Exercises 17–20, use the following information.

A model rocket is launched with a velocity of 64 feet per second. The equation $h = -16t^2 + 64t$ gives the height of the rocket t seconds after it is launched.

17. Write the equation of the axis of symmetry and find the coordinates of the vertex.

18. Graph the function.

19. What is the maximum height that the rocket reaches?

20. How many seconds is the rocket in the air?

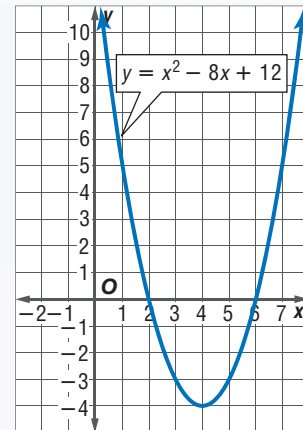**Example 1** Consider the graph of $y = x^2 - 8x + 12$.

a. Write the equation of the axis of symmetry.

$$x = -\frac{b}{2a}$$

$$x = -\frac{-8}{2(1)}$$

$$x = 4$$

The equation of the axis of symmetry is $x = 4$.

b. Find the coordinates of the vertex.

The x-coordinate for vertex is 4.

$y = x^2 - 8x + 12$ Original equation

$= (4)^2 - 8(4) + 12$ $x = 4$

$= 16 - 32 + 12$ Simplify.

$= -4$ Simplify.

The coordinates of the vertex are $(4, -4)$.

9-2 **Solving Quadratic Equations by Graphing** (pp. 480–485)

Solve each equation by graphing. If integral roots cannot be found, estimate the roots by stating the consecutive integers between which the roots lie.

21. $x^2 - x - 12 = 0$ **22.** $-x^2 + 6x - 9 = 0$

23. $x^2 + 4x - 3 = 0$ **24.** $2x^2 - 5x + 4 = 0$

25. $x^2 - 10x = -21$ **26.** $6x^2 - 13x = 15$

27. NUMBER THEORY Use a quadratic equation to find two numbers whose sum is 5 and whose product is -24.

Example 2 Solve $x^2 - 3x - 4 = 0$ by graphing.

Graph the related function $f(x) = x^2 - 3x - 4$.
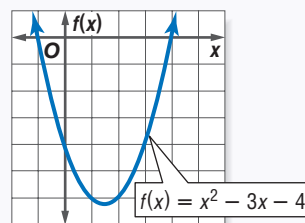
The x-intercepts are -1 and 4. Therefore, the solutions are -1 and 4.

9-3 Solving Quadratic Equations by Completing the Square (pp. 486–491)

Solve each equation by taking the square root of each side. Round to the nearest tenth if necessary.

28. $a^2 + 6a + 9 = 4$

29. $n^2 - 2n + 1 = 25$

Solve each equation by completing the square. Round to the nearest tenth if necessary.

30. $-3x^2 + 4 = 0$

31. $x^2 - 16x + 32 = 0$

32. $m^2 - 7m = 5$

33. GEOMETRY The area of a square can be tripled by increasing the length by 6 centimeters and the width by 3 centimeters. What is the length of the side of the square?

Example 3 Solve $y^2 + 6y + 2 = 0$ by completing the square. Round to the nearest tenth if necessary.

Step 1 Isolate the y^2 and y terms.

$y^2 + 6y + 2 = 0$ Original equation

$\quad y^2 + 6y = -2$ Subtract 2 from each side.

Step 2 Complete the square and solve.

$y^2 + 6y + 9 = -2 + 9$ $\left(\frac{6}{2}\right)^2 = 9$; add 9 to each side.

$\quad (y + 3)^2 = 7$ Factor $y^2 + 6y + 9$.

$\quad y + 3 = \pm\sqrt{7}$ Take the square root of each side.

$\quad y = -3 \pm \sqrt{7}$ Subtract 3 from each side.

The solutions are about -5.6 and -0.4.

9-4 Solving Quadratic Equations by Using the Quadratic Formula (pp. 493–499)

Solve each equation by using the Quadratic Formula. Round to the nearest tenth if necessary.

34. $x^2 - 8x = 20$

35. $r^2 + 10r + 9 = 0$

36. $4p^2 + 4p = 15$

37. $2y^2 + 3 = -8y$

38. $2d^2 + 8d + 3 = 3$

39. $21a^2 + 5a - 7 = 0$

40. ENTERTAINMENT A stunt person attached to a safety harness drops from a height of 210 feet. A function that models the drop is $h = -16t^2 + 210$, where h is the height in feet and t is the time in seconds. About how many seconds does it take to drop from 210 feet to 30 feet?

Example 4 Solve $2x^2 + 7x - 15 = 0$ by using the Quadratic Formula.

For this equation, $a = 2$, $b = 7$, and $c = -15$.

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ Quadratic Formula

$\quad = \dfrac{-7 \pm \sqrt{7^2 - 4(2)(-15)}}{2(2)}$ $a = 2, b = 7,$ and $c = -15$

$\quad = \dfrac{-7 \pm \sqrt{169}}{4}$ Simplify.

$\quad = \dfrac{-7 + 13}{4}$ or $\dfrac{-7 - 13}{4}$ Separate the solutions.

$x = 1.5$ or $x = -5$ Simplify.

The solutions are -5 and 1.5.

9–5 **Exponential Functions** (pp. 502–508)

Graph each function. State the y-intercept. Then use the graph to determine the approximate value of the given expression. Use a calculator to confirm the value.

41. $y = 5^x; 5^{0.7}$

**42.** $y = \left(\frac{1}{6}\right)^x; \left(\frac{1}{6}\right)^{0.2}$

Graph each function. State the y-intercept.

43. $y = 3^x + 6$

44. $y = 3^{x+2}$

45. BIOLOGY The population of bacteria in a Petri dish increases according to the model $p = 550 \cdot 2.7^{0.008t}$, where t is the number of hours and $t = 0$ corresponds to 1:00 P.M. Use this model to estimate the number of bacteria in the dish at 5 P.M.

Example 5 Graph $y = 2^x - 3$. State the y-intercept.

x	$2^x - 3$	y
−3	$2^{-3} - 3$	−2.875
−1	$2^{-1} - 3$	−2.5
0	$2^0 - 3$	−2
1	$2^1 - 3$	−1
2	$2^2 - 3$	1
3	$2^3 - 3$	5
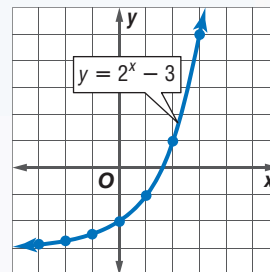
Graph the ordered pairs and connect the points with a smooth curve. The y-intercept is −2.

9–6 **Growth and Decay** (pp. 510–514)

Determine the final amount for each investment.

	Principal	Annual Interest Rate	Time (yr)	Type of Compounding
46.	$2000	3%	8	quarterly
47.	$5500	2.25%	15	monthly
48.	$15,000	2.5%	25	monthly
49.	$500	1.75%	40	daily

RESTAURANTS For Exercises 50 and 51, use the following information.
The total restaurant sales in the United States increased at an annual rate of about 5.2% between 1996 and 2004. In 1996, the total sales were $310 billion.

50. Write an equation for the average sales per year for t years after 1996.

51. Predict the total restaurant sales in 2008.

Example 6 Find the final amount of an investment if $1500 is invested at an interest rate of 2.5% compounded quarterly for 10 years.

$A = p\left(1 + \frac{r}{n}\right)^{nt}$ Compound interest equation

$= 1500\left(1 + \frac{0.025}{4}\right)^{4(10)}$ $P = 1500, r = 2.5\%$ or $0.025, n = 4$, and $t = 10$

≈ 1924.54 Simplify.

The final amount in the account is about $1924.54.

Write the equation of the axis of symmetry, and find the coordinates of the vertex of the graph of each function. Identify the vertex as a maximum or minimum. Then graph the function.

1. $y = x^2 - 4x + 13$ **2.** $y = -3x^2 - 6x + 4$
3. $y = 2x^2 + 3$ **4.** $y = -1(x - 2)^2 + 1$

Solve each equation by graphing. If integral roots cannot be found, estimate the roots by stating the consecutive integers between which the roots lie.

5. $x^2 - 2x + 2 = 0$

6. $x^2 + 6x = -7$

7. $x^2 + 24x + 144 = 0$

8. $2x^2 - 8x = 42$

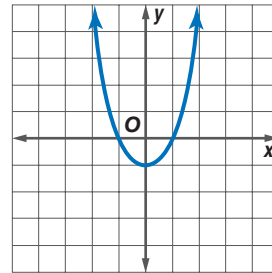**9. MULTIPLE CHOICE** Which function is graphed below?
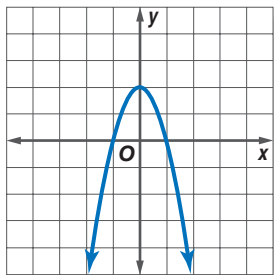
A $y = 2x^2 - 2$

B $y = 2x^2 + 2$

C $y = -2x^2 - 2$

D $y = -2x^2 + 2$

Solve each equation. Round to the nearest tenth if necessary.

10. $x^2 + 7x + 6 = 0$ **11.** $2x^2 - 5x - 12 = 0$

12. $6n^2 + 7n = 20$ **13.** $3k^2 + 2k = 5$

14. $y^2 - \frac{3}{5}y + \frac{2}{25} = 0$ **15.** $-3x^2 + 5 = 14x$

16. $z^2 - 13z = 32$ **17.** $7m^2 = m + 5$

18. MULTIPLE CHOICE Which equation best represents the parabola graphed below if it is shifted 3 units to the right?

F $y = x^2 - 1$

G $y = x^2 + 2$

H $y = x^2 - 6x + 8$

J $y = x^2 + 6x + 8$

Graph each function. State the y-intercept.

19. $y = \left(\frac{1}{2}\right)^x$ **20.** $y = 4 \cdot 2^x$

21. $y = 0.5(4^x)$ **22.** $y = 5^x - 4$

23. Graph $y = \left(\frac{1}{3}\right)^x - 3$ and state the y-intercept. Then use the graph to determine the approximate value of $y = \left(\frac{1}{3}\right)^{3.5} - 3$. Use a calculator to confirm the value.

24. CARS Ley needs to replace her car. If she leases a car, she will pay $410 a month for 2 years and then has the option to buy the car for $14,458. The current price of the car is $17,369. If the car depreciates at 16% per year, how will the depreciated price compare with the buy-out price of the lease?

25. FINANCE Find the total amount of the investment shown in the table if interest is compounded quarterly.
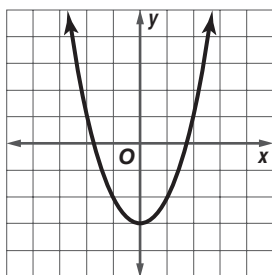
| Principal	$1500
Length of Investment	10 yr
Annual Interest Rate	6%

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. In the graph of the function $f(x) = x^2 - 3$, which describes the shift in the vertex of the parabola if, in the function, 3 is changed to 5?

    A 2 units up

    B 8 units up

    C 2 units down

    D 8 units down

2. The area of a rectangle is given by the equation $3\ell^2 + 10\ell = 25$, in which ℓ is the rectangle's length. What is the length of the rectangle?

$3\ell^2 + 10\ell = 25$

ℓ
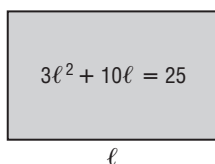
    F $\dfrac{3}{2}$

    G $\dfrac{5}{3}$

    H $\dfrac{10}{3}$

    J 5

3. Solve the equation $x(x + 2) = 224$ which represents two consecutive even integers whose product is 224. What is the smaller number?

    A 2   **C** 16

    B 14   **D** 24

4. Ben calculated the cost $f(x)$ to make and print x T-shirts in one month according to the function

$$f(x) = 9x - 3.00(x - 100),$$

where $x > 100$ T-shirts. The best interpretation for this function is that it costs Ben—

    F $9 for any number of T-shirts.

    G $6 for all shirts up to 100.

    H $3 less per T-shirt for all shirts over 100.

    J $3 for all shirts over 100.

5. What is the solution set of $3y^2 = 12$?

    A $\{3\}$

    B $\{-2, 2\}$

    C $\{-2, 2, 3\}$

    D $\{-2\}$

TEST-TAKING TIP

Question 5 Always write down your calculations on scrap paper or in the test booklet, even if you think you can do the calculations in your head. Writing down your calculations will help you avoid making simple mistakes.

6. Jennie hit a baseball straight up in the air. The height h, in feet, of the ball above the ground is modeled by the equation $h = -16t^2 + 64t$. How long is the ball above ground?

$h = -16t^2 + 64t$
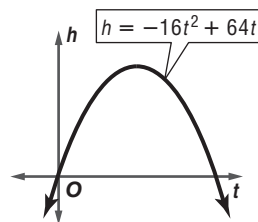
    F   1 second

    G   2 seconds

    H   4 seconds

    J   16 seconds

Standardized Test Practice at algebra1.com

7. Lisa has a savings account. She withdraws
half of the contents every year. After 4 years
she has $2000 left. How much did she have
in the savings account originally?

   A $32,000

   B $16,000

   C $8,000

   D $2,000
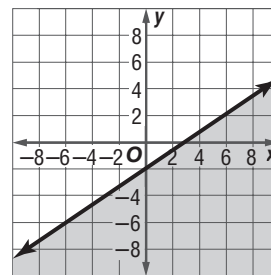
8. When is this statement true?

> The multiplicative inverse of a number is
> less than the original number.

   F This statement is never true.

   G This statement is always true.

   H This statement is true for numbers greater
than 1.

   J This statement is true for numbers less
than −1.

9. GRIDDABLE What is the slope of the line
graphed below?
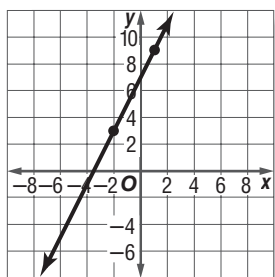
10. Which inequality is shown on the graph
below?

   F $y \le \frac{2}{3}x - 2$

   G $y < \frac{2}{3}x - 2$

   H $y \ge \frac{2}{3}x - 2$

   J $y > \frac{2}{3}x - 2$

11. Mr. Collins made a deck that was 12 feet
long. He is making a new deck that is
25% longer. What will be the length of
the new deck?

   A 9 feet

   B 12 feet

   C 15 feet

   D 18 feet

Pre-AP

**Record your answers on a sheet of paper.
Show your work.**

12. The path of Annika Sorenstam's golf ball
through the air is modeled by the equation
$h = -16t^2 + 80t + 1$, where h is the height in
feet of the golf ball and t is the time in seconds.

   a. Approximately how long was the ball in
the air?

   b. How high was the ball at its highest point?

NEED EXTRA HELP?												
If You Missed Question...	1	2	3	4	5	6	7	8	9	10	11	12
Go to Lesson or Page...	9–1	9–4	8–3	3–2	8–5	8–2	9–6	4-1	1-4	6-6	3–7	9–2